DESIGN OF CONCRETE STRUCTURES

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PREFACE

The intent of this work is to provide a text on concrete and reinforced concrete for both elementary and more advanced courses given in engineering schools. There has been no attempt to produce a handbook or to cover all the phases of concrete construction.

It is the aim of the authors to give sufficient development of the theory of concrete design with illustrative problems to insure the beginner a thorough understanding of the fundamentals. Complete designs of the more common concrete structures are given in order to furnish a vehicle for bringing together all the fundamental theory involved.

Since the last edition, the completion of the tests on columns made by the American Concrete Institute has resulted in a radical change in column design. This revision conforms to the recommendations of the Standard Building Code of the American Concrete Institute in that respect. It also follows the latest recommendations of that code for footing design.

The diagrams for T-beams, beams reinforced for compression and for bending and direct stress, have been entirely revised. All of these diagrams are constructed for values of pn, so that they may be used for concrete of any strength. The latter diagrams include circular sections and combine in one diagram cases of compression over the whole section and tension over part of the section.

A departure from previous editions is the collection of all tables and diagrams into Appendix D, which, it is thought, makes reference easier. Also, at the request of several of the users of the book, additional problems for class assignment are given throughout the elementary portions of the text.

Acknowledgment is made to those who have assisted in the preparation of the manuscript and to the many users of the book who have made valuable suggestions and constructive criticisms, many of which have been incorporated in this revision.

CORNELL UNIVERSITY,
ITHACA, NEW YORK,
August, 1940.

L. C. URQUHART,
C. E. O'ROURKE.

CONTENTS

DESIGN OF CONCRETE STRUCTURES

CHAPTER I

PLAIN CONCRETE

1. Introductory. Since the beginning of the twentieth century concrete has taken its place as one of the most useful and important structural materials. Owing to the comparative ease with which it can be molded into any desired shape its structural uses are almost unlimited; so wherever Portland cement and suitable aggregates can be obtained it has, for certain classes of work, displaced older materials. This apparent ease with which concrete may be prepared has led to its being employed by anyone who feels that the material is suited to his particular purpose. In many instances, proper knowledge of the substance and skill in its manufacture are not available, so that the resultant concrete is little more than a bulky, heavy material, lacking the strength and other properties which it should have attained, and often failing to fulfill that purpose for which it was intended.

To the individuals who obtain such results, concrete is merely a shoveled-together mass of cement, sand, stone, and water, which in a short time attains a varying degree of hardness and an uncertain strength. To the engineer who is more or less familiar with the many factors and variables entering into its manufacture, the process of making concrete does not appear quite so elementary. Experience shows that the quantity and quality of cement, aggregates, and water, and the processes of mixing and curing are all involved in the production of concrete. Results are dependent upon all of these variables. It is, therefore, the problem of the engineer so to study and control these factors that a concrete of the desired quality may be obtained.

2. Cements. Portland cement is the product obtained by finely pulverizing the clinker obtained by calcining to incipient fusion an intimate and properly proportioned mixture of argillaceous and calcareous materials with no additions subsequent to calcination except water and calcined or uncalcined gypsum. All Portland cement used in reinforced concrete construction should pass such standard specifications as those of the American Society for Testing Materials.[1] The color and rapidity of hardening of different brands of cement vary considerably and may be elements requiring special attention for the particular work involved.

Portland cement is manufactured by either the *dry* process or the *wet* process.[2] In the dry process the calcareous material is usually limestone and in the wet process marl. In all cases its content is principally calcium carbonate. The argillaceous material may be shale, clay, argillaceous limestone, or blast furnace slag which provides the silica and alumina.

In the dry process the raw materials are ground separately, mixed in the proper proportions, pulverized, and burned in a kiln. The resulting clinker is cooled and seasoned, materials are added to control the rate of setting, and the clinker is ground to a fine powder. In the wet process the procedure is similar except that the marl is stored in the form of a thin mud in vats and the argillaceous material in powder form is mixed with it before burning.

The finished product contains from 25 to 61 per cent of tricalcium silicate, from 7 to 44 per cent of dicalcium silicate, about 10 per cent of tricalcium aluminate, about 8 per cent of tetracalcium alumino ferrite, and about 3 per cent of calcium sulphate. The tricalcium silicate and the tricalcium aluminate are the predominating influences in producing strength up to 28 days.[3] The former continues to be effective in producing

[1] A.S.T.M. Standard C9-38.

[2] For description of cement manufacture see Johnson's "Materials of Construction," 8th ed., p. 310.

[3] For effect of cement composition on mortars and concretes see *Proc.*, A.S.T.M., vol. 34, part II, p. 244.

strength beyond that period, but the effect of the latter rapidly diminishes after 28 days and becomes zero or negative in one or two years. The dicalcium silicate contributes very little to the strength during the early stages but is the principal strength-producing compound beyond the 28-day period.

High early strength Portland cements are made from mixtures having a high lime content and are ground to a greater degree of fineness than the ordinary Portland. The finished product contains a large proportion of tricalcium silicate, the percentage by weight usually being at least 65 and often as high as 75, with a corresponding decrease in the amount of dicalcium silicate. With such a chemical content these cements harden and gain strength very rapidly and their use is economical where their extra cost is justified by the desire for high strength in a short period of time.

High-alumina cements contain about 40 per cent of lime, 40 per cent of alumina, and 15 per cent of ferric oxides. While they set at about the same rate as Portland, they gain strength much more rapidly on account of the high alumina content, so that in 24 hours a concrete made with a cement of this type will reach a strength equal to or greater than a concrete made with standard Portland will achieve in 28 days. However, as the age of such a concrete increases, there is a tendency toward a slight retrogression in strength.

Other special cements, such as *sand cement*, *tufa cement*, *puzzolan cement*, and *natural cement*, have been used in various projects, but in general they do not produce as strong or as durable a concrete as Portland cement. Reference to cement in future discussions means Portland cement.

3. Fine Aggregate. Fine aggregate should consist of sand, stone screenings, or other inert materials with similar characteristics, or a combination thereof, having clean, hard, strong, durable, uncoated grains, and free from injurious amounts of dust, lumps, soft or flaky particles, shale, alkali, organic matter, loam, or other deleterious substances. In general, all particles passing a No. 4 sieve (4 meshes per linear inch) are considered as fine aggregate. Most specifications, however, allow some degree of latitude from this requirement. The report of the

Joint Committee[1] recommends that not less than 85 per cent of the fine aggregate pass through a No. 4 sieve. Similarly it is in general advantageous that the fine aggregate be well graded from fine to coarse, and the report mentioned above recommends that not more than 30 nor less than 10 per cent of the fine aggregate pass through a No. 50 sieve. Extremely fine particles, if present in any great amount, are not beneficial to the strength of the resultant concrete, since they furnish so great an excess of surface area to be covered by the cement. Specifications vary as to the amount of these allowed, but in general not more than 6 per cent of the fine aggregate should pass through a No. 100 sieve.[2]

One method of classifying aggregates according to their grading is based on the surface area of the aggregate. In this method p_1 is taken as the percentage by weight of the sample finer than the No. 100 sieve as obtained by a sieve analysis, p_2 the percentage between the No. 100 and No. 50, p_3 the percentage between No. 50 and No. 30, etc., using the intervals between the No. 30, No. 16, No. 8, No. 4, 3/8-, 3/4-, 1 1/2-, and 3-in. sieves. Each percentage, except p_1, is multiplied by a coefficient. This

[1] The Joint Committee on Standard Specifications for Concrete and Reinforced Concrete consists of five representatives from each of the following: American Society of Civil Engineers, American Society for Testing Materials, American Railway Engineering Association, American Concrete Institute, Portland Cement Association. The Committee was organized in 1904, and presented progress reports in 1909 and 1912, and a final report in 1916, and was discharged. A new Committee of the same title and representing the same societies was organized in 1920, and presented a progress report in 1921 and a final report in 1924. A third committee was organized in 1930, with additional representation from the American Institute of Architects, and has since presented one progress report and a final report in June, 1940.

[2] The grading recommended by the A.S.T.M. is as follows:

	Percentage by Weight
Passing a 3/8-in. sieve	100
Passing a No. 4 sieve	95–100
Passing a No. 16 sieve	45– 80
Passing a No. 50 sieve	5– 30
Passing a No. 100 sieve	0– 8

coefficient is $\left(\frac{1}{2}\right)^{m-1}$ where m indicates the number of the term in the series. The surface modulus is then the sum of these products, *i.e.*,

$$p_1 + \frac{p_2}{2} + \frac{p_3}{4} + \frac{p_4}{8} + \frac{p_5}{16} + \frac{p_6}{32} + \frac{p_7}{64} + \frac{p_8}{128} + \frac{p_9}{256} + \frac{p_{10}}{512}$$

Another method of classifying aggregates according to their grading is based on the *fineness modulus.*[1] The same sieves are used as in the determination of the surface modulus, and the fineness modulus is the sum of the percentages by weight of the sample coarser than each sieve, divided by 100.

The surface modulus is large for fine materials and smaller for coarser ones and is most affected by a variation in the finer particles of an aggregate. On the other hand, the fineness modulus gives emphasis to the coarser content. Typical sands, used as fine aggregate for concrete have fineness moduli varying from 2.4 to 3.5, with an average value of about 3.0. In general, sands with a fineness modulus less than 2.5 should not be used for concrete.

Sand. All sands are derived from rocks which have broken down or disintegrated through the operation of physical agencies. In some cases, in addition to disintegration, there has been more or less decomposition involving the formation of new compounds. The principal disintegrating agencies are temperature changes and abrasion. The former cause cracking and a spalling off of particles of the constituent rock, because of the unequal expansion and contraction of the component minerals. The latter may be caused by the flow of water, wind, or glacial action. Chemical decomposition is brought about through the solvent power of water, aided often by the presence of various chemically active substances such as acids, which are carried by the water,

Quartz is the mineral which makes up the bulk of the particles of most sands. This is due to the fact that only the harder constituents of rocks survive disintegration and decomposition, and quartz is a common constituent of most rocks and capable

[1] *Bull.* I, Structural Materials Research Laboratory, Lewis Institute.

of resisting these destructive agencies. Not all quartz sands, however, make suitable concrete aggregates, for comparatively small amounts of organic impurities will render the sand unfit for use. Sandstone is a common source of quartz sand. Here the character of the binder of the original rock will determine the quality of the sand, since the individual particles of the sand are made up of still smaller particles of quartz, bound together by silica, iron oxide, lime carbonate, or clay.

Pyroxene and *hornblende* are complex silicates possessing a degree of hardness, strength, and durability slightly inferior to that of quartz. Hornblende has inferior weathering qualities. Feldspars are considerably less strong and durable than quartz. Mica is a very objectionable constituent of sand, being soft, low in strength, and because of its laminated structure, offering opportunity for the percolation of water.

Sand deposits, being usually the result of stream or glacial action, and also of such character that the percolation of surface water through the beds is very easy, are often contaminated by matter of organic origin carried in suspension by the water. Thus the coating of the grains by such substances as tannic acid,[1] sewage, manure, sugar, etc., is frequently encountered. Such a coating on the sand grains appears not only to prevent the cement from adhering, but also affects it chemically. Thus the effect of such substances is extremely detrimental, but at the same time their presence is hard to detect, a fact which increases the importance of carefully testing concrete sands.

Two tests may be made in the selection of a sand which may go far toward determining its suitability as an aggregate. It often happens that a sieve analysis of a dry sand will show a comparatively small percentage passing the No. 100 sieve, while in reality there are numberless other small particles loosely cemented together which by themselves would easily pass through the sieve. These particles usually consist of silt, loam, or clay, and are soluble or partially soluble in water. By taking a thoroughly dried sample of sand of known weight in a container,

[1] See *Proc.*, A.S.T.M., vol. 20, part 1, p. 309, for the "Effect of Tannic Acid on the Strength of Concrete."

adding sufficient water to cover the sample, and agitating thoroughly, a part of these fine particles is dissolved by the water. The whole contents of the container should then be poured into a nest of sieves. The material retained on the sieves should then be returned to the container and the process repeated until the wash water is clear. The washed sand may then be thoroughly dried and weighed, and the percentage of silt, loam, and clay in the original sample calculated. There are several exact specifications for making this decantation test,[1] and a similar test for field use, but it is usually specified that a sand showing more than 3 per cent of silt, clay, and loam by this test is not suitable for use as a concrete aggregate. Organic impurities in sand may be approximately detected by the colorimetric test.[2]

By far the best method of selection of a sand for concrete is the actual test of mortar briquettes made with the selected sand at the same time and under the same conditions as other briquettes in which standard Ottawa sand[3] is used. The report of the Joint Committee referred to above recommends that a sand shall preferably not be used as a fine aggregate unless such briquettes show at ages of 7 and 28 days a tensile and compressive strength at least equal to those made with standard Ottawa sand. Such a test as this often eliminates a sand which appears to be clean, well graded, and entirely suitable for a concrete aggregate, but which, owing to the presence of organic matter in its constituent grains, or to a coating of the grains with tannic acid, is actually a very poor sand.

Screenings. Limestone screenings or crusher dust are sometimes used as fine aggregates, but the concrete made therefrom is usually inferior in quality to that made with an average sand and is apt to be or become more permeable.

4. Coarse Aggregate. Coarse aggregate should consist of crushed stone, gravel, or other approved materials of similar characteristics or combinations thereof, having clean, hard,

[1] A.S.T.M. Standard C117-37.

[2] A.S.T.M. Standard C40-33.

[3] Standard Ottawa sand is a natural sand obtained at Ottawa, Ill., passing a screen having 20 meshes and retained on a screen having 30 meshes per linear inch, prepared and furnished by the Ottawa Silica Co., Ottawa, Ill.

strong, durable uncoated particles free from injurious amounts of soft, friable, thin, elongated, or laminated pieces, alkali, organic or other deleterious matter.

Crushed Stone. The quality of an aggregate of this type obviously depends upon the character of the original rock. The principal classes of rocks from which aggregates are derived are granites, trap rocks, limestones, and sandstones. Granite is an igneous rock whose principal mineral constituents are quartz and feldspar, with varying amounts of mica, hornblende, and other materials. The structural qualities of granite vary greatly, but granites as a class rank among the hardest, strongest, and most durable stones. Trap rock includes basalt, diabase, and a number of other igneous rocks possessing similar chemical and physical properties. The principal mineral constituents of most of these rocks are pyroxene and feldspar. They are generally rather fine grained, hard, tough, and durable. Limestone is a sedimentary rock which contains carbonate of lime, calcite, or carbonate of lime together with a double carbonate of lime and magnesia, dolomite, as the essential constituent. Sand and clay are common impurities, some varieties of which contain large amounts of shells and other fossils. Limestones vary greatly in structure, strength, hardness, and durability, and, although there are some limestones which have superior structural qualities, the average limestone is inferior to average granites and trap rocks as a concrete aggregate. Sandstones consist of grains of varying sizes, chiefly quartz, bound together by various cementing agencies or binders. A silicious binder produces a sandstone of the greatest structural strength, while an iron oxide or lime carbonate binder is much less efficient. A sandstone whose binder is clay is the least valuable of all as a concrete aggregate.

The maximum size of coarse aggregate advisable depends upon the character of the work. Since the stone is one of the strongest constituents of concrete, it is desirable to have as many and as large particles as possible. The greater the size of the particles, the less surface area there is to be coated, and the smaller amount of cement required for a concrete of given strength. When,

however, the maximum size is comparatively large, it is very important that it be well graded down to the minimum size in order to make a dense, compact mass. In small reinforced concrete members the maximum size available is as small as ¾ in. in diameter, and rarely in any reinforced work is a diameter greater than 1½ in. advisable. On the other hand, for large, massive work, with no structural reinforcement, much larger sizes may be used to advantage. The specification for minimum size recommended by the Joint Committee is that not more than 10 per cent by weight pass a No. 4 sieve, nor more than 5 per cent by weight a No. 8 sieve.

Gravel. Gravel of good quality makes a suitable concrete aggregate. Gravel is nothing more than pieces of natural rock broken away from the parent ledges and worn down by stream or glacial action. Its strength as an aggregate depends upon the rock from which it came, provided it has not become decayed or coated with objectionable organic matter. Too much emphasis cannot be given to the necessity of determining the cleanness of the gravel. A clayey coating is easily detectable, but the transparent organic coatings which prevent adhesion are not so easily discerned, without chemical analysis, so that many weak and inferior concretes result from the use of an apparently clean, but really "dirty" gravel as an aggregate.

Natural gravel may have a large proportion of particles so small as to be classed as "fine aggregate." These may be screened out before using, or a sieve analysis of the natural gravel being made, the proper amount of additional fine aggregate (if any) to add to obtain the desired proportions may be determined.

Slag. Slag from blast furnaces is a hard though porous material of high compressive strength, which in some localities can be obtained much more cheaply than stone of good quality. It offers a very rough surface for the adhesion of the cement, and, provided the sulphur content is low, it may make an excellent aggregate for massive concrete construction. Generally it should not be used in thin sections exposed to any action from water on account of its porosity.

Cinders. Cinders as an aggregate have the advantage of making a resultant concrete considerably lighter in weight than that made from stone or gravel. Formerly it was thought that well-burned cinders made a more fire-resisting concrete than other aggregate, but more recent experiences have shown that cinder concrete is little, if any, better in this respect. Cinder concrete is inferior in strength to other concrete, and on account of the danger from the probable sulphur content, it is not used where any great structural strength is required. Its principal use occurs for filling where no great strength is necessary. When used, cinders should be free from unburnt coal or fine ashes.

5. Water. Water used for concrete should be clean and free from oil, acid, alkali, organic matter, or other deleterious matter. Sea water is not so desirable as fresh water, although where it has been used in structures subject to the weathering action of sea water no greater disintegration has resulted.

6. Proportioning of Ingredients. Haphazard and careless proportioning of the ingredients of concrete was formerly the rule rather than the exception, but the number of resultant failures is surprisingly small. In the early years of concrete construction little attention was paid to any of the ingredients except the cement. Specifications might require a clean, sharp sand and a coarse aggregate of a specified crushed stone, but there was little emphasis on the grading of the aggregates or on the amount of water to be used. On the other hand, on almost all work of any magnitude, frequent and exhaustive tests of the cement were required. Under those conditions the practice grew of specifying certain arbitrary proportions for the mix, such as 1:2:4, or 1:3:6. When a concrete of high structural strength was desired, a rich mix was specified, and mixtures as rich as 1:1:2 have occasionally been used, while for less important or more massive work, mixtures as lean as 1:4:8 have been specified. Unfortunately this practice of arbitrary proportioning has persisted to some extent to the present day, although it is now generally recognized that the relative consistency (amount of water used) has some relation to the strength of the resultant concrete.

The measuring of the quantities of the constituent materials is done either by volume or by weight. The latter method is much more accurate, but the former method is more frequently used. The proportions referred to above are proportions by volume, *i.e.*, a 1:2:4 mixture indicates 1 cu. ft. of cement, 2 cu. ft. of fine aggregate, and 4 cu. ft. of coarse aggregate. Since the volume occupied by a given weight of cement varies by as much as 30 per cent for different degrees of compactness, it is necessary that the degree of compactness when measured be known. For this reason it is usual to specify that one bag of cement weighing 94 lb. shall be considered as 1 cu. ft. Cement is sold by the bag, or in barrels of four bags each, or in bulk. Where used in bulk, means must be provided for weighing the loose cement in order to insure the proper proportions.

The space occupied by a given number of sand grains varies considerably with the moisture content. Increases in volume of 25 per cent or more, caused by the addition of water to dry sand, are not uncommon.[1] Natural sand as it comes from the bank ordinarily contains from 2 to 4 per cent moisture by weight. Sand used in the laboratory is often entirely free from moisture, and sand used on the work may have either more or less moisture content than natural sand. For accurate proportioning, the moisture content must be taken into consideration when the measuring is done by volume. The bulking factor at the usual degree of moisture must be ascertained and proper allowances made, while at the same time the degree of moisture must be maintained as nearly constant as possible.

7. Design of Mortar Mixtures. In selecting the proportions of materials that are to go into a mortar it is often necessary and usually desirable to be able to predict the strength, permeability and the amount of mortar that will result from a given proportion of the ingredients. If there are no air voids in a wet mortar (and tests show that this is practically true for consistencies used in practice), the resultant volume[2] of the mortar is equal to the

[1] See *Proc.*, A.S.T.M., vol. 20, part II, p. 147.

[2] The absolute volume is the volume of the solid material, or the gross volume minus the volume of voids.

absolute volume of the cement plus the absolute volume of the sand, plus the volume of water used. In the average cubic foot of cement (1 sack of 94 lb.) there is 0.487 cu. ft. of solids. In 1 cu. ft. of sand the amount of solids depends upon the grading and the size of the grains. In beach sand there are from 0.60 to 0.63 cu. ft. and in commercial building sands about 0.68 cu. ft. of solids. The exact amount may be determined for any sand by obtaining its weight per cubic foot and its specific gravity.[1]

Strength. The water in a mixture chemically reacts upon the cement and produces a colloidal coating on the particles. The finest particles of cement become completely colloidal, while some of the larger particles are never completely penetrated. This colloidal coating is the whole cementing action, and once sufficient water is present to produce this reaction, additional water tends to weaken the cementing material. Therefore, it is reasonable to expect that the strength of a mortar at any given age will vary with the amount of mixing water per unit quantity of cement and the larger the amount of water the weaker the mortar.

In Fig. 1 are shown the results of studies of several investigators. The increased strength shown by the later studies is due to the stronger cements produced today in comparison with those of ten or more years ago. Figure 1 brings out clearly the direct relation between strength and *water-cement ratio.* Figure 2 shows the same relation, but here the water-cement ratio is expressed in gallons of water per sack of cement. This relation was first stated by Abrams[2] in 1918, whose curve of that date is shown in the figure. A more recent curve[3] by the same investigator shows the improvement in cement in recent years, and since the strength of plastic mixtures is independent of the amount and gradation of the aggregates, this later curve is applicable to the design of mortars for strength, which use present-

[1] A.S.T.M. Standard C128-39.
[2] *Eng. News-Rec.*, May 2,1918.
[3] *Proc.*, A.C.I., 1931, p. 1330.

day Portland cements. This curve cannot be used unless damp curing conditions are maintained.

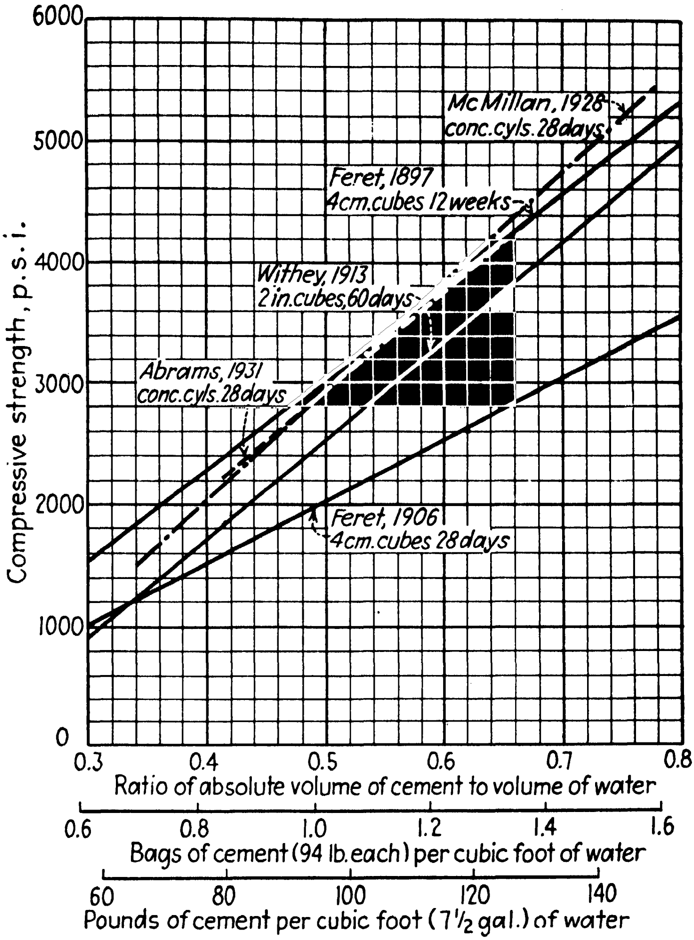


FIG. 1.—Relation between compressive strength and absolute volume of cement to volume of water.

In the measurement of water in a mixture all water outside of the solid particles is included. Most sand is delivered in a more or less damp condition and the moisture that is carried must be determined and allowance made in determining the amount of water to add.

Permeability. The permeability of a mortar is measured by the rate at which water under a given pressure will pass through a given thickness of the material. When the mortar is mixed, not all the water reacts with the cement and a large portion of it is really free water necessary in the mix to make it workable. As the mortar hardens, this free water gradually evaporates, thus providing air voids. The longer the mortar is cured in a damp condition, the greater the amount of water in permanent combination with the cement and the less the amount of air voids. Excess water in the original mix will, of course, increase the air voids and make the mortar more permeable. Repeated tests have shown that a reasonably water-tight mortar cannot be obtained with a water-cement ratio greater than 7½ gal. of water per sack of cement. In addition to a low water-cement ratio and prolonged curing, as dense a mixture as possible should be used.

Density. Other things being equal, a coarse-graded sand will produce a denser mortar than fine-graded sand. For any given proportion of sand to cement the voids in the mortar will vary with the volume of the mixing water used. In a very dry mortar the percentage of voids may be greater than in a wetter mix so that the voids decrease as water is added until the point of minimum voids (minimum amount of mortar) is reached. The addition of more water increases the voids. The exact proportions of cement, sand, and water that will produce the densest mortar vary for each sand depending upon its gradation. In general, coarse sands take less water and less cement than fine sands.

Workability. There are so many factors affecting the workability of a mortar that no satisfactory method of measuring it has ever been developed. Gradation, shape of particles, and proportions all have their effect. A mortar allowed to stand for 1 or 2 hours and then reworked often is found more workable than the original mortar. Fine sand makes a more workable mortar than coarse sand, but since its water requirement is higher, the strength is decreased. In providing a workable mortar care must be taken that strength and durability are not sacrificed.

8. Determination of Yield and Strength. It is desired to supply 100 cu. yd. of 1:3 mortar which will reach a strength of 3000 p.s.i. in 28 days. The sand available has 35 per cent voids, weighs 100 lb. per cu. ft. when dry, and contains 2 per cent of

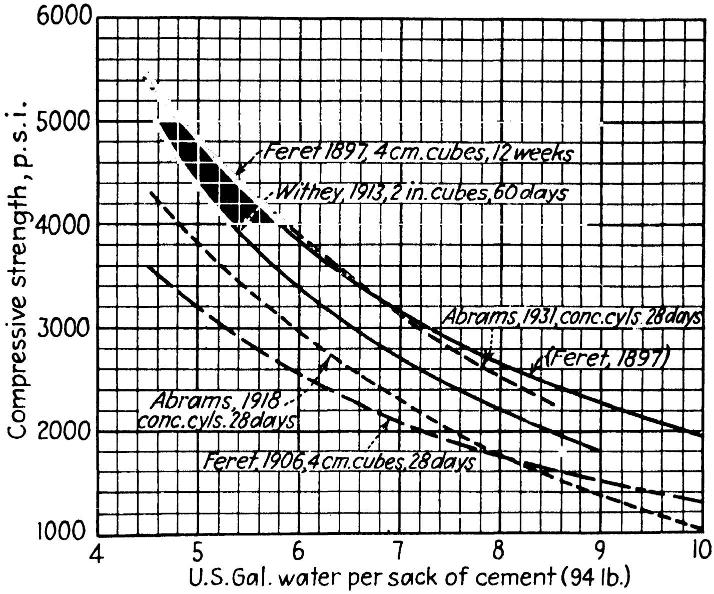


FIG. 2.—Relation between compressive strength and water-cement ratio.

moisture by weight, in which condition it weighs 83 lb. per cu. ft. What quantities of materials are required?

Reference to Abram's curve in Fig. 2 shows that a 3000-lb. mortar cured damp for 28 days should have 7.2 gal. of water per sack of cement. A one-sack batch will yield $0.487 + 3 \times 0.65 + 7.2 \times 0.134 = 3.40$ cu. ft. $= 0.126$ cu. yd. In 100 cu yd. there will be

$$100 \div 0.126 = 793 \text{ one-sack batches}$$

Therefore the quantities required are

$$\text{Cement} = 793 \text{ sacks}$$
$$\text{Sand } 3 \times 793 \div 27 = 88.1 \text{ cu. yd. dry}$$
$$\text{Water } 7.2 \times 793 = 5710 \text{ gal.}$$

However, since the sand contains 2 per cent moisture, the number of cubic yards in its moist condition is

$$100/83 \times 88.1 = 106 \text{ cu. yd.}$$

If this mortar were to be mixed in approximately 1-cu. yd. batches (8 sacks) the sand and water for each batch would be as follows.

$$\text{Sand } 3 \times 8 \times 100/83 = 28.9 \text{ cu. ft.}$$
$$\text{Water } 7.2 \times 8 - (0.02 \times 2400) \div 8.35 = 52 \text{ gal.}$$

In actual work some of the water in the mix is lost either by absorption by adjacent materials or by evaporation so that the actual yield is reduced from 2 to 4 per cent. This should be considered in making estimates of quantities of materials required for a given volume of mortar.

9. Concrete Mixtures. Concrete may be considered as a mortar mixture into which the coarse aggregate particles have been mixed, or it may be considered as a combination of the separate ingredients. In any case the factors governing the strength of a mortar hold true for a concrete provided a suitable coarse aggregate is used. Since the water in the mixture has to wet the surfaces of the coarse aggregate particles as well as those of the cement and sand grains, it follows that the larger the size of the coarse aggregate, the less the water that will be required for a given consistency; hence greater strength and durability.

Design specifications have not kept pace with the improvement in cement in recent years. For this reason caution should be exercised in designing for low specified strengths. With fair curing conditions a compressive strength of 2000 p.s.i. in 28 days may be obtained with a very lean mixture, but the resulting concrete will be neither water-tight nor capable of withstanding ordinary weather conditions. Experience has shown that concrete exposed to the elements should not have a water content of more than $7\frac{1}{2}$ gal. per sack of cement.

A concrete must have a certain degree of workability, but, as for mortars, no method has yet been developed which is

an accurate measure of workability. In the case of a concrete, this quality is influenced by the nature and amount of the coarse aggregate as well as by the amounts of cement and water, and the amount and nature of the fine aggregate. A method of measuring the workability, or more properly the consistency, is the *slump test.*[1] A conical shell of 16-gauge galvanized metal with a base 8 in. in diameter, a top 4 in. in diameter, and an altitude of 12 in. is filled to overflowing with concrete rodded into the shell in three separate layers, each receiving 25 strokes of a 5/8-in. bullet-end rod 24 in. in length. The excess is carefully struck off and the mold is at once lifted slowly vertically. The amount of drop of the top of the mass below the original 12-in. height, measured in inches, is known as the slump. Mass concrete and highway mixtures are workable with a slump of from 1 to 3 in. Concretes for reinforced beams and columns require a greater degree of workability, with slumps of from 4 to 6 in.

As in mortar, excess water in concrete dries out as the concrete ages, so that a water-tight concrete should not have a water content of more than 7 1/2 gal. per sack of cement. Careful grading of aggregates to obtain as dense a mixture as possible also tends to decrease the permeability, but the waste and consequent additional expense rarely warrants such a procedure.

10. Design of Concrete Mixtures. In the actual selection of the proportions that are to go into a given mix, the first step must be the selection of the water-cement ratio which will produce the strength required. In making this selection this ratio must in no case exceed 7 1/2 gal. per sack of cement and if the concrete is to be subjected to severe weather conditions a decrease to 6 or 6 1/2 gal. will be worth-while insurance of producing a durable concrete.

Abram's curve shown in Fig. 2 presupposes ideal conditions of mixing, placing, and curing, which usually are not met on the average job. Therefore the selection of the water-cement ratio for the desired strength should be taken from Fig. 3 which is a graphical representation of the specification for strength of the American Concrete Institute (1928).

[1] A.S.T.M. Standard C143-39.

Since part of the water will usually be contained in the aggregates, this amount must be determined. A portion of the moisture in the aggregates is in the interior of the particles. Such moisture, if it remains in the interior until after the cement has set, does not affect the water-cement ratio. If a bone-dry aggregate is used, a certain amount of water will be absorbed by the aggregates and the water-cement ratio will be changed. The average aggregate will absorb about 1 per cent of water by weight, trap rock and granite about one-half that amount, while a

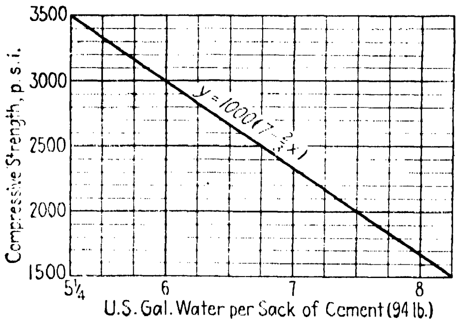


FIG. 3.

very porous aggregate may absorb several per cent by weight.[1] It may then be desirable to make up several trial batches, with variable quantities of fine and coarse aggregate, until the desired degree of workability is obtained. From the record of these trial batches, the amount of water, fine aggregate, and coarse aggregate for each batch finally may be selected.

From the above paragraph it is obvious that an arbitrary specification of the quantities of cement, sand, and coarse aggregate is not desirable. It is far better to specify the strength desired, for some types of work the maximum size of aggregate

[1] The exact amount of water absorbed by any aggregate may be obtained by determining the percentage of surface moisture (A.S.T.M. Standard C128-39) and also the percentage of total moisture. The latter may be obtained by drying out a moist sample of known weight and noting its weight when thoroughly dried. The difference between the percentages obtained by these two tests is the percentage of water absorbed by the aggregate.

which may be used, and possibly a limiting ratio between the amounts of fine and coarse aggregate. The following example shows the procedure under such a specification:

11. Example of the Design of a Concrete Mixture. Certain aggregates have been approved for a specific job. They have the following characteristics: The sand has 35 per cent voids and weighs 104.4 lb. per cu. ft. dry and rodded.[1] One cubic foot measured in its natural moist condition weighs 88.1 lb. and this same quantity when thoroughly dried weighs 84.4 lb. The percentage of moisture by weight is therefore $(88.1 - 84.4) \div 84.4 = 4.4$ per cent and the bulking factor is $104.4 \div 84.4 = 1.24$. The coarse aggregate has 33 per cent voids and weighs 107.5 lb. per cu. ft. dry and rodded. One cubic foot measured in its natural moist condition weighs 105.0 lb. and this same quantity when thoroughly dried weighs 103.3 lb. The percentage of moisture by weight is therefore $(105.0 - 103.3) \div 103.3 = 1.6$ per cent, and the bulking factor is $107.5 \div 103.3 = 1.04$.

The specified strength of the concrete is 2000 p.s.i. at 28 days. From Fig. 3, $7\frac{1}{2}$ gal. of water are to be used for each sack of cement. Trial batches using 5 lb. of cement and 3.3 lb. of water (which maintains the desired water-cement ratio) are made up using various proportions of surface-dry aggregates. The desired degree of workability, as measured by the slump, is obtained with 12.1 lb. of sand and 21.4 lb. of coarse aggregate.

For a one-bag batch with the surface-dry aggregates the following quantities must be used:

Cement = 1 sack	=	94 lb.
Sand = $\frac{94}{5} \times 12.1$	=	227.5 lb.
Coarse aggregate = $\frac{94}{5} \times 21.4$	=	402.3 lb.
Water	=	7.5 gal.

However, the aggregates are to be used in the field in their natural condition as determined above. It will be assumed that each of the aggregates naturally absorbs 1.0 per cent of moisture by weight. The amount of sand must then be increased by

[1] See A.S.T.M. Standards C128-39 and C69-30.

4.4 − 1.0 = 3.4 per cent to 235.2 lb. The free water in the sand is 7.7 lb. = 0.92 gal. Likewise the amount of coarse aggregate must be increased by 1.6 − 1.0 = 0.6 per cent to 404.7 lb. and the free water in the coarse aggregate is 2.4 lb. = 0.29 gal.

The field mix by weight using the natural aggregates is then:

Cement = 1 sack	= 94.0 lb.
Sand	= 235.2 lb.
Coarse aggregate	= 404.7 lb.
Water = 7.5 − 0.92 − 0.29	= 6.3 gal.

The field mix by volume is:

Cement = 1 sack	= 1.00 cu. ft.
Sand = 235.2 ÷ 88.1	= 2.67 cu. ft.
Coarse aggregate = 404.7 ÷ 105.0	= 3.85 cu. ft.

The yield for a one-bag batch is:

Cement = 1 cu. ft. (1 − 0.513)	= 0.49 cu. ft.
Sand = 2.67 cu. ft.* 2.67 (1 − 0.35) ÷ 1.24	= 1.40 cu. ft.
Coarse aggregate = 3.85 cu. ft.	
3.85 (1 − 0.33) ÷ 1.04	= 2.48 cu. ft.
Water = 7.5 gal.	1.00 cu. ft.
	5.37 cu. ft.

The yield from a one-bag batch being known, it is possible to compute the quantities required for a given volume of concrete. As in mortars, from 2 to 4 per cent should be added on account of the actual loss, after placing, of a portion of the water.

12. Control of Concrete Mixtures. Where a small quantity of concrete is to be placed, elaborate examination and inspection of each batch are not warranted and economy is best served by keeping the water-cement ratio below the theoretical value.

On the other hand, where large quantities of concrete are to be placed, special considerations are desirable to insure economy and a uniform quality of concrete. Once the proportions are selected, as long as the aggregates come from the same source little trouble is usually encountered (except variable water content) in providing the proper amount of fine aggregate for

* With the water within the particles assumed as 1 per cent, a given number of sand grains will occupy the same space whether this amount of moisture is actually present or not.

each batch. The coarse aggregate may segregate in the stock pile and considerable care must be taken to insure uniformity.

The control of the water content is the most difficult. As shown in Art. 11, the amount of moisture in the aggregates has a considerable effect upon the volume and some effect upon the weight. Unfortunately, the percentage of moisture in the stock piles is not constant. Where measurement is to be made by volume, a large job warrants a "job curve" for each aggregate. This curve shows the volume per cubic foot for different percentages of moisture, and by making periodic determinations of the amount of moisture in the stock piles, the correct amount of aggregates and water can be assured for each batch. Where measurement is made by weight, a similar curve may be constructed, showing the amount of water for various weights of aggregate with different moisture contents.

Another method of control is the use of an inundator. The volume of sand is measured while inundated in just sufficient water to fill the previously determined voids, and then the necessary additional amount of water is added. Any moisture originally present in the sand thus becomes a part of the amount required to inundate the sand and no readjustment is necessary. The only inaccuracy of this method is caused by the amount of moisture in the coarse aggregate, but as this may easily be determined and is often fairly constant, a proper allowance may readily be made.

13. Mixing. Practically all concrete is machine-mixed. The ordinary mixer consists of a rotating drum, in the interior of which are blades so placed that, as the drum rotates, they lift the ingredients, which in turn slide off the blades and drop to the mass then at the bottom of the drum. The time of mixing depends upon the character of the mixture and the speed of the mixer, but in general little is to be gained by continuing the operation beyond 2 minutes, as is shown by Fig. 4.[1] The Joint Committee recommends that "the mixing of each batch shall continue not less than 1 min. after all the materials are in the mixer, during which time the mixer shall rotate at a peripheral speed of about 200 ft. per min."

[1] ABRAMS, *Proc.*, A.C.I., 1918, p. 22.

14. Placing. Concrete is conveyed to the forms from the mixer by means of buggies or barrows or by inclined chutes or pipes. It is important that the method used does not allow segregation of the ingredients. It should not all be dumped in one place and allowed to flow horizontally, but should be deposited in approximately uniform layers. Forms should be tight

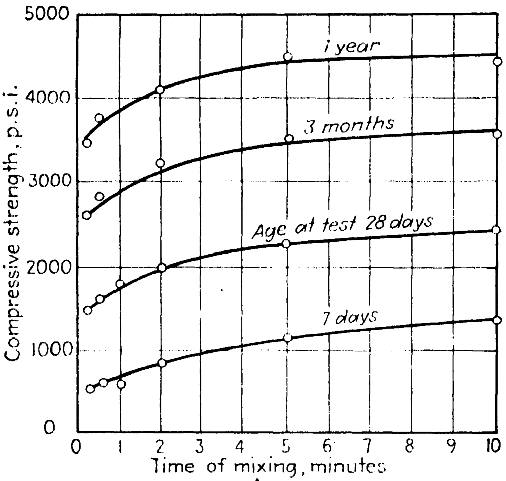


FIG. 4.—Effect of time of mix upon strength, for stationary mixers.

in order to prevent the loss of cement carried by the escaping water. In hot dry weather, wooden forms should be thoroughly wetted on the contact faces to prevent them from robbing water from the mortar faces in contact with the forms, which causes these faces to be pulled away with the forms upon stripping.

The filling of the forms should be continuous where possible in order to prevent the formation of laitance or "day's work" planes. Laitance is a whitish substance consisting of the finest particles of the cement together with some of the silt and clay from the aggregates. It is brought to the surface of freshly mixed concrete where excess water is used (as it usually is in reinforced concrete) and, as it hardens very slowly and never acquires much strength, it constitutes a plane of weakness. Where such continuous deposition is impossible, the laitance should be scraped off and the surface of the old concrete roughened and wetted before placing is resumed.

Where forms have considerable height with reinforcement continuous in either a vertical or a horizontal direction over the full height, some means should be provided of depositing the concrete without dropping it through too great a distance. In addition to the separation that is bound to take place, both forms and reinforcement become coated with hardened concrete long before they are completely filled, which may cause planes of weakness in the top of the structure. Properly constructed enclosed chutes or pipes extending down into the forms will insure against these weakness planes and also produce a satisfactory surface finish.

Spading or puddling of the concrete is necessary to cause it to spread laterally after it has been deposited in the forms. This puddling should be done in such a manner as to force the coarse aggregate against the forms. In a properly designed mix sufficient mortar will follow and encase the coarse aggregate to insure a good surface finish, but if all of the coarse aggregate is forced into the interior, away from the forms, the surface is less durable.

Vibrators have been used in the past few years to aid in the flow of the concrete in the forms. They are operated either by compressed air or by electricity. They are used both internally and against the outside of the forms. Since they allow the use of a very sluggish mixture, with its consequent lower water-cement ratio, higher strength concretes may be expected.

15. Curing. The principal variations in curing conditions which affect the process of hardening and the strength of the concrete are variations in moisture and temperature conditions. While it is important that the amount of water used in mixing be controlled so that the consistency is as nearly normal as practical, it is just as important that the concrete be not allowed to dry out immediately, if the maximum strength obtainable is to be attained. All concrete should be protected against premature drying out for at least 1 week, and for a longer time if the temperature is near the freezing point. This may be done by sprinkling with water at intervals, or by covering with damp or wet burlap. In road construction, water may be held over the entire surface by damming the edges with loose earth and forming a series of

ponds. The importance of keeping the concrete moist while hardening cannot be too strongly emphasized. Tests show that a concrete allowed to dry out immediately will usually reach a strength of not more than 50 per cent of the strength of similar concrete kept moist over the entire period of curing. Figure 5 shows this relation[1] graphically. All test specimens were tested at 4 months, having had various intervals of storage in damp sand. Each value is the average of 24 tests (four each for six consistencies).

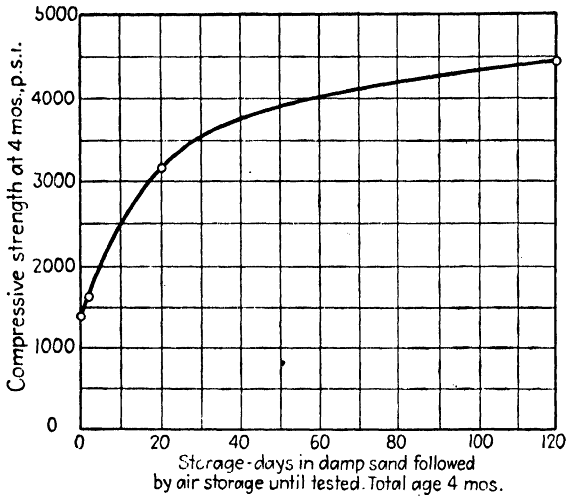


FIG. 5.

The relation between the mean temperature during the curing period and the strength of concrete is illustrated by Fig. 6.[2] The tests from which the curves were plotted covered a wide range of temperature conditions, and the results were fairly consistent. A knowledge of the effect of the mean temperature upon the strength is very necessary in determining the time when forms may be removed and loads applied, and a careful study of Fig. 6 will furnish the necessary information for deter-

[1] Taken from *Bull.* 2, Structural Materials Research Laboratory, Lewis Institute, Chicago, Ill.

[2] Taken from *Bull.* 81, Engineering Experiment Station, University of Illinois.

mining the relative length of time the forms should be kept in place under different temperature conditions.

By combining high temperatures with a saturated condition of the atmosphere, it would follow that accelerated hardening of the concrete would be obtained. These conditions are brought into being by the application of live steam to concrete while

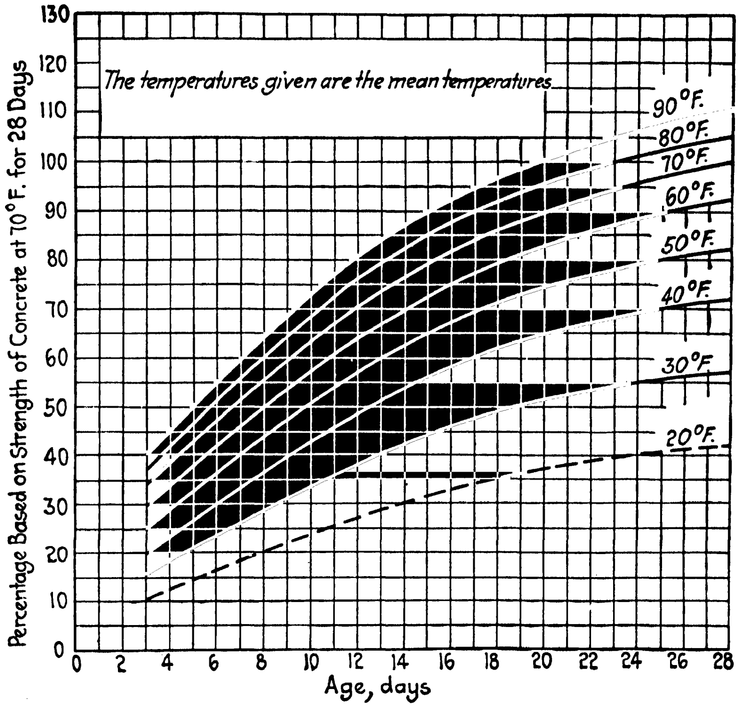


FIG. 6.

hardening. This method is especially useful in the manufacture of concrete blocks, tile, small pipe, etc., where the saving in forms, storing space, and time is important. By placing the concrete products in a confined space, and applying the steam under pressure, a still more rapid increase in strength will be attained. The steam should not be applied until after the concrete has obtained an initial set. Results of tests show that up to 80 p.s.i. gauge pressure, steam has an accelerating action on the hardening of concrete, and that the compressive strength increases

with the pressure and time of exposure. The application of the steam, too, permanently accelerates the hardening after the exposure to steam ceases. Concrete so treated has reached a compressive strength in 2 days (exposure to steam under pressure 24 hours) greater in some cases by 100 per cent than unsteamed concrete has reached in 28 days.

16. Freezing. The effect of low temperatures in delaying the hardening of concrete is shown in Fig. 6. When water reaches a temperature of 39 deg. Fahrenheit some subtle change occurs which decreases its chemical ability for combination. This change becomes more marked as the freezing point is approached, and concretes placed with the temperature near the freezing point take several times as long to obtain a final set as concretes cured at normal temperatures. In case of dry atmospheric conditions much of the water may evaporate before the final set takes place, and insufficient water be left to combine chemically with the cement. In case the temperature falls below the freezing point before final set, the expansion of the water while freezing exerts a force sufficient to destroy the cohesion between the particles of the green concrete.

The injurious effect of freezing is lessened by two factors, namely, that concrete is a very poor conductor of heat, and that the chemical action of setting and hardening generates a certain amount of heat to combat the freezing action of the atmospheric conditions. Thus the serious injury is usually confined to the surface of the concrete, and rarely penetrates more than an inch or two in depth. In massive members this may not seriously impair the strength, but be harmful only to the appearance. In the smaller members, however, a large percentage of the strength may be lost.

Various methods are used to prevent the freezing of concrete, namely, heating the materials, covering the green concrete, adding certain salts to the mixture to lower the freezing point of water, etc. The Joint Committee recommends that:

"In freezing weather suitable means shall be provided for maintaining a temperature of at least 50 deg. Fahrenheit for not less than 72 hours after placing, or until the concrete has thor-

oughly hardened . . . Salt, chemicals, or other foreign materials shall not be mixed with the concrete for the purpose of preventing freezing, unless approved by the Engineer."

Effects of Miscellaneous Agencies and Conditions on the Strength and Durability of Concrete

17. Salts, Hydrated Lime, and Water-proofing Compounds. Common salt is often added to the mix to lower the temperature at which the water will freeze. The addition of salt lowers the freezing point about 1 deg. Fahrenheit for each 1 per cent of salt added to the mixing water. This has been shown to be beneficial to the strength of concretes cured at freezing temperatures up to 12 per cent. More than this amount of salt has generally proved injurious. With normal temperatures the addition of common salt is injurious, the decrease in strength being roughly proportional to the percentage of salt added. The set is retarded in all cases, and in reinforced concrete the salt is likely to cause corrosion of the steel.

Calcium chloride is used for the same purpose as common salt. Not such a large percentage is beneficial to strength as in the case of common salt, but up to 4 per cent concretes cured at any temperature are benefited, and the setting is accelerated. Tests made at the University of Wisconsin[1] indicated that the best effect was obtained with a mixture of 2 per cent of calcium chloride and 9 per cent of common salt.

The use of hydrated lime in quantities up to 15 per cent of the weight of the cement has been advocated by various authorities on the theory that it improved the workability of the concrete or increased its strength and water-tightness. In lean mixtures it is true that the addition of hydrated lime does have a marked effect in producing a more plastic and better working concrete. In the richer mixes this effect is less pronounced. Some tests have shown a slight increase in strength with the use of a small percentage of hydrated lime. It appears that if the hydrated lime is added without decreasing the amount of cement or increasing the amount of water, such an increase usually occurs,

[1] *Wisconsin Eng.*, October, 1913.

but if some of the cement is replaced by hydrated lime, a reduction in strength can be expected.

Various water-proofing compounds in powdered or liquid form are sometimes used to make a more impervious concrete. They are either added to the mixing water, mixed with the cement on the job, or added to the cement during its manufacture. Their function is to fill the voids or pores of the concrete with a more or less soapy, insoluble filler, and thus prevent the percolation of water through the concrete. The results obtained are varied. Some practically impervious concrete has been produced, while on other work the water-proofing has not been successful. Practically all of the compounds in use detract from the compressive strength of the concrete. Fully as impervious concrete can generally be obtained by using a slightly richer mix, well-graded fine aggregate, as stiff a consistency as possible, and thoroughly puddling the concrete as it is placed.

Certain classes of mineral oils have been used in concretes for damp-proofing them or reducing their permeability, but the results obtained do not warrant their use.

18. Alkali. The action of alkali on concrete is a problem peculiar to the prairie regions of the west. These regions, because they have a low rainfall and poor drainage, present extraordinary conditions in respect to the amounts of soluble salts present in the soil. These salts are mainly sodium, magnesium, and calcium sulphates. Seepage water from shallow excavations commonly shows concentrations of from 1 up to 6 per cent or more. Chemical action between these sulphates and the cement causes the decomposition of the concrete, the physical action resembling exactly that of frost. Cases have been cited where concrete immediately above and free from such contact was in first-class condition. The action is slow and retarded by impermeability, but while there is reason to believe that well-made structures of dense concrete will stand up indefinitely in sea water where constantly immersed, there is no doubt that equally well-made concrete will not stand up long in contact with ground waters of high alkali content. The life of a structure in contact with such waters may be lengthened by following the same precautions that

should be used in making and placing concrete which is subject to the action of sea water, and by providing drainage such that as small an amount of alkali water as possible shall come in contact with the finished concrete.[1]

19. Oils, Acids, and Sewage. Concrete thoroughly hardened is unaffected by mineral oils such as ordinary petroleum or engine oils. Various animal and vegetable oils may slightly weaken and disintegrate a concrete, but such cases are rare.

Acids which seriously injure other materials will also injure concrete. The condition where this is most likely to occur is in the discharge of acids in sewage. Strong sulphuric acid in contact with the concrete converts the carbonate of lime into sulphate of lime, which is soft and easily corroded. Two factors, however, tend to make this effect less marked; first, the likelihood of the acid being so much diluted by the water of the sewage as to be practically harmless, and second, the greasy substance which is usually found to coat the perimeter of a sewer under the water line prevents the full action of the acid upon the cement.

20. Manure. Manure is sometimes used to cover fresh concrete in freezing weather. Since dry manure is a poor conductor of heat, and since during decomposition it generates heat, it is quite effective in preventing freezing of the concrete. Unless the work is first covered by some impermeable material, the uric acid in the manure is likely not only to discolor the green concrete but partially to disintegrate it. If the manure becomes wet during the early stages of the hardening of the concrete, unless the latter is efficiently protected, the disintegrating effect may be quite marked. Thoroughly hardened concrete is sometimes discolored by contact with manure, but its strength is not impaired.

[1] The Joint Committee specifies that: "Concrete in alkali waters or below the ground line of alkali soils shall contain a minimum of 1¾ bbl. (7 bags) of Portland cement per cubic yard in place . . . Concrete shall be placed in such a manner as to minimize the number of horizontal or inclined seams, or work planes. Metal reinforcement or other corrodible metal shall not be placed closer than 2 in. to the surface of members exposed to alkali soils or waters. In foundations and in heavy structures the metal reinforcement shall not be placed closer than 3 in. to the surface."

21. Electrolysis. Although in most structures the danger of action by electrolysis is negligible, there are certain conditions where reinforced concrete may be seriously damaged by the flow of an electric current between the concrete and the steel. If electrically positive reinforcement is in contact with concrete, it will become corroded, provided the concrete is sufficiently moist and the voltage sufficiently high. The corrosion of the reinforcement with its consequent expansion causes cracking in the concrete. Strong currents, however, of high enough voltage are not usually found under actual conditions, so danger from this source is very rare. Common salt, even in amounts less than 1 per cent, increases the rate of corrosion of the reinforcement since it increases the conductivity of the concrete. This rate of increase is so tremendous that special care should be taken in structures exposed to contact with sea water to prevent the flow of stray electric currents. In constructions where stray electric currents may be expected, no salt should be used in the concrete. Electrically negative reinforcement in contact with concrete produces a softening effect upon the latter, which may extend for $\frac{1}{4}$ in. or more all around the reinforcement. This softening effect eventually completely destroys the bond between the concrete and the steel. It manifests itself at all voltages, the rate being approximately proportional to the voltage.

22. Sea Water. Almost invariably, specifications forbid the use of sea water in mixing concretes. The detrimental effect to the concrete itself is usually not so large except in very lean mixtures, but in reinforced concrete construction where sea water is used in mixing, the corrosion of the steel is likely to be serious, and may eventually result in the complete destruction of the work.

The reliability of concrete and reinforced concrete when exposed to the action of sea water is variable, but, under favorable conditions and with proper care, structures comparing favorably in durability with those of timber or steel can be constructed. A richer and denser mix should be used than for ordinary construction, in order to insure against the infiltration of water into the concrete, which causes, in the case of plain con-

crete, complete disintegration due to chemical reaction, or in the case of reinforced concrete, failure due either to electrolysis or to rusting of the steel. Just enough fresh water should be used in mixing so that the concrete settles around the reinforcing rods with light tamping. The forms should be oiled to insure a smooth surface, and if practical, a richer mortar coat should be applied next the forms at the same time the other concrete is placed. Reinforcing material should be protected by at least 3 in. of concrete. Construction joints should be avoided, and the concrete should not be exposed to the sea water until it is thoroughly hardened.[1]

[1] The Joint Committee specifies that: "Plain concrete in sea water from 2 ft. below low water to 2 ft. above high water, or from a plane below to a plane above wave action, shall contain a minimum of 1¾ bbl. (7 bags) of Portland cement per cubic yard in place. Other plain concrete in sea water or exposed directly along the sea coast shall contain a minimum of 1½ bbl. (6 bags) of Portland cement per cubic yard in place. Porous or weak aggregates shall not be used . . .

"Sea water shall not be allowed to come in contact with the concrete until it has hardened for at least 4 days. Concrete shall be placed in such a manner as to minimize the number of horizontal or inclined seams or work planes. The placing of concrete between tides shall be a continuous operation, . . . where it is impossible to avoid seams or joints the surface of the set concrete shall be roughened . . . , thoroughly cleaned of foreign matter and laitance, and saturated with water. The new concrete placed in contact with the hardened or partially hardened concrete shall contain an excess of mortar to insure bond. To insure this excess mortar at the juncture of the hardened and newly deposited concrete, the cleaned and saturated surfaces of the hardened concrete, including vertical and inclined surfaces, shall first be slushed with a coating of neat cement grout against which the new concrete shall be placed before the grout has attained its initial set. Concrete shall be deposited in sea water only when so directed by the Engineer . . .

"Metal reinforcement shall be placed at least 3 in. from any plane or curved surface, except at corners when it shall be at least 4 in. from adjacent surfaces. Metal chairs, supports, or ties shall not extend to the surface of the concrete. Where unusually severe conditions of abrasion are anticipated, the face of the concrete from 2 ft. below low water to 2 ft. above high water, or from a plane below to a plane above wave action, shall be protected by creosoted timber, dense vitrified shale brick, or stone of suitable quality, as designated on the plans or as required by the Engineer."

23. Compressive Strength. The ultimate strength of a concrete normally increases with age. This increase proceeds very rapidly for the first few days after the concrete is placed, but becomes more gradual as time goes on, though continuing at a more reduced rate for an indefinite period. The compressive strength of concrete at the age of 28 days is generally used as a measure of the quality of the concrete. This assumes proper mixing and placing and suitable curing conditions. The compressive strength of the concrete is based on tests of 6- by 12-in. or 8- by 16-in. cylinders made in accordance with the Standard Methods of Making and Storing Specimens of Concrete in the Field, of the American Society for Testing Materials,[1] and tested in a well-equipped laboratory by a competent operator.

The ultimate compressive strength expressed in pounds per square inch is used as a basis for determining the unit stresses to be used in design, for it has been found that practically all of the other structural properties of a concrete are proportional to the compressive strength.

24. Tensile Strength. The tensile strength of concrete is a property of little importance, because it is so low in comparison with the compressive strength that it is usually neglected altogether in the design of reinforced concrete structures. It may roughly be estimated as having a value of about 10 per cent of the compressive strength.

25. Transverse Strength. The transverse or flexural strength of concrete is low as compared with its compressive strength, but much greater than the strength in pure tension. The transverse strength is measured by the stress developed in beam action. In a reinforced concrete member this strength is usually disregarded, and steel reinforcement is placed in the member to develop the flexural stresses on the tension side. Load tests, however, on reinforced concrete structures have shown that the transverse strength of the concrete contributes to a marked degree in increasing the capacity of the structure. Tests made by Duff A. Abrams[2] indicate that, at 28 days, the transverse

[1] A.S.T.M. Standard C31-39.

[2] See *Bull.* 11 Structural Materials Research Laboratory, Lewis Institute, Chicago.

strength varies from 26 per cent of the compressive strength for a 1000-lb. concrete to 15 per cent for a 4000-lb. concrete.

26. Shearing Strength. The shearing strength of concrete is important in that failure by shear on a diagonal plane often occurs in short compression specimens. The direct shear must not be confused with the combination of shear and diagonal tension that occurs in the web of a beam. The resistance of concrete to direct shear is difficult to determine, as it is almost impossible to eliminate the effect of bearing, diagonal tension, and other stresses, so that different series of tests show quite a variation from one another. For most concrete the shearing strength is at least 60 per cent of the compressive strength, and will need to be considered in design only in exceptional cases.

27. Elasticity. Concrete is not a perfectly elastic material, there being a slight decrease in the ratio of stress to strain as the stress increases. Concrete also shows a permanent set under the smallest loads, but within working limits there is a fairly constant relation between temporary stress and strain, which may be considered as the modulus of elasticity of the concrete.

Since the stress-strain line is curved almost from the beginning, the method of calculating the modulus of elasticity needs to be considered. Figure 7 is a typical stress-strain diagram with the curvature somewhat exaggerated. A load producing a stress *Oc* has been applied and removed a sufficient number of times until the permanent set *Oa* shows no further appreciable increase, and all points on the stress-strain line *ad* fall on an approximately straight line. The deformation measured from the original position, for a stress *Oc*, is *Ob*, and the slope of the line *Od* is called the "secant modulus." The slope of the tangent to the curve, represented by *OT*, is known as the "initial modulus" or "initial tangent modulus."

In reinforced concrete design the principal use of the modulus of elasticity is to determine the value of the relative stresses carried by the steel and concrete, assuming that there is perfect bond between the two materials. For such a computation the deformation should be measured from the original position, and the secant modulus should be used. In the case of a beam

where the stress in the concrete varies according to the "straight-line" theory, the secant modulus, while not exactly representing the conditions, is nearer to the exact conditions than the initial modulus. The relation between the secant modulus and the initial modulus is not a constant, but for all but the smallest loads the former is the smaller.

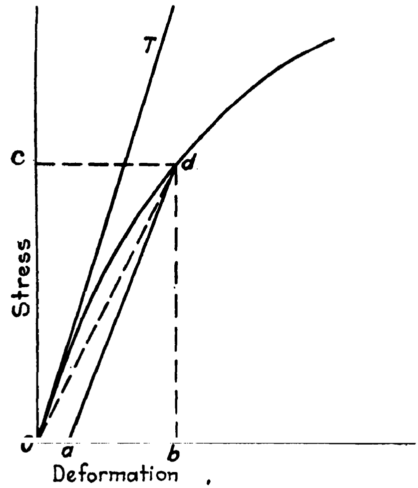


FIG. 7.

There is a relation between the modulus of elasticity of a concrete and its compressive strength, but this relation is not linear. Stanton Walker[1] expresses this relation for usual concretes as $E = 33{,}000\ S^{5/8}$, where E = the initial modulus, and S the compressive strength of the concrete. The tests from which this relation was determined covered a wide range of consistencies, mixes, times of mixing, curing conditions, and age at time of test. These tests also showed that the modulus of elasticity increases as the aggregate becomes coarser (within certain limits), that it increases with age, the richness of the mix, and time of mixing, and is less for wet than for dry consistencies.

The value for this initial modulus for concrete at the age of 28 days varies from about 1,500,000 to 5,500,000 p.s.i. with

[1] See *Bull.* 5, Structural Materials Research Laboratory, Lewis Institute, Chicago.

a somewhat narrower range for the usual concretes. The values most often used in design are generally somewhat smaller, the Joint Code specifying that the modulus of elasticity of the steel be taken as 30,000,000 p.s.i. and that of the concrete as $1000f'_c$, where f'_c is the ultimate compressive strength determined as described in Art. 23.

28. Elastic Limit. For the same reasons as those given at the beginning of the previous section, there can be no *elastic limit* in the true sense of the term. There appears to be a stress, however, below which repetition of the same load does not cause appreciable increase in set, while beyond this stress, repetition of load causes increased set indefinitely, and final failure far below the normal ultimate strength. This stress may be considered as the *elastic limit.* Tests show quite a range of values, varying from 25 to 90 per cent of the ultimate compressive strength, but for the average concrete it is probably in the neighborhood of from 40 to 60 per cent of the ultimate compressive strength.

29. Plastic Flow. Tests have shown that all concrete under a sustained load continues to deform or "plastically flow" for a long period of time. This deformation is independent of that due to shrinkage caused by the decrease in moisture content. The results of tests by R. E. and H. E. Davis[1] on 4- by 14-in. cylinders of 1:5.05 concrete with a water-cement ratio of 1.03, loaded at 28 days and stored in air of 70 per cent humidity are shown in Fig. 8. For specimens stored under water the deformations due to flow were much smaller.

The effect of this plastic flow in a reinforced concrete column is, under sustained load, to gradually relieve the compressive stress in the concrete until the yield point of the steel is reached. Tests of reinforced concrete columns, sponsored by the American Concrete Institute, made at Lehigh University and at the University of Illinois[2] have shown increases in the steel stress of from two to four times that obtained under the initial loading.

In beams and slabs the effect of the shrinkage and plastic flow is to move the neutral axis toward the tensile reinforcement,

[1] *Proc.*, A.C.I., vol. 27, p. 837, 1931.
[2] *Proc.*, A.C.I., vol. 27, pp. 677 and 761, 1931, and vol. 28, p. 157, 1932.

thus decreasing the stress in the concrete and increasing that in the steel.

In all cases the removal of the load shows some recovery or backward flow, which often continues for an appreciable period, but such values are always less than the deformations under the sustained load.

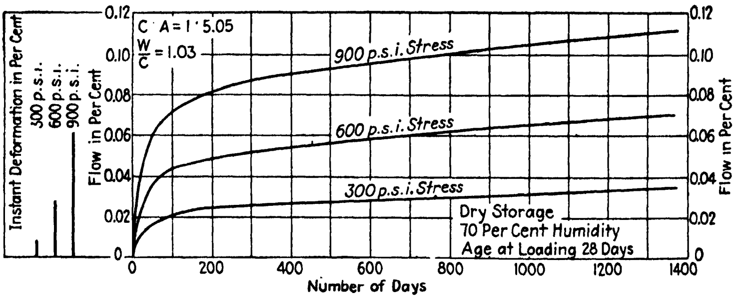


Fig. 8.

30. Contraction and Expansion. Concretes expand as the temperature is raised, and contract as the temperature is lowered. The coefficient of expansion per degree of temperature change increases somewhat with the richness of the mix, but the range of values is small. Tests made in the laboratories of Cornell University gave a range of values of from 0.00000677 for a 1:1½:3 concrete to 0.00000537 for a 1:3:6 concrete, with an average for all tests of 0.00000604. Other tests have shown a close agreement. The value generally used is 0.000006 per degree Fahrenheit.

Concretes expand in volume if kept wet or immersed in water, and contract if exposed to air. This property is not confined to freshly placed concrete but is characteristic of concretes of many years' service. A concrete which dries out in air may be expected to contract from 0.02 to 0.05 per cent, and when immersed in water may expand at least one-half of this amount.

This tendency to change in volume with different moisture conditions and changes in temperature does, of course, set up stresses of both tension and compression in a restrained reinforced concrete structure. The tensile stresses often exceed the amount that the concrete can sustain, and cracks result.

31. Bond. The adhesion of new concrete to work previously placed, becomes an important consideration in certain classes of construction. Very few tests of this property, however, have been made. Tests made by Hector St. George Robinson in 1912 indicate that a thorough cleaning of the old surface is beneficial to bond. When the old surface was roughened, cleaned, either treated with hydrochloric acid or coated with cement grout, a joint of about 80 per cent efficiency was obtained. Merely wetting the surface gave an efficiency of about 40 per cent, and wetting and roughening something more than 50 per cent. If the old concrete is not thoroughly wetted, it will draw the moisture from the new concrete, often leaving not enough in the latter for a normal consistency, and resulting in weak concrete near the joint as well as producing a joint of low efficiency. (For bond between concrete and steel see Art. 41.)

32. Weight. The weight of a concrete varies somewhat with the proportions of the mix, the consistency, and the character of the aggregate. The richer concretes are slightly heavier, and the wetter consistencies are lighter, except when cinders are used as the coarse aggregate. A stone or gravel concrete will usually weigh between 140 and 150 lb. per cu. ft., with an average of about 145 lb. per cu. ft. In reinforced concrete the steel adds from 3 to 5 lb. per cu. ft., and the weight of reinforced concrete (including the steel) is usually taken as 150 lb. per cu. ft. The weight of cinder concrete may be taken as 115 lb. per cu. ft.

Other Properties of Concrete

33. Resistance to Fire. Concrete is not only incombustible, but also a poor conductor of heat. Hence it is a splendid fire-resisting and fire-proofing material. Clay products and building stones are equally non-combustible, but they possess greater conductivity and a higher coefficient of expansion. The low coefficient of expansion lessens the tendency to crack when heated, and the low conductivity prevents the transference of the heat of the fire to the interior of the mass and to the reinforcing steel. Tests of conductivity have shown that when the surface of a mass of concrete is exposed for hours to a high heat, the

temperature at a depth of 1 in. beneath the surface is considerably lower, while at a depth of 3 in. or more the rise in temperature is very slight.

The low thermal conductivity of concrete is to a large extent due to voids in the material. Neat cement, with a void content about twice that of the average concrete, shows a corresponding decrease in its conductivity. It is also partly due to the absorption of the heat of vaporization by the water of combination in the hardened cement. The absorption of heat by the surface material as it becomes dehydrated retards the dehydration of the concrete beneath. The surface concrete which is injured by heat, but which remains in place, affords protection for the material farther in, as it is a poorer conductor than the original concrete.

The experience gained from some of the great fires, for example, those of Baltimore and the Edison Plant, etc., has shown that concrete exposed to high heat for a considerable length of time becomes calcined to a depth of from ¼ to ¾ in. but shows no tendency to spall off except at exposed corners and edges.

The Joint Committee specifies as follows for concrete covering over steel reinforcement:

"Metal reinforcement in fire-resistive construction shall be protected by not less than 1 in. of concrete in slabs and walls, and not less than 2 in. in beams, girders, and columns, provided aggregate showing an expansion not materially greater than that of limestone or trap rock is used; when impracticable to obtain aggregate of this grade, the protective covering shall be 1 in. thicker and shall be reinforced with metal mesh having openings not exceeding 3 in., placed 1 in. from the finished surface."

An idea of the severe test which concrete may be expected to pass as a fire-resisting material may be obtained from the following specification of the Building Code of the City of New York for fire-proof partition walls.

"A vertical panel of not less than 14 ft. long and 9 ft. high shall be subjected to a fire continuous for not less than 1 hour at an average temperature of 1700 deg. Fahrenheit during the latter half hour, followed by an application for not less than 2½ min-

utes of a hose stream from a $1\frac{1}{8}$-in. nozzle at 30 lb. nozzle pressure, without passage of flame during the test."

34. Weathering Qualities. The principal weathering agencies affecting the durability of concrete are variation in temperature, wind, rain, and variation in moisture conditions. Changes in temperature and moisture conditions cause more or less expansion and contraction in concretes, which in turn are apt to cause cracking that may result in ultimate failure. Cracking due to variations in temperature is likely to be confined principally to the surface of a structure, and may be made less harmful by the use of steel reinforcement so placed that a multitude of small cracks, which often are not visible to the naked eye, replace a few large and deep cracks. Expansion and contraction due to moisture changes are oftentimes more serious, as the moisture may penetrate the concrete farther and cause dangerous stresses to be introduced. The expansion and contraction of rich mixes are considerably more than those of the leaner mixtures, when moisture and temperature conditions vary. This circumstance is often responsible for the cracking off of a rich surface coat floated or plastered on a leaner base. The surface material not only tends to expand and contract more on account of its comparative richness, but it protects the underlying material from going through the extensive temperature and moisture changes which it itself is experiencing. To prevent the ultimate spalling off of this surface layer, as lean and as thin a surface coat as possible should be used, and where practicable, it should be applied before the leaner base has set so as to make the bond between the two as strong as possible.

35. Abrasive Resistance. The extensive use of concrete in the construction of roads, pavements, and floors makes its resistance to wear or abrasion an important consideration. In general, a concrete of high compressive strength will have a high resistance to abrasive action. Abrasion either wears away the cement and sand grains or it pulls the sand grains out of the cement matrix. It follows, therefore, that with soft aggregates more cement will be needed in order to keep the wear low, while with hard and durable aggregates just sufficient cement is needed

to hold the aggregate against the abrasive action. The quantity of mixing water used, however, the length of time of the mixing, and the curing conditions have more effect on the abrasive resistance than the hardness of the aggregate, and a good wearing surface can be produced with inferior aggregates if other conditions are favorable. Provided the proper precautions are taken, and a good quality concrete produced, the actual wear on the surface of either a pavement or a floor will not be of any serious amount, no matter how heavy the traffic.

CHAPTER II

GENERAL PROPERTIES OF REINFORCED CONCRETE

36. Types of Reinforcement. The reinforcing steel in reinforced concrete construction must be of such form and size that it easily may be incorporated as a part of the structure and provide sufficient surface to bond thoroughly together the two materials. In order to prevent the great concentration of stress at any point in the concrete, and in order to furnish sufficient area for bond strength, it is necessary to use the steel in comparatively small sections. With the small sections required, economy of manufacture requires the use of steel in the form of round or square bars. These vary in size from ¼ in. in diameter up to 1½ in. in diameter. Bars of all diameters are not always readily obtainable, and designers should confine their selections to the sizes manufactured by all bar companies. These are indicated in Table I (Appendix D). In order to insure prompt delivery the number of sizes and lengths of bars to be used on a job should be kept to a minimum. The following extras in cents per 100 lb. are standard with all mills for both round and square bars.

Size Extras

¾ in. and larger	Base
⅝ in.	10 cts.
½ in.	20 cts.
⅜ in.	40 cts.
¼ in.	$1.00

Lengths less than 10 ft. are subject to the following extras:

Lengths over 60 in. and less than 120 in.	5 cts.
Lengths 48 in. to 60 in. inclusive	10 cts.
Lengths 24 in. to 48 in. inclusive	15 cts.
Lengths 12 in. to 24 in. inclusive	30 cts.
Length 12 in. or less . . . not less than	40 cts.

Small quantities of the same shape and size are subject to the following extras:

Less than 2000 lb. to 1000 lb.	20 cts.
Less than 1000 lb.	50 cts.

There is also an extra charge for cutting less than 2000 lb. of any size to a specific length, whether or not the above extras apply. They are as follows:

Less than 2000 lb. to 1500 lb.	10 cts.
Less than 1500 lb. to 1000 lb.	20 cts.
Less than 1000 lb. to 500 lb.	40 cts.
Less than 500 lb.	60 cts.

Plain round and square bars are sometimes used, the necessary bond strength being furnished by the adhesion of the steel and concrete. Plain flat bars are not desirable, as the adhesion between them and the concrete is considerably less than for round or square bars. Deformed bars have been devised to furnish a bond between the concrete and steel in addition to the normal surface adhesion. This is accomplished by providing projections or depressions or both on the surface of the bar. Some deformed bars are so shaped that the area of the section is constant throughout the length, while others have considerable difference between sectional areas taken at different points. On most reinforced concrete work of the present day some form of deformed bar is used. Some of the common types of deformed bars in use are illustrated in Fig. 9.

Wire fabric and expanded metal in various forms are used to a considerable extent in slabs and other thin concrete sections. These types of reinforcement are easy to place, and since the metal is so well distributed in small sections, it is especially well adapted to resist the cracking likely to occur from changes in temperature and moisture conditions. Some of the forms of this type of reinforcement are shown in Fig. 10.

37. Grade of Steel. Three grades of steel[1] are in use for reinforcing bars, namely, structural steel, intermediate, and hard. Some authorities prefer that bars of the structural steel grade

[1] A.S.T.M. Standard A15-33.

only be used but the modern tendency is to use the intermediate grade. The brittleness of the hard or high-carbon steel is to be

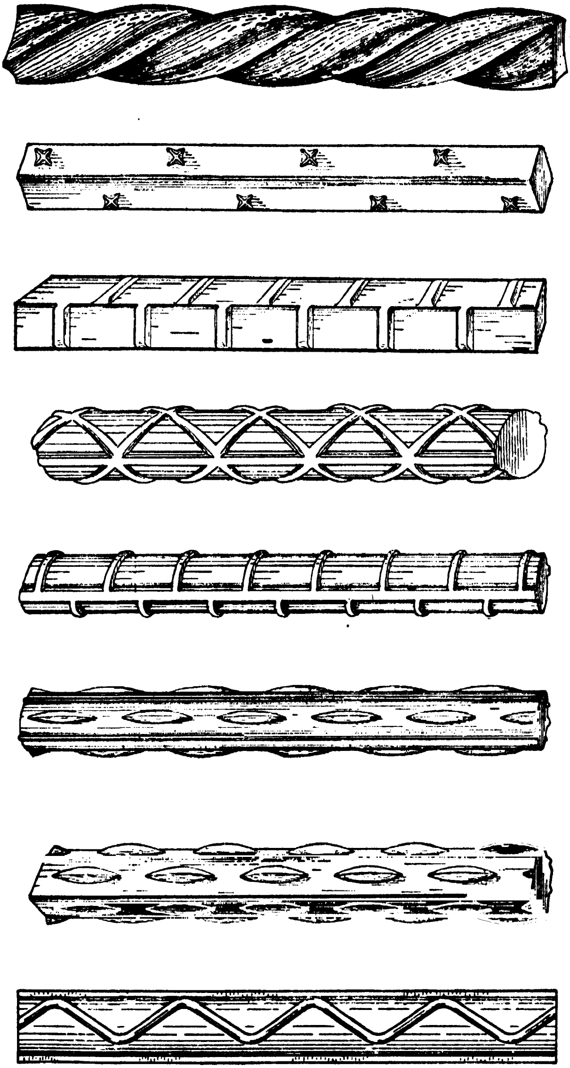


Fig. 9.

feared especially in light members subject to sudden impact stresses. High-carbon steel when used should be thoroughly

inspected and tested in order to prevent brittle or cracked material from being used in the completed structure. Structural

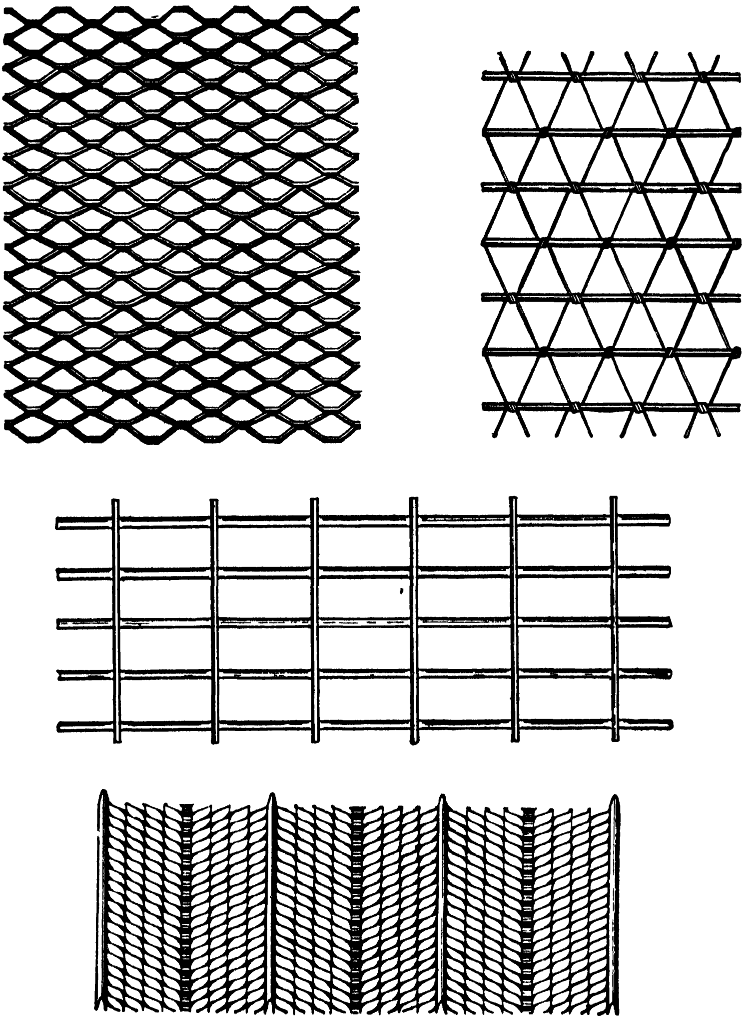


FIG. 10.

steel should have an ultimate strength of from 55,000 to 70,000 p.s.i. The intermediate grade should range from 70,000 to

90,000 p.s.i., and the hard grade should have an ultimate strength of 80,000 p.s.i. or greater.

38. Coefficient of Expansion. The coefficient of expansion of steel is approximately 0.0000065 per degree Fahrenheit, and this value may be used in all design.

39. Modulus of Elasticity. The modulus of elasticity of all grades and kinds of steel is nearly the same, and may be taken as 30,000,000 p.s.i.

40. Advantages of Concrete and Steel in Combination. Since concrete is only about one-tenth as strong in tension as in compression, it cannot be used economically by itself for the construction of any member sustaining or likely to sustain flexural stresses. Its compressive strength is sufficiently high to be of structural importance, and it is a good fire-proof material; it is durable, and materials for its manufacture can be obtained in almost any locality.

Steel, on the other hand, when not embedded in concrete, cannot withstand successfully great heat and is subject to corrosion. Its tensile strength is high in almost any shape of section. To resist compression by itself it must be made in forms of less concentrated cross-section than the bar, in order to have lateral rigidity.

When the two materials are so arranged in a structural member, subject to both tension and compression, that the steel will resist the tension and the concrete the compression, the greatest advantage over other types of construction occurs. The member is more fire-proof than one constructed of steel or timber alone, and is often more economical. The steel is used in its cheapest form, the bar, and protected by its covering of concrete. In compression members such as columns, the use of the steel is not so economical, but a reinforced concrete column is again the most permanent and fire-proof construction obtainable. Columns of structural steel incased in concrete may be as durable, but the initial cost is greater. Plain concrete columns are not safe construction on account of the possible bending or shearing forces which may develop.

41. Bond between the Concrete and the Steel. All reinforced concrete construction is based on the assumption that the two materials are thoroughly bonded together. The high value of the adhesion of concrete to steel rods embedded in it was known long before the days of reinforced concrete, and use of this property was made in anchor bolts, rods, etc. Most of the tests of bond have been made by embedding a short reinforcing bar in a block or cylinder of concrete and pulling it out in a testing machine. In such tests the concrete surrounding the bar is in compression, and the conditions do not correspond to those ordinarily existing in beams or slabs. Other tests have been made with the two bars embedded, one in each end, of a concrete cylinder, and tension applied to each rod to determine the bond stress. Still other tests have been made with rods embedded in small reinforced concrete beams, the middle portion of the rods being left exposed. The results of tests of different types seem to show that a correct value of the bond resistance can be obtained by properly made tests of the simple kind first mentioned.

From an extensive series of bond tests made at the University of Illinois[1] conclusions were reached as follows:

Bond between concrete and steel may be divided into two principal elements, adhesive resistance and sliding resistance. The source of adhesive resistance is not known, but its presence is a matter of universal experience with materials of the nature of mortar and concrete. Sliding resistance arises from inequalities of the surface of the bar and irregularities of its section and alignment together with the corresponding conformations in the concrete. The adhesive resistance must be overcome before sliding resistance comes into action. In other words, the two elements of bond resistance are not effective at the same time at a given point. Many evidences of the tests indicate that adhesive resistance is much the more important element of bond resistance.

Pull-out tests with plain bars show that a considerable bond stress is developed before a measurable slip occurs. After the adhesive resistance is overcome, a further slip without an opportunity of rest is accompanied by a rapidly increasing bond

[1] *Bull.* 71, Engineering Experiment Station, University of Illinois.

stress until a maximum bond resistance is reached at a definite amount of slip.

Pull-out tests with plain round bars show end slip to begin at an average bond stress equal to about one-sixth the compressive strength of 6-in. cubes from the same concrete; the maximum bond resistance is equal to about one-fourth the compressive strength of 6-in. cubes. These values were about the same for a wide range of mixes, ages, and conditions of storage. In terms of the compressive strength of 8- by 16-in. concrete cylinders these values would be about 13 per cent for first end slip and 19 per cent for the maximum bond resistance.

The tests indicate that bond stress is not uniformly distributed along a bar embedded any considerable length and having the load applied at one end. Slip of bar begins first at the point where the bar enters the concrete, and the bond stress must be greater here than elsewhere until a sufficient slip has occurred to develop the maximum bond resistance at this point. Slip of bar begins last at the free end of the bar. After slip becomes general, there is an approximate equality of bond stress throughout the embedded length.

The maximum bond resistance was not materially different for bars of different diameters. Rusted bars gave bond resistances about 15 per cent higher than similar bars with ordinary mill surface. The tests with flat bars showed wide variations of bond resistance and were not conclusive. Square bars gave values of unit stress about 75 per cent of those obtained with plain round bars.

Adhesive resistance must be destroyed, sliding resistance largely overcome, and the concrete ahead of the projections must undergo an appreciable compressive deformation before the projections on a deformed bar become effective in taking bond stress. The tests indicate that the projections do not materially assist in resisting a force tending to withdraw the bar until a slip has occurred approximating that corresponding to the maximum sliding resistance of plain bars. As slip continues a larger and larger portion of the bond stress is taken by direct bearing of the projections on the concrete ahead.

A working bond stress equal to 4 per cent of the compressive strength of the concrete tested in the form of 8- by 16-in. cylinders at the age of 28 days (equivalent to 100 p.s.i. in concrete having a compressive strength of 2500 p.s.i.) is as high a stress as should be used. This stress is equivalent to about one-third that causing the first slip of bar and one-fifth the maximum bond resistance of plain round bars as determined from pull-out tests. The use of deformed bars of proper design may be expected to guard against local deficiencies in bond resistance due to poor workmanship and their presence may properly be considered as an additional safeguard against ultimate failure by bond. It does not seem wise, however, to place the working bond stress for deformed bars much higher than that used for plain bars.

The Joint Committee recommends a bond stress of 0.04 of the ultimate compressive strength for plain bars and 0.05 for deformed bars.

42. Length of Embedment of Reinforcing Bars to Develop Full Strength in Bond.

Let

f_s = allowable unit tensile stress in the steel.
A_s = the area of the bar.
o = the circumference or perimeter of bar.
i = diameter or thickness of bar.
u = allowable unit bond stress between the concrete and the steel.
l_1 = required length of embedment.

For any bar $$l_1ou = A_sf_s$$

For round bars $$\pi l_1iu = \frac{\pi i^2 f_s}{4}$$

For square bars $$4iul_1 = i^2fs$$

For round or square bars $$l_1 = \frac{f_si}{4u}$$

43. Reinforced Concrete in Tension. Early tests of reinforced concrete seemed to indicate that the ultimate strength in tension was far greater than that of plain concrete. This was

due to the fact that the bond between the concrete and the steel causes a uniform stretching of the concrete, and the cracks which occur when the concrete is stressed are so numerous and minute as to be difficult to detect when they first begin to open up, and do not become visible until a stretching occurs corresponding to a tensile stress much greater than the ultimate strength of concrete.

A reinforced concrete beam carrying its design load is more heavily stressed on the tension side than the ultimate strength of plain concrete, provided enough steel is embedded on the tension side to develop the full allowable compressive strength of the concrete. The presence of the cracks above referred to, therefore, greatly decrease the tension that can be taken by the concrete, and most moment formulas now in use for the design of reinforced concrete beams neglect entirely the tensile strength of the concrete.

The effect of temperature and moisture changes on plain concrete is discussed in Arts. 15 and 34. If a structure having a large area of exposed surface is restrained by outside forces, these changes cause stresses to be set up in the concrete which will in turn cause cracks to appear on the exposed surface. In order to prevent the appearance of large and unsightly cracks, such surfaces should be reinforced with sufficient steel (generally about 0.25 per cent of the cross-section of the concrete) to cause the stretching due to the tension in the concrete to be distributed uniformly over the whole surface, and thus make the cracks so numerous as to be invisible.

CHAPTER III

BEAMS AND SLABS

44. Stresses in Homogeneous Beams. Reinforced concrete beams are non-homogeneous members in that they are made of two entirely different materials. The formulas that are used in the analysis of reinforced concrete beams are therefore different from those that are used in the design or investigation of beams composed entirely of steel, wood, or any other structural material. The fundamental principles involved in the derivation of these formulas are, however, essentially the same as those relating to homogeneous beams, although many modifications are required before definite applications can be made. Briefly, these fundamental principles are as follows:

1. At any cross-section there exist internal forces which may be resolved into components normal and tangential to the section. Those components which are normal to the section are stresses of tension and compression; their function is to resist the bending moment at the section. The tangential components added together constitute a stress known as the resisting shear.

2. The neutral axis passes through the center of gravity of the cross-section.

3. The intensity of stress normal to the section increases directly with the distance from the neutral axis, and is a maximum at the extreme fiber. The intensity of stress at any given point in the cross-section is represented by the equation

$$f = \frac{My}{I}$$

in which f = the unit fiber stress at a distance y in. from the neutral axis.

M = the external bending moment at the section in inch-pounds.

I = the moment of inertia of the cross-section about the neutral axis in biquadratic inches.

4. The longitudinal shear in pounds per square inch (v) at any point in the cross-section is given by the equation

$$v = \frac{VQ}{Ib}$$

in which V = the total shear at the section in pounds.

Q = the statical moment about the neutral axis of that portion of the cross-section lying between an axis through the point in question parallel to the neutral axis, and the nearest face (upper or lower) of the beam in inches cubed.

I = the moment of inertia of the cross-section about the neutral axis in biquadratic inches.

b = the width of the beam at the given point in inches.

The statical moment mentioned above is the product of the area of the portion considered and the distance of its center of gravity from the neutral axis.

5. In a beam with constant cross-section, the maximum values of f and v will occur where M and V, respectively, are a maximum.

6. At any point in the beam there exists a vertical shear, the intensity of which is equal to that of the longitudinal or horizontal shear.

7. The intensity of shear (horizontal and vertical) along a vertical cross-section in a rectangular beam varies as the ordinates of a parabola, the intensity being zero at the top and bottom of the beam and a maximum at the neutral axis (see Fig. 26). The maximum is one and one-half times the average intensity or $\frac{3}{2} \times \frac{V}{ba}$, since at the neutral axis $Q = \frac{ba^2}{8}$ and $I = \frac{ba^3}{12}$, in equation $v = \frac{QV}{Ib}$.

8. Owing to the action of shearing forces (horizontal and vertical) and flexure stresses, at any point in a beam there are inclined stresses of tension and compression, the maximum values

of which form an angle of 90 degrees with each other. The intensity of the inclined stress at any point is given by the equation

$$t = \frac{f}{2} \pm \sqrt{\frac{f^2}{4} + v^2}$$

in which f = the intensity of horizontal fiber stress.

v = the intensity of vertical or horizontal shearing stress at the point.

The inclined stress makes an angle α with the horizontal of such an amount that $\tan 2\alpha = \frac{2v}{f}$.

9. Since the horizontal and vertical shearing forces are equal and the flexural stresses are zero at the neutral plane, the inclined tensile and compressive forces at any point in that plane form an angle of 45 degrees with the horizontal, the intensity of each being equal to the unit shear at the point. At the end of a simply supported beam where the bending moment is zero, these stresses act at practically 45 degrees with the horizontal for the entire depth of the beam. Since the shear is zero at the point of maximum moment, the stresses there are horizontal.

45. Assumptions in the Theory of Flexure. The common theory of flexure assumes:

1. A plane cross-section before loading remains a plane cross-section after loading.

2. The stress is proportional to the deformation.

The first of these two assumptions implies that the unit deformations of the fibers at any section are proportional to their distances from the neutral axis, and the second that the unit stresses in the fibers vary as the distances of the fibers from the neutral axis.

The common theory of flexure does not apply for wide ranges of stress. In the design of structures, however, the stresses used are only a comparatively small percentage of the ultimate, and the errors in the above assumptions are small and on the side of safety. For stresses in excess of those commonly used in design, the relation between stress and deformation is not constant; the stress-deformation diagram for such cases assumes more nearly the form of a parabola.

In the following discussions, a straight-line variation between stress and deformation is assumed. Furthermore, the tensile strength of the concrete is neglected.

46. Plain Concrete Beams. Plain concrete beams are inefficient as flexural members since failure on the tension side of the beam occurs when but a small portion of the ultimate compressive strength of the concrete has been developed on the compression side of the beam. The resisting moment of a plain concrete beam may be expressed by the equation

$$M = \frac{f_t I}{c}$$

in which f_t = the working unit stress of concrete in tension.
c = the distance from the neutral axis to the extreme tension fiber.

For a rectangular section of width b and depth a, $I = \frac{1}{12}ba^3$ and $c = \frac{a}{2}$, so that $\frac{I}{c} = \frac{ba^2}{6}$.

Illustrative Problems. I. How great a moment can be developed by a plain rectangular concrete beam whose cross-section is 8 × 14 in. if the safe working stress of concrete in tension is 125 p.s.i.?

$$M = \frac{f_t I}{c} = \frac{f_t ba^2}{6} = \frac{125 \times 8 \times 14^2}{6} = 32{,}670 \text{ in.-lb.}$$

II. A simply supported plain concrete beam with a span of 8 ft.-0 in. is to support a uniform live load of 200 lb. per lin. ft. Determine the size of beam required. The allowable extreme fiber stress is 125 p.s.i.

Assume the weight of the beam as 115 lb. per lin. ft. The total load is then 315 lb. per lin. ft.

$$M = \tfrac{1}{8} \times 315 \times 8^2 \times 12 = 30{,}300 \text{ in.-lb.}$$

Since $M = f_t(\tfrac{1}{6}\, ba^2)$,

$$ba^2 \text{ (required)} = \frac{6 \times 30{,}300}{125} = 1460 \text{ in.}^3$$

If b = 8 in., a (required) = 13.5 in. The weight of the beam is 113 lb. per lin. ft., which agrees closely with the assumed weight.

Rectangular Beams with Tension Reinforcement

47. Flexure Formulas. The tensile and transverse strength of plain concrete is very low and unreliable (see Art. 43), and its practical uses are limited to structures or parts of structures in which no tensile stresses are induced, *i.e.*, to arches, piers, and certain massive constructions. In order to make concrete available for use in structural members involving tension, such as beams, for example, steel bars are embedded in the tension side of the beam. It is, of course, assumed that the bars are embedded so that the union between the steel and concrete is

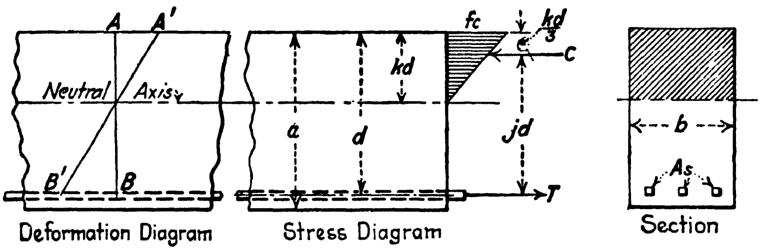


Fig. 11.

sufficient to make the two materials act as one. The purpose of the steel is to carry the tensile stresses. The concrete sustains the compressive and shearing stresses, because its resistance to these is comparatively large.

Figure 11 represents a portion of a rectangular reinforced concrete beam. Let *AB* represent any cross-section before the load is applied to the beam, and *A'B'* the same cross-section after the load is applied. The upper fibers of the beam (the compression fibers) will tend to shorten, and the lower fibers will lengthen. According to Assumption 1, Art. 45, the deformation at any horizontal plane in the beam is proportional to the distance from that plane to the neutral axis. With proper interpretation, the distance *AA'* may be considered to represent the shortening of the extreme upper fibers for a unit length of beam, and *BB'* the unit elongation of the steel. In the stress diagram, f_c represents the unit compressive stress in the extreme fiber at

the section AB. The total compression $C = (\frac{1}{2}f_ckd)b = \frac{1}{2}f_ckbd$, and the total tension, neglecting that in the concrete, is $T = A_sf_s$, in which f_s is the unit stress in the steel and A_s the cross-sectional area of the steel. For the general notation used in the following discussion, see Appendix A.

For equilibrium, the total compressive resistance of a beam must equal the total tensile resistance. From Fig. 11,

$$\tfrac{1}{2}f_ckbd = A_sf_s \tag{a}$$

From the assumption that deformations vary as the distance from the neutral axis,

$$\frac{AA'}{BB'} = \frac{kd}{d - kd} \tag{b}$$

Since $E = \dfrac{\text{unit stress}}{\text{unit deformation}}$, it follows that

$$AA' = \frac{f_c}{E_c} \quad \text{and} \quad BB' = \frac{f_s}{E_s}$$

Hence

$$\frac{AA'}{BB'} = \frac{E_s}{E_c} \cdot \frac{f_c}{f_s} = \frac{nf_c}{f_s} \tag{c}$$

Equating (*b*) and (*c*),

$$\frac{nf_c}{f_s} = \frac{kd}{d - kd} \tag{d}$$

from which

$$f_s = \frac{nf_c(1 - k)}{k} \tag{1}$$

or

$$f_c = \frac{f_sk}{n(1 - k)} \tag{1a}$$

Equations (1) and (1*a*) give the relation between the actual simultaneous stresses in the steel and the concrete in any beam at any stage of loading, provided the value of k is known. The equation for k is derived in the following paragraph.

The actual ratio of the area of the steel to the effective cross-sectional area of the concrete is

$$p = \frac{A_s}{bd}$$

Hence, from equation (*a*),

$$k = \frac{pf_s}{\frac{1}{2}f_c} \tag{e}$$

Substituting in equation (*e*) the value of f_s from equation (1)

$$k = \frac{2pn(1 - k)}{k}$$

Solving for k,

$$k = \sqrt{2pn + (pn)^2} - pn \tag{2}$$

This value of k is independent of the unit stresses in the steel and concrete but is dependent upon the proportion of steel in the beam and the ratio of the moduli of elasticity of the two materials. It is to be used in reviewing, *i.e.*, in calculating unit stresses or resisting moments of a beam whose dimensions and amount of reinforcement are known.

From Fig. 11, the lever arm jd of the internal stress couple is

$$jd = d - \frac{kd}{3}$$

hence

$$j = 1 - \frac{k}{3} \tag{3}$$

The resisting moment of a beam is dependent upon the strength of either the steel or the concrete. The resisting moment of each is equal to the total stress in each, *i.e.*, compression in concrete and tension in steel, multiplied by the lever arm jd of the couple, or

$$M_c = (\tfrac{1}{2}f_ckbd)jd = \tfrac{1}{2}f_ckjbd^2 \tag{4}$$

$$M_s = A_sf_sjd \tag{5}$$

Since $A_s = pbd$, equation (5) may also be written as follows:

$$M_s = pf_sjbd^2 \tag{5a}$$

The actual internal moments as expressed by equations (4) and (5) are each equal to the external bending moment at all stages of loading (for equilibrium), but if the maximum allowable

value of the resisting moment of the concrete, M_c, is reached before that of the steel, M_s, it means that the beam will be overstressed on the compression side before the maximum allowable fiber stress in the steel is reached; *i.e.*, the beam has more steel than is theoretically required—it is overreinforced.

In designing a reinforced concrete beam, it is desirable to place in the beam an amount of steel such that the limiting unit stresses or limiting resisting moments as expressed by equations (4) and (5) or (4) and (5*a*) shall be reached simultaneously. If this ideal steel ratio is obtained

$$M_c = M_s = \tfrac{1}{2} f_c kjbd^2 = A_s f_s jd = pf_s jbd^2 \qquad (f)$$

or

$$M = Kbd^2 \qquad (6)$$

in which $K = \tfrac{1}{2} f_c kj$ or $pf_s j$

If the ratio $\dfrac{f_s}{f_c} = r$, equation (*d*) reduces to the form $\dfrac{n}{r} = \dfrac{k}{1-k}$, and solving for k,

$$k = \frac{n}{n+r} \qquad (7)$$

This value of k depends only upon the unit stresses in the steel and in the concrete and upon the value of the ratio n. Therefore *equation* (7) *cannot be used in review,* since the *simultaneous* values of f_s and f_c are not known.

An expression for the ideal steel ratio may be obtained as follows: Since with this ideal percentage of steel $\tfrac{1}{2} f_c kj = pf_s j$, (equation f), it follows that $p = \dfrac{k}{2r}$ and since for any given values of f_s and f_c, $k = \dfrac{n}{n+r}$, the equation for p becomes

$$p = \frac{n}{2r(n+r)} \qquad (8)$$

The values of M_c and M_s will be equal to each other only when the amount of steel placed in the beam is such that the actual steel ratio, $p = \dfrac{A_s}{bd}$, is equal to the value given by equation (8).

On account of the commercial sizes of reinforcing steel in use, the actual ratio will usually be greater or less than the ideal. In the former case M_s will be greater than M_c and the strength of the beam will be limited by that of the concrete. For under-reinforced beams, in which the actual steel ratio is less than the ideal ratio, the reverse will be true.

48. Application of Equations to Design and Review Problems. The equations previously developed are summarized below.

$$f_s = \frac{nf_c(1 - k)}{k} \tag{1}$$

$$f_c = \frac{f_s k}{n(1 - k)} \tag{1a}$$

$$k = \sqrt{2pn + (pn)^2} - pn \text{ (review)} \tag{2}$$

$$j = 1 - \frac{k}{3} \tag{3}$$

$$M_c = \tfrac{1}{2} f_c k j b d^2 \tag{4}$$

$$M_s = A_s f_s j d \tag{5}$$

$$= p f_s j b d^2 \tag{5a}$$

$$M = K b d^2 \tag{6}$$

$$k = \frac{n}{n + r} \text{ (design only)} \tag{7}$$

$$p = \frac{n}{2r(n + r)} \tag{8}$$

To design a beam, either equation (4) or (6) may be used to determine the cross-section required to insure against crushing of the concrete under any given bending moment, and equation (5) to determine the area of steel necessary to develop the full strength of the concrete in compression. Values of k and j are obtained from equations (7) and (3).

Economic and constructional considerations are usually best served when the cross-section of a rectangular beam is so proportioned that b is from one-half to three-quarters of d. Construction limitations make it undesirable to select d in multiples of less than ½ in., and sometimes 1-in. multiples are used. Also, in order to keep the amount of mill or carpenter work as small as possible, the width of beam b should be chosen so that a plank

of standard width may be used for the bottom form. In fulfilling this requirement, most designers consider the nominal width of the plank and hence proportion all beams for widths of even integral inches. Some designers, however, prefer to consider the actual width of the lumber available after dressing and proportion their beams accordingly. The nominal width is used in all of the problems in this text.

To determine the resisting moment of a given beam of known dimensions and reinforcement, equations (4) and (5) or (4) and (5*a*) should be solved for M_c and M_s; the smaller value is the required moment. To determine the maximum unit stresses f_s and f_c in a beam of known dimensions and condition of loading, equations (4) and (5), or (4) and (1), or (5) and (1*a*) can be used, substituting for M_s or M_c the maximum external bending moment M. In any review problem, values of k and j are obtained from equations (2) and (3).

The value of the external bending moment varies according to the method of supporting the beams and the type of loading. For example, a simply supported beam, *i.e.*, one resting on two supports, one at each end, and not restrained in any way, with a uniformly distributed load, may be assumed as having a moment equal to $\frac{1}{8}wl^2$; a partially continuous beam (continuous over one support only), with the same type of loading $\frac{1}{10}wl^2$; and a fully continuous beam (continuous over two or more supports) $\frac{1}{12}wl^2$, in which w = the load per unit of length and l = the span. For other loadings and methods of support see Chap. VI.

The span length l of freely supported beams and slabs is generally taken as the distance between the centers of supports, but need not exceed the clear span plus the depth of the beam or slab. The span length for continuous or restrained members built monolithically with the supports is often considered as the clear distance between faces of supports. Many designers use the distance between the centers of supports as the effective span length, for both continuous and simply supported beams.

The above equations and methods refer to flexural stresses only, (tension and compression) and do not provide for the shearing stresses that exist in the beam. These are considered separately in Arts. 67 to 84.

49. Placing the Reinforcement. In placing the reinforcement, three general requirements must be fulfilled. First, there must be sufficient space between the rods to permit proper placing of the concrete around them; second, there must be sufficient concrete in the plane of the rods properly to transmit the stresses of tension and shear; third, there must be sufficient concrete below the steel to afford ample protection for the steel against moisture and fire damage.

The Standard Building Code[1] of the American Concrete Institute (hereafter referred to as the Joint Code) specifies that the clear distance between parallel bars shall be not less than: (*a*) 1 in.; (*b*) 1½ times the diameter for round bars or 2 times the side dimension for square bars; (*c*) 1⅓ times the maximum size of the coarse aggregate.

The Joint Code also specifies that the concrete protective covering for reinforcement at surfaces not exposed directly to the ground or weather shall be not less than ¾ in. for slabs and walls and not less than 1½ in. for beams, girders, and columns. If the concrete surface is to be exposed to the weather or in contact with the ground, a protective covering of at least 2 in. is required, except that if the concrete is to be poured in direct contact with the ground, without the use of forms, a covering of at least 3 in. must be furnished.

In general, the centers of bars in beams should be at least 2½ in. from the bottom surface of the beam, in order to furnish at

[1] The Standard Building Code refers to the report of Committee 318 (the Standard Building Code Committee, formerly 501) of the American Concrete Institute. This committee presented a report at the 32nd Annual Convention of the Institute on Feb. 25, 1936, containing tentative building regulations for reinforced concrete. This report was subsequently approved as a tentative standard, No. 501-36T. The Committee, cooperating with the Committee on Engineering Practice of the Concrete Reinforcing Steel Institute, presented a revised report in 1940, containing revisions based upon several years of intensive study of the numerous advances in both design and construction that had been developed subsequent to the presentation of the original report. This revised report was returned to the Committee for further consideration of some of the recommendations in the report. Text references to the Standard Building Code (the Joint Code) quote from this revised report, except for two-way slabs and flat-slab floors.

least $1\frac{1}{2}$ in. of clear insulation below the bars and the stirrups that are used for web reinforcement (see Art. 75). If no stirrups are required, this distance from the center of the bar to the surface can be made 2 in., provided that the bars are not larger than 1 in. in size. In slabs, 1 in. to the center of the bars is ordinarily sufficient to give the required $\frac{3}{4}$-in. insulation. These conditions are illustrated in Fig. 12. Although the distances $2\frac{1}{2}$, 2, and 1 in., shown in Fig. 12, are not the exact distances required

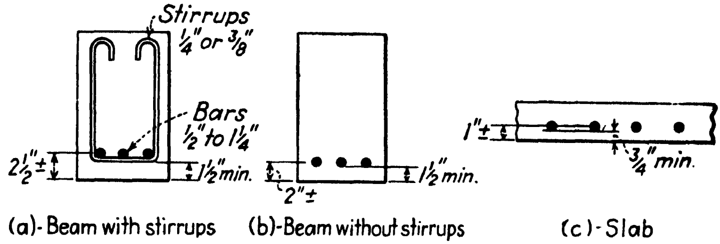


FIG. 12.

to furnish the clear insulation specified in the code, they will in practically all cases satisfy the minimum requirements and are sufficiently exact for all design purposes. Total depths of beams should be taken in multiples of not less than $\frac{1}{2}$ in., and preferably 1 in., so that effective depths (d) will also be in multiples of $\frac{1}{2}$ in., or 1 in., if the above-mentioned distances to the centers of the bars are used.

50. Allowable Unit Stresses. The allowable unit working stresses in the concrete, as specified in the Joint Code, are given in Appendix B. These stresses are given as a percentage of the ultimate compressive strength, f'_c, of the concrete at the age of 28 days (see Art. 23). Thus, the safe working unit stress in flexure is $0.40f'_c$* which gives for 2500-lb. concrete, a value of 0.40×2500, or 1000 p.s.i. A somewhat lower unit stress in flexure is specified in many municipal building codes. To emphasize this fact, some of the following problems are solved with assumed working stresses which do not agree with the recommendations as given in Appendix B. Where such exception is taken, the assumed allowable unit stresses are given in the data of the problem.

* A proposed revision of the Code (1941) will increase this to $0.45f'_c$.

Concretes used in ordinary building construction are proportioned so as to develop ultimate compressive strengths varying usually from 2500 to 3500 p.s.i. The lower strength concretes are normally used for the floor systems and for footings, while the higher strength concretes are used for the columns. In special cases, 3750-lb. or greater strength concretes might be used.

51. *Illustrative Problems.* I. A rectangular reinforced concrete beam has a total cross-section of 12 × 24 in. and a length of 20 ft.-0 in. It is reinforced with four ⅞-in. round bars in one row, the centers of the bars being 2½ in. above the lower surface of the beam. Assuming a 2500-lb. concrete structural-grade steel, and following the specifications of the Joint Code, Appendix B, what is the resisting moment of the beam?

$$p = \frac{A_s}{bd} = \frac{2.41}{12 \times 21.5} = 0.0093$$

$$k = \sqrt{2 \times 0.0093 \times 12 + (0.0093 \times 12)^2} - 0.0093 \times 12 = 0.375$$

$$j = 1 - \frac{0.375}{3} = 0.875$$

From equations (4) and (5),

$$M_c = \tfrac{1}{2} \times 1000 \times 0.375 \times 0.875 \times 12(21.5)^2 = 911{,}000 \text{ in.-lb.}$$

$$M_s = 2.41 \times 18{,}000 \times 0.875 \times 21.5 = 818{,}000 \text{ in.-lb.}$$

The beam is therefore underreinforced, the strength of the steel governs, and the resisting moment is 818,000 in.-lb.

II. Use the beam of the preceding problem, and compute the value of the unit stress in the steel (f_s) and in the concrete (f_c), if a uniform live load of 1000 lb per lin ft is placed upon it

$$\text{Weight of the beam} = \frac{12 \times 24}{144} \times 150 = 300 \text{ lb. per ft.}$$

$$\text{Total load} = 1000 + 300 = 1300 \text{ lb. per ft.}$$

$$M = \tfrac{1}{8} \times 1300 \times 20^2 \times 12 = 780{,}000 \text{ in.-lb.}$$

From Problem I, $k = 0.375$, and $j = 0.875$.

Substituting in equation (5),

$$780{,}000 = 2.41 \times f_s \times 0.875 \times 21.5$$

$$f_s = 17{,}200 \text{ p.s.i.}$$

From equation (1*a*),

$$f_c = \frac{17{,}200 \times 0.375}{12(1 - 0.375)} = 860 \text{ p.s.i.}$$

The value of f_c could also have been obtained from equation (4), as follows.

$$780{,}000 = \tfrac{1}{2} \times f_c \times 0.375 \times 0.875 \times 12 \times (21.5)^2$$
$$f_c = 860 \text{ p.s.i.}$$

III. If the beam of Problem I were to support a single concentrated load at the mid-span, what would be the maximum safe load that could be so placed?

From the solution of Problem I, the resisting moment is 818,000 in.-lb. The weight of the beam is 300 lb. per ft., and the moment caused by this dead load is $\frac{1}{8} \times 300 \times 20^2 \times 12 =$ 180,000 in.-lb. The maximum moment available for the concentrated load is 818,000 − 180,000 = 638,000 in.-lb. The maximum moment caused by the concentrated load is $M = \frac{1}{4}Pl$, hence

$$\tfrac{1}{4}P \times 20 \times 12 = 638{,}000$$

from which

$$P \text{ (Maximum)} = 10{,}630 \text{ lb.}$$

It should be noted that if the concentrated load were not at the mid-span, the maximum moments due to the dead load and the concentrated load would not occur in the same cross-section. If the concentrated load were near the mid-span, it is very probable that the section of maximum moment would coincide with the section at which the concentrated load was placed. The dead-load moment *at this section*, instead of the maximum dead-load moment, should be deducted from the resisting moment of the beam in order to obtain the maximum moment available for the concentrated load (see Problem III, Art. 53).

IV. Determine the cross-section of concrete and area of steel required for a simply supported rectangular beam with a span of 18 ft.-0 in. which is to carry a uniform live load of 1000 lb. per lin. ft. A 2500-lb. concrete is to be used, and the allowable

unit stresses are to be as specified in the Joint Code. The allowable unit stress in the steel is 18,000 p.s.i.

Assuming that the weight of the beam is 200 lb. per lin. ft., the total load to be carried is 1200 lb. per lin. ft., and the actual external bending moment is

$$M = \tfrac{1}{8} \times 1200 \times 18^2 \times 12 = 583{,}200 \text{ in.-lb.}$$

$$r = \frac{18{,}000}{1000} = 18.0$$

$$k = \frac{12}{12 + 18.0} = 0.400$$

$$j = 1 - \frac{0.400}{3} = 0.867$$

Substituting in equation (4),

$$583{,}200 = \tfrac{1}{2} \times 1000 \times 0.400 \times 0.867 \times bd^2$$

from which

$$bd^2 = 3370 \text{ in.}^3$$

Let b be taken as 10 in.; then d (required) = 18.4 in., and d (selected) = 19 in. Adding 2 in. below the center of the steel, the total cross-section is 10 × 21 in., and the weight of the beam is 220 lb. per lin. ft.

The actual weight of the beam does not agree with the assumed value. Hence it is necessary to check back to see whether any revision should be made in the design. The revised bending moment is

$$M = \tfrac{1}{8} \times 1220 \times 18^2 \times 12 = 593{,}000 \text{ in.-lb.}$$

$$593{,}000 = \tfrac{1}{2} \times 1000 \times 0.400 \times 0.867 \times bd^2$$

$$bd^2 = 3420 \text{ in.}^3$$

With b = 10 in., d = 18.5 = 19 in. Since these results agree with those assumed in the revision, the design is satisfactory.

With these values of b and d, the required area of steel is obtained from equation (5) as follows:

$$593{,}000 = A_s \times 18{,}000 \times 0.867 \times 19$$

$$A_s = 2.00 \text{ sq. in.}$$

Two 1-in. square bars, area 2.00 sq. in., are selected.

If the ideal steel ratio had been used in determining the quantity of steel to be placed in the beam, the procedure would have been as follows:

From equation (8),

$$p = \frac{12}{2 \times 18.0(12 + 18.0)} = 0.0111$$

and

$$A_s = 0.0111 \times 10 \times 19.0 = 2.11 \text{ sq. in.}$$

This, it will be noticed, is slightly in excess of the required area as determined by the first method. This difference may be accounted for as follows: The value of A_s, as computed by this latter method, represents an area of steel that will develop in tension the full compressive strength of a beam whose effective dimensions are 10×19 in. But in order fully to develop a moment of 593,000 in.-lb. as required by the problem, an effective cross-section of only 10×18.5 in. was needed. The value of $d = 19.0$ in. was selected to simplify the dimensioning of the plans. The development of the full strength of a 10- $\times$ 19-in. beam therefore furnishes an excess of steel over that required to provide for the maximum external bending moment in the beam, and hence is a waste of material. In this case the two 1-in. square bars would not be sufficient. A greater difference between the theoretical value of d required and that furnished would emphasize this to a greater extent. *The latter method involving the ideal steel ratio is therefore not recommended for general use.*

ADDITIONAL PROBLEMS

1. A simply supported rectangular beam has a total cross-section of 10×16 in. and a length of 20 ft.-0 in. It is reinforced with four ⅝-in. round bars in one row. The distance from the centers of the bars to the lower surface of the beam is 2½ in. With 2500-lb. concrete and intermediate grade steel, what is the resisting moment of the beam?

2. If a concentrated load of 3500 lb. were placed on the beam of Problem 1, at a distance of 7 ft.-0 in. from the support, what would be the maximum unit stress in the concrete and the maximum unit stress in the steel?

3. A simply supported rectangular beam with a span of 18 ft.-0 in. supports a uniform live load of 975 lb. per lin. ft. and a concentrated load of 3000 lb. at the middle of the span. With $f'_c = 3000$ p.s.i. and with intermediate-grade reinforcing steel, determine the required cross-section and steel area.

4. A simply supported rectangular beam with a span of 17 ft.-0 in. supports a live load which varies in amount from zero at the left support uniformly to an amount of 1000 lb. per lin. ft. at the right support. A 2500-lb. concrete and structural-grade reinforcing bars are to be used. Design the beam.

5. A rectangular beam, simply supported, has a width of 14 in. and an effective depth of 26 in. If $n = 12$ and if the allowable unit stresses are 1000 p.s.i. and 20,000 p.s.i. for the concrete and steel, respectively, what steel area must be used in order that the resisting moment with respect to the strength of the concrete may be the same as the resisting moment with respect to the steel?

6. What would be the resisting moment of the beam in Problem 5, if the reinforcement consisted of four 7/8-in. round bars?

52. Tables for Rectangular Beams. Many of the computations involved in the design and review of rectangular beams may be eliminated by the use of previously prepared tables. For example, in design, the value of $k = \frac{n}{n + r}$ depends only upon the ratio of the moduli of elasticity and the allowable unit stresses of the two materials. Table 6 (Appendix *D*) gives, for the most common values of n and for all practical combinations of f_s and f_c, the values of k as determined by the above equation. Corresponding values of $j = 1 - \frac{k}{3}$, $K = \frac{1}{2}f_ckj = pf_sj$ (for use in the formula $M = Kbd^2$), and $p = \frac{n}{2r(n + r)}$ are also given.

Similarly, Table 7 (Appendix *D*), for use in the review of beams, gives the value of $k = \sqrt{2pn + (pn)^2} - pn$, and $j = 1 - \frac{k}{3}$ for sufficient values of the variables p and n to make the solution of ordinary problems possible with but a slight amount of interpolation.

53. *Illustrative Problems Involving the Use of Tables.* I. The use of Table 6 in designing a beam may be shown by its application to Problem IV of Art. 51.

From this table (for values of $n = 12$, $f_c = 1000$, and $f_s = 18{,}000$), $K = 173$, and $j = 0.867$. Therefore from equation (6),

$$593{,}000 = 173bd^2$$

and

$$bd^2 = 3420 \text{ in.}^3$$

Let $b = 10$ in.; then $d = 18.5$ or 19 in.

From equation (5),

$$593{,}000 = A_s \times 18{,}000 \times 0.867 \times 19$$

from which

$$A_s = 2.00 \text{ sq. in.}$$

The use of the equation $M = Kbd^2$ is identical with the use of $M_c = \frac{1}{2}f_ckjbd^2$, since $K = \frac{1}{2}f_ckj$.

II. Problem I of Art. 51 may be solved by combining Tables 6 and 7. The problem might be reworded as follows: A rectangular reinforced concrete beam has a total cross-section of 12 × 24 in. and a length of 20 ft.-0 in. It is reinforced with four ⅞-in. round bars in one row, the centers of the bars being 2½ in. above the lower surface of the beam. As the load is increased, which will reach the specified stress limit first, the concrete or the steel, *i.e.*, is the beam over- or underreinforced? What is the resisting moment of the beam? Use a 2500-lb. concrete, structural-grade steel, and allowable unit stresses as given in the Joint Code.

The ideal percentage of steel required to give equal strengths in tension and compression, with allowable unit stresses of 18,000 and 1000, and with $n = 12$, is given in Table 6 as 0.0111. The actual value of p in the beam $= \dfrac{2.41}{12 \times 21.5} = 0.0093$. The beam is, therefore, underreinforced and its strength is limited by that of the steel.

From Table 7, $k = 0.375$ and $j = 0.875$ and the resisting moment is

$$M = 2.41 \times 18{,}000 \times 0.875 \times 21.5 = 818{,}000 \text{ in.-lb.}$$

NOTE: Table 6 has been used *only* to determine the relative strengths of the steel and the concrete in the beam, and *not* to determine the values of k and j.

III. The concrete used in constructing the beam shown in Fig. 13 has an ultimate 28-day strength in compression of 2500 p.s.i., and the reinforcing bars are of intermediate grade steel. What maximum concentrated load P can be placed in the position shown without exceeding the bending stresses specified in the Joint Code?

$$p = \frac{2.41}{12 \times 19.5} = 0.0103$$

From Table 7, $k = 0.390$, and $j = 0.870$. Table 6 shows that the beam is overreinforced (ideal $p = 0.0094$), and the strength is governed by that of the concrete in compression.

$$M_c = \tfrac{1}{2} \times 1000 \times 0.390 \times 0.870 \times 12 \times (19.5)^2 = 776{,}000 \text{ in.-lb.}$$

Since the principal load is the concentrated load (the only other load is the weight of the beam, which is comparatively small) and since this load is fairly close to the center of the span, it is

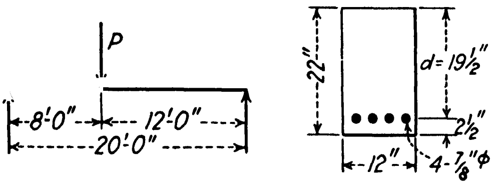


FIG. 13.

probable that the maximum moment will occur under the load, *i.e.*, at a distance of 8 ft. from the left support. At this section, the moment M_D due to the weight of the beam (275 lb. per ft.) is

$$M_D = \left(275 \times \frac{20}{2} \times 8 - \frac{275 \times 8^2}{2}\right)12 = 158{,}400 \text{ in.-lb.}$$

The moment available for the concentrated load is then 776,000 − 158,400 = 617,600 in.-lb. The moment under the concentrated load, caused by this load, is

$$P \times {}^{12}\!/_{20} \times 8 \times 12 = 57.6P \text{ in.-lb.}$$

Equating this to its maximum allowable numerical value,

$$57.6P = 617{,}600$$

from which

$$P = 10{,}700 \text{ lb. (maximum)}$$

With this load on the beam, in addition to the weight of the beam, the shear passes through zero under the concentrated load ($V_8 = 10{,}700 \times {}^{12}\!/_{20} + 275 \times {}^{20}\!/_{2} - 275 \times 8 - 10{,}700 = -3730$ lb.), which indicates that the maximum moment occurs at this section, as assumed.

54. Analysis of Rectangular Beams by the Principle of the Transformed Section. In a homogeneous beam the neutral

axis passes through the center of gravity of the cross-section. A reinforced concrete beam can be treated as a homogeneous beam, if the steel is considered to be replaced by concrete, so placed, and of such an amount, as to produce the same effect as the steel in resisting the bending moment. The equivalent amount of concrete and the required location of this concrete are obtained from the following analysis.

According to the theory of flexure, the unit stress on any fiber of a homogeneous beam at a given distance from the neutral axis is the same as the unit stress on any other fiber at the same distance from the neutral axis. Hence, if the moduli of elasticity of the steel and the concrete were equal to each other, the steel could be replaced by the same area of concrete in the same horizontal plane as the steel. The moduli of elasticity are not equal, however, and in order to resist the same total tensile stress the equivalent area of concrete must be modified accordingly. The deformation of the hypothetical equivalent concrete, which will be assumed in the same horizontal plane as the steel, is the same as that of the original steel, and, in Fig. 11 the deformation of a unit length of beam can be represented by the distance BB'. Since E = unit stress divided by unit deformation, if the unit stress in the equivalent concrete is f_{ec} and the unit stress in the steel is f_s, the following relations are obtained:

$$E_c = \frac{f_{ec}}{BB'}, \qquad \text{or} \qquad BB' = \frac{f_{ec}}{E_c}$$

$$E_s = \frac{f_s}{BB'}, \qquad \text{or} \qquad BB' = \frac{f_s}{E_s}$$

Hence

$$\frac{f_{ec}}{E_c} = \frac{f_s}{E_s}, \qquad \text{and} \qquad f_s = \frac{E_s}{E_c} \times f_{ec} = nf_{ec}$$

It is thus seen that the unit stress in the steel is n times the unit stress in the equivalent concrete, and in order to resist the same total stress (fiber stress caused by a given bending moment) every square inch of steel must be replaced by n square inches of concrete all in the same horizontal plane as the steel. The resulting equivalent homogeneous beam will then be as shown in the cross-section in Fig. 14, all of the tension being resisted by the

shaded strip of concrete, of area nA_s, at a distance d below the extreme compression fiber, and all of the compression being resisted by the shaded concrete area above the neutral axis. The unshaded concrete area below the neutral axis is assumed to serve only as a means of connecting the effective tension and compression areas so as to permit the development of the stresses on these areas. The cross-section shown in Fig. 14 is called the transformed section and represents the homogeneous section which would be equivalent, as far as resisting bending stresses is concerned, to the original composite steel-and-concrete beam.

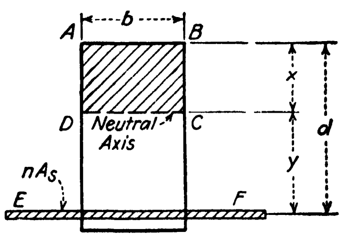


FIG. 14.

Since the neutral axis of the homogeneous beam passes through the center of gravity of the cross-section, the moment of the area $ABCD$ in the transformed section, Fig. 14, about the neutral axis DC must equal the moment of the equivalent area EF about DC, or

$$bx \times \frac{x}{2} = nA_s \times y = nA_s(d - x)$$

This one equation, in conjunction with others which are obvious from the fundamental flexure theories and assumptions, is all that is necessary to complete the analysis of beams with tension reinforcement.

55. *Illustrative Problems.* I. Solve Problem I of Art 51 by means of the principle of the transformed section. This problem involves the determination of the resisting moment of a beam whose dimensions and reinforcement are known. The neutral axis is located from the equation in the preceding article, as follows:

$$\frac{12x^2}{2} = 12 \times 2.41(21.5 - x)$$

$$6x^2 = 621.8 - 28.9x$$

$$x = 8.05 \text{ in.}$$

The arm of the resisting couple (see Fig. 15) is equal to $21.5 - \frac{8.05}{3} = 18.82$ in. If the full strength of the steel is developed, the total tension will be $18{,}000 \times 2.41 = 43{,}400$ lb., and the simultaneous total compression will be the same. If the maximum allowable extreme compression fiber stress is developed, the total compression will be $\frac{1}{2} \times 1000 \times 8.05 \times 12 = 48{,}300$ lb., and the simultaneous total tension will also be 48,300 lb. The full strength of the concrete obviously cannot be developed without overstressing the steel in tension. The beam is therefore underreinforced, and the resisting moment is equal to the

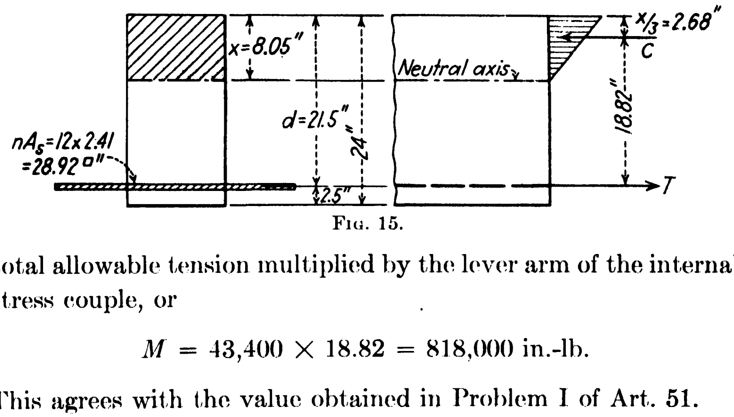


FIG. 15.

total allowable tension multiplied by the lever arm of the internal stress couple, or

$$M = 43{,}400 \times 18.82 = 818{,}000 \text{ in.-lb.}$$

This agrees with the value obtained in Problem I of Art. 51.

II. By means of the principle of the transformed section, design the beam which is described in Problem IV of Art. 51.

Assuming that the weight of the beam is 220 lb. per lin. ft. the maximum bending moment is

$$M = \frac{1}{8} \times 1220 \times 18^2 \times 12 = 593{,}000 \text{ in.-lb.}$$

In an ideal beam, the steel and the concrete are both stressed to the limit. When the beam is fully loaded, the extreme fiber stress in compression at the section of maximum moment should be 1000 p.s.i. and the simultaneous stress in the equivalent tension concrete should be $\frac{18{,}000}{12} = 1500$ p.s.i. The stress diagram is shown in Fig. 16. In order that these stresses may be

realized, the following proportion must be true:

$$\frac{x}{d} = \frac{1000}{1000 + 1500}$$

from which

$$x = d\left(\frac{1000}{2500}\right) = 0.4d$$

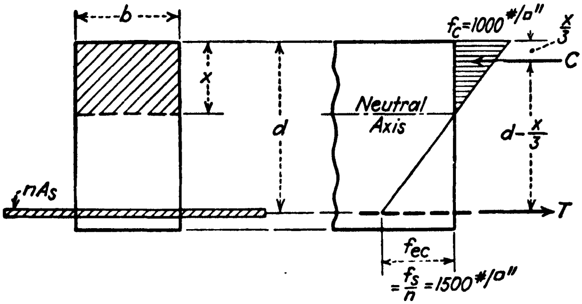


FIG. 16.

The lever arm of the stress couple is then $d - \frac{0.4d}{3} = 0.867d$, and the resisting moment of the beam, expressed in terms of the strength of the concrete in the compression area, is

$$M = \tfrac{1}{2} \times 1000 \times 0.4d \times b \times 0.867d = 173bd^2$$

Since the resisting moment must be equal to the maximum bending moment, the following equation can be written:

$$593{,}000 = 173bd^2$$

from which

$$bd^2 = 3420 \text{ in.}^3$$

With $b = 10$ in., $d = 18.5$, or 19 in., and the weight of the beam is 220 lb. per ft. as assumed.

With $d = 19$ in., the maximum compressive stress in the concrete will be slightly less than 1000 p.s.i. and the value of x as computed above will no longer be the true value of x. The error will not be great, however, unless the discrepancy between the true value of d and the actual value is of an appreciable amount. For design purposes, the neutral axis can be

assumed to be in the same position relative to the top of the beam as computed above, and the lever arm of the stress couple is $0.867 \times 19 = 16.47$ in. The total tension in the steel is then

$$T = \frac{593{,}000}{16.47} = 36{,}000 \text{ lb.}$$

The required steel area is therefore

$$A_s = \frac{36{,}000}{18{,}000} = 2.00 \text{ sq. in.}$$

These results agree with those obtained in Problem IV of Art. 51.

If a theoretically correct solution were desired for the steel area, the true value of x, for $d = 19$ in., could be obtained by equating the resisting moment with respect to the strength of the compression concrete to the maximum bending moment. The maximum unit compressive stress in the extreme fiber in this equation would be $1500 \times \dfrac{x}{19 - x}$ instead of 1000 lb. The remainder of the solution is similar to that given above. The essential computations are as follows:

$$\left(\frac{1}{2} \times 1500 \times \frac{x}{19 - x} \times 10x\right)\left(19 - \frac{x}{3}\right) = 593{,}000 \text{ in.-lb.}$$

$$x = 7.44 \text{ in.}$$

$$f_c = 1500 \times \frac{7.44}{19 - 7.44} = 965 \text{ p.s.i.}$$

$$C = T = \tfrac{1}{2} \times 965 \times 10 \times 7.44 = 35{,}900 \text{ lb.}$$

$$A_s = \frac{35{,}900}{18{,}000} = 1.99 \text{ sq. in.}$$

Slabs

56. Types of Slabs. Slabs may be supported on two sides only, or they may rest on beams along all four edges. A slab which is supported only on two sides is essentially a rectangular beam of comparatively large ratio of width to depth. There are, however, certain modifications entering into the design and review of such slabs which it was not necessary to consider in the solution of rectangular beams. Slabs which are supported

on four sides, with reinforcement in two directions, present the additional problems of determining the proportion of the total load that is transmitted in each direction to the supporting beams, and also allocating this proportion of the total load to the various strips into which the slab is assumed to be divided. These problems are considered in Arts. 61 to 66.

Floor slabs in buildings are usually designed for a uniform live load covering the entire slab area. The following discussions

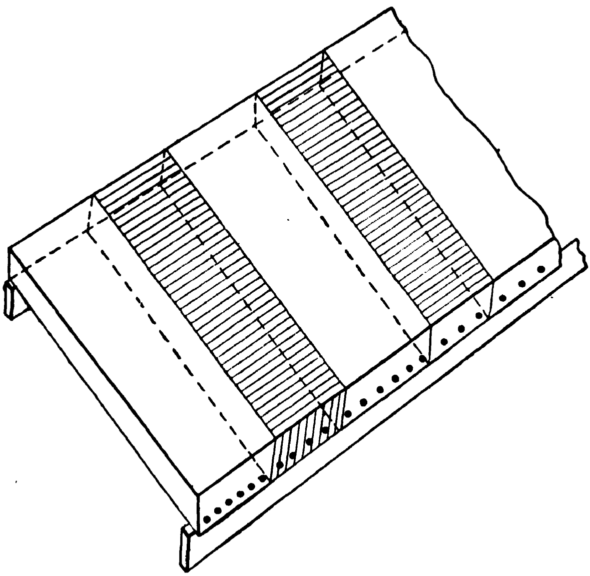


FIG. 17.

are intended to apply only to the analysis of such uniformly loaded slabs. Concentrated loads on concrete slabs are supported by a greater width of slab than the mere contact width. Methods of computing the probable distribution of concentrated loads are explained in Chap. XI. Multiples of ¼ in. may be used in selecting the total thickness of slabs.

57. Slabs Supported on Two Sides Only. The simplest form of slab is one of indefinite width, supported by only two beams, one at each edge of the slab. If a 12-in. strip of slab were cut out at right angles to the supporting beams, such as either of the shaded areas in Fig. 17, a rectangular beam 12 in. in width

would result, with a depth equal to the thickness of the slab, and a length equal to the distance between supports. This strip could then be analyzed by the same formulas which were used in problems dealing with rectangular beams, the bending moment being computed for a width of 1 ft. The load per square foot on the slab would then be the load per linear foot on the imaginary beam. Since all of the load on the slab must be transmitted to the two supporting beams, it follows that all of the reinforcing steel should be placed at right angles to these beams, with the exception of any bars that may be placed in the other direction to take care of shrinkage and temperature stresses. A slab which is supported on two sides only thus consists (in theory) of a series of rectangular beams side by side.

The ratio of steel in a slab may be determined by dividing the sectional area of one bar by the area of concrete between two successive bars, the latter area being the product of the depth to the center of the bars and the distance between them, center to center. The ratio of steel may also be determined by dividing the average area of steel per foot of width by the effective area of concrete in a 1-ft. strip. The average area of steel per foot of width is equal to the area of one bar times the average number of bars in a 1-ft. strip (12 divided by the spacing in inches), and the effective area of concrete in a 1-ft. (or 12-in.) strip is equal to 12 times the effective depth *d*.

To illustrate the latter method of obtaining the steel ratio p, assume a 5-in. slab with an effective depth of 4 in., and with ½-in. round bars spaced 4½ in., center to center. The average number of bars in a 12-in. strip of slab is $\frac{12}{4\frac{1}{2}} = 2.7$ bars, and the average steel area in a 12-in. strip is $2.7 \times 0.1963 = 0.53$ sq. in. Hence $p = \frac{0.53}{12 \times 4} = 0.0110$. By the other method,

$$p = \frac{0.1963}{4\frac{1}{2} \times 4} = 0.0110.$$

The spacing of bars which is necessary to furnish a given area of steel per foot of width is obtained by dividing the number of bars required to furnish this area, into 12. For example, to

furnish an average area of 0.444 sq. in. per ft., with ½-in. round bars, requires $\frac{0.444}{0.1963} = 2.3$ bars per foot; the bars must be spaced not more than $\frac{12}{2.3} = 5.2$ in., center to center.

If the slab is of one span only and if it rests freely on its supports, the maximum positive moment M, assuming a uniform load of w lb. per sq. ft., is $M = \frac{1}{8}wl^2$. The span length l is taken as the distance center to center of supports, but it need not exceed the clear span plus the depth of the slab. If a single-span slab is built monolithically with the supporting beams, provision must be made for the negative moment which is developed at the supports by the condition of restraint there. The maximum positive moment would be less than for a corresponding freely supported slab. Positive and negative moments of $\frac{1}{10}wl^2$ may be used.

If the slab is of more than one span, built monolithically with the supporting beams or walls, both positive and negative moments exist, which should be computed by the principles of continuity. The span length l of such slabs may be taken as the clear distance between faces of supports. Consideration should be given to the relative lengths of adjoining spans, the comparative stiffnesses of the supports, and the relation between the magnitudes of the dead and live loads. The Joint Code recommends the following maximum moments and shears where the spans are approximately equal (actually, where the longer of two adjacent spans does not exceed the shorter by more than 20 per cent) and where the intensity of the live load does not exceed three times the intensity of the dead load:

Negative moment at face of first interior support:

For spans greater than 10 ft.

Two spans, $M = \frac{1}{8}wl^2$
More than two spans, $M = \frac{1}{10}wl^2$

For spans less than 10 ft.

Two spans, $M = \frac{1}{10}wl^2$
More than two spans, $M = \frac{1}{12}wl^2$

Negative moment at face of other interior supports:

$$M = \frac{1}{12}wl^2$$

Positive moment at center of span:

End spans, $M = \frac{1}{10}wl^2$

Interior spans, $M = \frac{1}{12}wl^2$

Shear in end spans at first interior support:

$$V = 1.20\frac{wl}{2}$$

Shear at other supports:

$$V = \frac{wl}{2}$$

58. Placing the Reinforcement. The insulation at the bottom should follow the recommendation of the Joint Code unless conditions warrant some change (see Art. 49). In the average slab, a depth of 1 in. below the center of the steel may be used.

The lateral spacing of bars, except those which are used only to prevent shrinkage and temperature cracks, should not exceed three times the thickness of the slab; the minimum spacing is given in Art. 49.

59. Temperature Reinforcement. Reinforcement for shrinkage and temperature stresses normal to the principal reinforcement shall be provided in slabs where the principal reinforcement extends in one direction only. The Joint Code specifies the following minimum ratios of reinforcement area to effective concrete area, but in no case shall such reinforcement bars be placed farther apart than five times the slab thickness or more than 18 in.

Floor slabs where plain bars are used...............	0.0025
Floor slabs where deformed bars are used...........	0.0020
Floor slabs where wire fabric is used, having welded intersections not farther apart in the direction of stress than 12 in.............................	0.0018
Roof slabs where plain bars are used...............	0.0030
Roof slabs where deformed bars are used............	0.0025
Roof slabs where wire fabric is used, having welded intersections not farther apart in the direction of stress than 12 in.............................	0.0022

In general, ¼-in. round bars at 12 in. on centers or ⅜-in. round bars at 18 in. on centers will be satisfactory.

60. *Illustrative Problems.* I. Design a fully continuous reinforced concrete slab, supported on two sides only, to sustain a live load of 120 lb. per sq. ft. The span of the slab is 11 ft.-0 in. A 2000-lb. concrete is to be used; $f_s = 18{,}000$ p.s.i.

Assuming a 5-in. slab and considering a 12-in. strip at right angles to the supporting beams, the maximum external bending moment on this strip, which may be considered as a rectangular beam 12 in. in width, is

$$M = \tfrac{1}{12} \times 182 \times 11^2 \times 12 = 22{,}000 \text{ in.-lb.}$$

Since $M = Kbd^2$, in which $K = 139$ (Table 6), the required effective cross-section of the imaginary beam is

$$bd^2 = \frac{22{,}000}{139} = 158.0 \text{ in.}^3$$

Since $b = 12$ in., $d = 3.6$ in. Selecting d as a multiple of ½ in., the depth to the center of the steel is made 4 in., and the total thickness 5 in. This agrees with the assumed value, and no revision is necessary.

From Table 6, $j = 0.867$. The area of steel per foot of slab width is obtained from equation (5) as follows:

$$22{,}000 = A_s \times 18{,}000 \times 0.867 \times 4$$
$$A_s = 0.353 \text{ sq. in.}$$

Selecting ½-in. round bars, $\dfrac{0.353}{0.1963} = 1.8$ bars are required per foot of width. The maximum spacing is then $\dfrac{12}{1.8} = 6.7$ in. In order to simplify the construction, a spacing of 6½ in. is used throughout the slab. Since this gives a suitable arrangement, the ½-in. round bars are satisfactory. The necessary arrangement of the steel to provide for the negative moments at the supports is explained in a later article. Temperature and shrinkage stresses in the direction perpendicular to the main reinforcement will be provided for by placing ¼-in. round bars

at 12 in. on centers, or ⅜-in. round bars at 18 in. on centers, at right angles to the main reinforcement.

The required effective cross-section could have been obtained from equations (7), (3), and (4), in the order named, if no tables had been available.

II. Review the slab which was designed in Problem I, to determine the maximum unit stresses in the steel and in the concrete, when the slab is loaded with its full live load of 120 lb. per sq. ft.

The maximum bending moment on a 12-in. strip of slab, as computed in Problem I, is 22,000 in.-lb. The average steel ratio is

$$p = \frac{0.1963}{6.5 \times 4} = 0.0075$$

From Table 7, $k = 0.374$ and $j = 0.875$, and substituting in equation (5*a*), the unit stress in the steel is obtained as follows:

$$22{,}000 = 0.0075 \times f_s \times 0.875 \times 12 \times 4^2$$
$$f_s = 17{,}500 \text{ p.s.i.}$$

The unit stress in the concrete is obtained from equation (1*a*).

$$f_c = \frac{17{,}500 \times 0.374}{15(1 - 0.374)} = 697 \text{ p.s.i.}$$

ADDITIONAL PROBLEMS

1. Design a fully continuous slab, supported on two sides only, to support a uniform live load of 200 lb. per sq. ft. The span of the slab is 10 ft.-0 in., and a 2500-lb. concrete is to be used, with structural-grade reinforcing steel.

2. A slab of one span, freely supported on two sides only, has a span of 11 ft.-0 in. and a total thickness of 5 in. and is reinforced with ½-in. round bars 7 in. on centers, the centers of the bars being 1 in. above the lower surface of the slab. If $f'_c = 2500$ p.s.i. and $f_s = 20{,}000$ p.s.i., what is the safe uniform live load, in pounds per square foot, that can be placed upon the slab?

Slabs Supported on Four Sides

61. General Considerations. When a slab panel is square, or nearly so, and there are beams at the four edges of the panel, the slab should be reinforced in two directions so as to transmit the total load to all four beams. The amount of load that is trans-

mitted in each direction will depend upon the relative lengths of the sides of the panel and the conditions of continuity that exist at the four edges.

If the panel is square and if the construction is such that the same degree of restraint exists at each edge, one-half of the total load will be transmitted to each pair of beams. If the panel is longer in one direction than in the other, more than one-half of the load will be transmitted in the shorter direction, and the remainder will be transmitted in the longer direction. If, however, one side of the panel is very much longer than the other, such a large proportion of the total load will be transmitted in the shorter direction that reinforcement parallel to the longer side would be of little practical value.

When a slab is reinforced in two directions, the value of d that is established for one set of bars definitely fixes the corresponding value to be used in the computations of the other set. The two sets of bars are placed one above the other, the upper resting directly on the lower. In a square slab it is customary to use the effective depth for the upper row in all computations and to place the same reinforcement, similarly spaced, in the lower row. On account of the larger value of d, this provides a slight excess of steel in the lower row, but it simplifies the details of construction. In rectangular slabs, it will generally be found economical to place the shorter bars, which carry the larger part of the load, underneath the longer bars, thus making the d for the short bars as great as possible with a given slab thickness.

62. Distribution of Load Based on Deflections. A theoretical analysis of a slab supported on four sides is complicated by many factors, such as the condition of restraint or continuity at each edge of the panel and the effect of the stiffening action of the beams on the portions of the slab adjacent to these beams.

A strip of slab such as strip A, Fig. 18, parallel and adjacent to a supporting beam supports practically none of the load in the middle portion of the span of the strip (for example, the load on area ab); the strip B at right angles to A supports the entire load in that region. The load on an area such as ac, near the corner of the panel, would be supported more or less equally by

the strips A and C. In a square slab, the loading curve on any strip actually approximates a parabola with ordinates varying from a minimum at the center to w lb. per sq. ft. at the ends, w being the total load per square foot on the slab. The center minimum depends upon the location of the strip in the slab, decreasing from $\frac{w}{2}$ for a strip at the center of the panel to zero for a strip close to one of the supporting beams.

In a rectangular slab of length l and width l', the amount of load carried by each system of reinforcement is found analytically by equating the deflections of two strips, one parallel to and midway between each pair of beams. The deflection of each strip is proportional to the load per foot multiplied by the fourth power of the span. Hence, since the deflections of the two strips must be equal at the center, if w_1 represents that part of the load w which is transmitted in the short direction and w_2 that part which is transmitted in the long direction, the following equation can be written

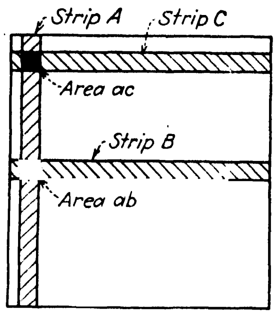


FIG. 18.

$$w_1 l'^4 = w_2 l^4$$

from which

$$\frac{w_1}{w_2} = \frac{l^4}{l'^4}$$

The amount of load per square foot that is carried by each of the two sets of bars is thus inversely proportional to the fourth power of the span of the strip in which the bars are placed. Moment computations based on this theory, which assumes a uniform distribution of the load on each strip, are not theoretically correct, since for points near the short edges of the panels the proportion of the load carried by the longitudinal strips will be greater than indicated above, and for points near the long edges of the panels it will be smaller. The theory does not recognize any effect of conditions of continuity on dis-

tribution, and another error is thus introduced in practical application.

63. Load Distribution According to Joint Code Specifications (1928). The distribution that was recommended in the Joint Code (1928) is that the part w_1 of the total load w which is trans ferred in the short direction is represented by the equation

$$w_1 = \left(\frac{l}{l'} - \frac{1}{2}\right)w$$

in which l = longer dimension of the slab.
l' = shorter dimension of the slab.

According to this equation, if the ratio l/l' is greater than $1\frac{1}{2}$, all of the load must be transmitted in the short direction to the longer beams: the only reinforcement required in the long direction is then that necessary to resist shrinkage and temperature stresses and to assist in distributing the load to the shorter bars.

Experimental analyses indicate that a strip of slab which is adjacent and parallel to one of the beams is not actually stressed as much as a strip more remote from the beams. Hence the bars in the outer quarters of the slab could be spaced farther apart than those in the middle portion. Tests indicate also that the

DISTRIBUTION OF LOADS ON SLABS SUPPORTED ON FOUR SIDES

Proportion of load carried in:	Method of distribution	Ratio l/l'					
		1.0	1.1	1.2	1 3	1 4	1 5
Short direction	Defl. theory	0.50	0.59	0 67	0 75	0 80	0 83
	Joint Code	0.50	0.60	0 70	0.80	0 90	1 00
Long direction	Defl. theory	0 50	0.41	0 33	0.25	0.20	0 17
	Joint Code	0.50	0.40	0 30	0.20	0 10	0 00

computed moments, assuming an arbitrary continuity coefficient of $\frac{1}{12}$ (*i.e.*, $M = \frac{1}{12} w_1 l^2$) for fully continuous panels or $\frac{1}{10}$ for corner panels, are greater than those that actually exist, and as a result a spacing equal to twice the minimum computed spacing is generally permitted in the outer quarters of the slab.

There is a fairly close agreement between the distribution indicated in Art. 62 and that recommended above, as is shown in the table on page 82.

The method of load distribution described above is no longer considered a generally accepted means of analyzing two-way slabs. However, the design is easily made, the resulting details are known to be on the safe side, and, if only a small floor area is involved, the added cost of the construction is not prohibitive. The use of the method may therefore be justified in some cases, and the problems in Art. 64 are given to illustrate the fundamental principles involved. A more rational method is suggested in Art. 66.

64. *Illustrative Problems Based on Load Distribution of Art. 63.* I. A floor panel is to be 9 ft.-0 in. by 10 ft.-0 in. in plan and the slab is to be fully continuous and reinforced in two directions. Design the slab to carry a uniform live load of 300 lb. per sq. ft. Assume $f_c = 1000$, $f_s = 18{,}000$, and $n = 12$.

Let w_1 be the part of the total load per square foot that is transmitted in the short direction.

$$w_1 = \left(\frac{l}{l'} - \frac{1}{2}\right)w = \left(\frac{10}{9} - \frac{1}{2}\right)w = 0.61w$$

Assuming the weight of the slab as 50 lb. per sq. ft. (or a total thickness of 4 in.), the total load on the slab is $w = 300 + 50 = 350$ lb. per sq. ft.

Design of Transverse or Short Direction.

$$w_1 = 0.61 \times 350 = 215 \text{ lb. per sq. ft.}$$

The actual external bending moment per foot of slab width is

$$M_1 = \tfrac{1}{12} \times 215 \times 9^2 \times 12 = 17{,}500 \text{ in.-lb.}$$

From Table 6, $K = 173$ and $j = 0.867$; hence, from equation (6),

$$bd^2 \text{ (required)} = \frac{17{,}500}{173} = 101 \text{ in.}^3$$

Since $b = 12$ in., the required value of d is 2.9 in. A value of

$d = 3$ in. will be used; allowing 1 in. of concrete below the center of the bars, the total thickness of the slab is 4 in. as assumed, and no revision is necessary.

From equation (5)

$$A_s = \frac{17{,}500}{18{,}000 \times 0.867 \times 3} = 0.374 \text{ sq. in.}$$

This is the area required per foot of width; with ½-in. round bars the number of bars required in each 12-in. strip is $\frac{0.374}{0.1963} = 1.9$, and the required spacing is $\frac{12}{1.9} = 6.3$ in.

Design of Longitudinal or Long Direction. The part w_2 of the total load w that is transmitted in the long direction is 350 − 215 = 135 lb. The actual external bending moment M_2 on a 12-in. strip in this direction is

$$M_2 = \tfrac{1}{12} \times 135 \times 10^2 \times 12 = 13{,}500 \text{ in.-lb.}$$

$$bd^2 \text{ (required)} = \frac{13{,}500}{173} = 78 \text{ in.}^3$$

Since the long bars are placed directly on top of the transverse bars, the actual value of d for the longitudinal bars is equal to 2½ in. Therefore

$$bd^2 \text{ (furnished)} = 12 \times (2\tfrac{1}{2})^2 = 75$$

This is less than that required, and the strength of the concrete is not sufficient to carry the load in this direction. Increase the thickness of the slab to 4¼ in. The additional weight (about 3 lb. per sq. ft.) can be disregarded without material error. The product of bd^2 furnished in the long direction is now $12 \times (2\tfrac{3}{4})^2 = 91$, which is ample.

From equation (5), the area of steel required per foot of width is

$$A_s = \frac{13{,}500}{18{,}000 \times 0.867 \times 2.75} = 0.314 \text{ sq. in.}$$

The spacing required for ½-in. round bars is

$$\frac{12}{\dfrac{0.314}{0.1963}} = 7.5 \text{ in.}$$

The revised spacing of the bars in the short direction (the effective depth of the bars is now 3¼ in.) is $6.3 \times \frac{3.25}{3} = 6.8$ in. The bars will be spaced 6½ in. center to center in both directions. As stated in Art. 63, a spacing of $2 \times 6.8 = 13.6$ in. could be used for the bars in the outer quarters of the slab, but a maximum spacing of three times the slab thickness is normally specified. The bars in the outer quarters of the slab will be spaced 12 in. on centers.

II. A typical floor panel, 10 ft.-0 in. by 12 ft.-0 in., is reinforced in two directions with ½-in. square bars 8 in. center to center, the center of the lower row of bars being placed 1 in. above the lower surface of the slab. The total thickness of the slab is 5 in., $f_s = 20{,}000$ p.s.i., $f_c = 800$ p.s.i., and $n = 15$. What live load per square foot will the panel sustain?

Investigation of Short Direction. The bars in the short direction are placed beneath the others. Their effective depth, therefore, is 4 in. The actual ratio of steel in the short direction is

$$p = \frac{0.25}{8 \times 4} = 0.0078$$

and from Table 7, $k = 0.380$ and $j = 0.873$.

For a 12-in. strip parallel to the short sides of the slab, the resisting moment of the concrete is

$$M_c = \tfrac{1}{2} \times 800 \times 0.380 \times 0.873 \times 12 \times 4^2 = 25{,}500 \text{ in.-lb.}$$

The resisting moment of the steel is

$$M_s = 0.0078 \times 20{,}000 \times 0.873 \times 12 \times 4^2 = 26{,}200 \text{ in.-lb.}$$

The smaller of these two resisting moments must not be exceeded by the actual external bending moment. The values shown above indicate that the slab is slightly overreinforced in the short direction, *i.e.*, there is more steel than is required to develop the full compressive strength of the concrete. This fact could have been determined by comparing the actual steel ratio with the ideal ratio for the given allowable unit stresses. The ratio

furnished, 0.0078, is greater than the ideal, 0.0075, as given in Table 6.

The external bending moment equals $\frac{1}{12}w_1l'^2$, and its maximum allowable value is 25,500 in.-lb. Therefore,

$$25{,}500 = \frac{1}{12} \times w_1 \times 10^2 \times 12$$

from which $w_1 = 255$ lb. per ft.

This is the total load per linear foot that can safely be carried by a 12-in. strip in the short direction. Since this is $\frac{l}{l'} - \frac{1}{2} = \frac{7}{10}$ of the total load w on the slab, the total load that can be placed on the slab before the short bars will reach their allowable stress (as governed by the concrete in this case) is

$$\frac{255}{0.7} = 364 \text{ lb. per sq. ft.}$$

Investigation of Long Direction. The effective depth of the long bars is $3\frac{1}{2}$ in.

$$p = \frac{0.25}{8 \times 3\frac{1}{2}} = 0.0089 \qquad k = 0.400 \qquad j = 0.866$$

For a 12-in. strip of slab parallel to the long sides of the panel, Table 6 shows that the slab is overreinforced in this direction and the resisting moment of the 12-in. strip is limited by the strength of the concrete.

$$M_c = \frac{1}{2} \times 800 \times 0.400 \times 0.866 \times 12 \times (3.5)^2 = 20{,}400 \text{ in.-lb.}$$

The actual bending moment is $\frac{1}{12}w_2l^2$; hence

$$20{,}400 = \frac{1}{12} \times w_2 \times 12^2 \times 12$$

from which

$$w_2 = 142 \text{ lb. per ft.}$$

This represents the load that can be carried safely in the long direction; it is equal to but $\frac{3}{10}$ of the total load on the slab. Hence,

$$142 = 0.3w, \qquad \text{and} \qquad w = 473 \text{ lb. per sq. ft.}$$

From the two investigations above, it is seen that a total load of 473 lb. per sq. ft. could be placed on the slab without overstressing it in the long direction. This load, however, would considerably overstress the slab in the short direction. The maximum total load per square foot that can be placed on this slab is thus determined by the strength of the transverse direction, and equals 364 lb. as computed above.

The slab itself weighs 62 lb. per sq. ft.; hence the safe live load is $364 - 62 = 302$ lb. per sq. ft.

III. A fully continuous floor panel 9 ft.-0 in. by 9 ft.-0 in. in plan is to support a live load of 300 lb. per sq. ft. Determine the required thickness of slab and the arrangement of the reinforcement. Assume maximum allowable unit stresses of 750 p.s.i. and 16,000 p.s.i. for the concrete and steel, respectively, and $n = 15$.

Assume $t = 4\frac{1}{2}$ in.; then the total load per square foot is 356 lb. Since the slab is square, one-half of this load will be transmitted in each direction. Hence $w_1 = 178$ lb. For a 12-in. strip of slab in either direction,

$$M = \frac{1}{12} \times 178 \times 9^2 \times 12 = 14{,}400 \text{ in.-lb.}$$

Since $M = Kbd^2$, in which K (Table 6) is 134,

$$bd^2 = \frac{14{,}400}{134} = 107.3 \text{ in.}^3$$

$b = 12$ in.; therefore d must be at least 2.98 in.

In a square slab, since the moments for strips in either direction are equal, the strength of the strip parallel to the upper row of bars will govern the design. Selecting an effective depth of 3 in. for this row, allowing 1 in. of insulation below the center of the lower row, and assuming $\frac{1}{2}$-in. bars, the total thickness of slab required is $3 + \frac{1}{2} + 1$, or $4\frac{1}{2}$ in. as assumed.

For the upper row,

$$A_s = \frac{14{,}400}{16{,}000 \times 0.862 \times 3} = 0.348 \text{ sq. in. per ft.}$$

The number of $\frac{1}{2}$-in. round bars required per foot of width

$= \frac{0.348}{0.1963} = 1.8$, and the maximum spacing of the bars is $\frac{12}{1.8} =$ 6.6, or 6½ in.

The spacing of bars in the lower row is made the same, for uniformity. Since the effective depth for this row is ½ in. greater than for the upper row, this arrangement is safe but not excessively uneconomical. The bars in the outer quarters of the slab may be spaced about 13 in. center to center in accordance with the recommendation of Art. 63.

ADDITIONAL PROBLEMS

1. A fully continuous floor panel 11 ft.-0 in. by 12 ft.-0 in. is to support a uniform live load of 250 lb. per sq. ft. A 3000-lb. concrete and intermediate-grade reinforcing bars are to be used. Design the slab: (*a*) in accordance with the load distribution recommended in Art. 63; (*b*) in accordance with the proposed Joint Committee method described in Art. 66.

2. A fully continuous floor slab 10 ft.-0 in. by 11 ft.-0 in. has a total thickness of 5 in. and is reinforced with ½-in. round bars spaced 7 in. on centers in both directions. The centers of the shorter bars are 1 in. above the lower surface of the slab, and the longer bars are placed directly on the shorter bars. If $f'_c = 2000$ p.s.i. and $f'_s = 18{,}000$ p.s.i., what safe live load, in pounds per square foot, can be placed on the floor, if the load distribution in Art. 63 is assumed?

3. If the longer bars in Problem 2 were spaced 8 in. on centers instead of 7 in., what safe live load could be placed on the slab?

4. Design a fully continuous floor slab, 10 ft. square, to support a uniform live load of 250 lb. per sq. ft. Assume $n = 15$, $f_c = 800$ p.s.i., and $f_s = 16{,}000$ p.s.i.

65. Theoretical Distribution of Moments in Two-way Slabs. An exact analysis of a concrete slab panel which is supported on four sides should take into consideration the conditions of continuity along each edge of the panel, the relative stiffness of the beams as compared with that of the slab, and the relative stiffness of each adjoining panel. Some of the conditions of continuity are represented in the following types of panels: (1) a single panel, (2) the end panel in a single row of panels, (3) an intermediate panel in a row, (4) a corner panel, (5) a wall panel in a rectangular group of panels, (6) an interior panel in a group.

Such so-called exact analyses result in moments which are very much in excess of those indicated in tests of slabs of the same type, and for this reason the moment coefficients obtained in the analyses are usually reduced by a definite percentage in order to obtain rational design values. The importance of a theoretical analysis is not so much, therefore, to determine moment coefficients, as it is to obtain information concerning the effect of construction details on the distribution of the moments.

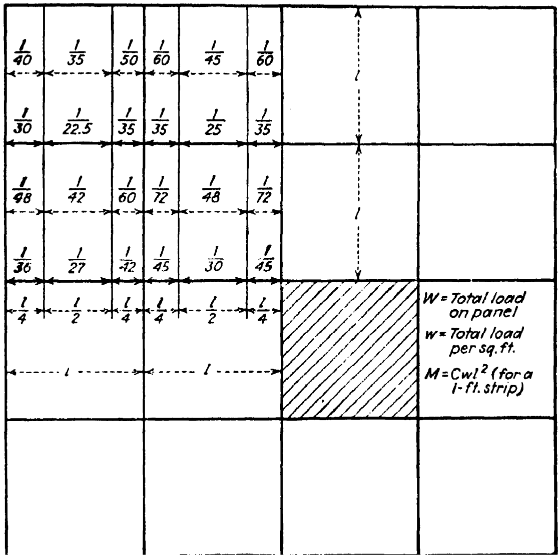


Fig. 19.

Formulas for the critical moments in square and rectangular panels, and in the supporting beams, reduced in accordance with comparisons with test results, have been advanced by H. M. Westergaard.[1] The coefficients for square panels are shown in Fig. 19. These coefficients are those of a constant term Wl, in which W is the total load on one panel and l is the length of the square panel.

In order to make a comparison between the coefficients shown in Fig. 19 and those used in the problems of Art. 64, which are

[1] "Formulas for the Design of Rectangular Floor Slabs and the Supporting Girders," *Proc.*, A.C.I., vol. 22, p. 26, 1926.

based on the (1928) Joint Code Specifications, the former should be multiplied by 2. This is because the Joint Code coefficients are those of a constant term $\frac{W}{2}$ (for square slabs), assuming that one-half of the total load on the panel is transmitted in each direction. The revised positive-moment coefficient as proposed by Westergaard for the middle half of an interior panel is then $2 \times 1/48 = 1/24$ and the revised negative-moment coefficient is $2 \times 1/30 = 1/15$. The coefficient used in Art. 64 for both positive and negative moments in an interior panel is $1/12$. The (1928) Joint Code specification is thus seen to be very conservative, particularly in regard to positive moment. For the side strips in a similar panel the comparative values of positive moment are $2 \times 1/72 = 1/36$ by Westergaard, and $1/2 \times 1/12 = 1/24$ by the Joint Code; the comparative values of negative moment are $2 \times \frac{1}{45} = \frac{1}{22.5}$ and $\frac{1}{2} \times \frac{1}{12} = \frac{1}{24}$. The latter coefficients govern the steel area only; the (1928) Joint Code value is based on the fact that the Code permits a spacing of bars in the side strips equal to twice the spacing computed for the middle strips. The comparison indicates that this increased spacing is thoroughly justified. It indicates also the necessity of making proper provision for the negative moments at the edges of the slab.

66. Design of Two-way Slabs According to Joint Committee (1940). A reasonably simple method of two-way slab analysis has been proposed in the Joint Committee Report of June, 1940, which takes into consideration the effect of discontinuity at one or more edges of the panel. In this method, the slab is considered to be built monolithically with the supporting walls or beams, so that a negative moment is developed on all exterior edges.

The panel is divided into middle strips and outer strips as in the case of flat-slab floors (see Art. 212). If the panel is nearly square, the middle strip has a width of one-half panel, and each outer strip has a width of one-quarter panel, as shown in Fig. 20. In panels where the ratio of short span L_s to long span L_L is less than 0.5, the width of the middle strip that extends in the short

direction is equal to the difference between the long and short spans, the remaining area being divided equally between the two outer strips.

The span lengths are taken as the distance between the centers of supports or as the clear span plus twice the slab thickness, whichever value is the smaller. The critical sections for bending moment are along the center lines of the panel for positive

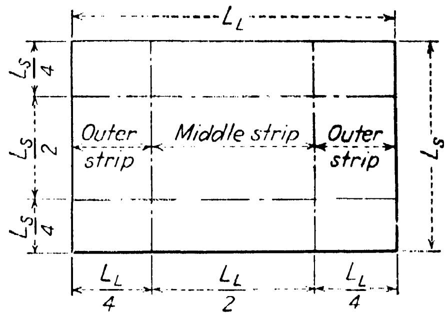


Fig. 20.

moment and along the faces of the supporting beams or walls for negative moment.

The bending-moment coefficients shown in Figs. 21 to 23 are for a strip of slab 1 ft. wide and are in terms of wL_s^2, where w is the uniform load per square foot on the panel and L_s is the short-span length. Moments, whether for strips in the long or short direction, are computed in terms of the short-span length.

The recommended coefficients, as in the case of the design provisions for flat slabs, are based partly on analysis and partly on test data.

Positive Moments. Positive moments at the middle of the short span are given in Fig. 21 for all ratios of short to long span. For an interior panel the lower curve is used. For a side panel where the slab is discontinuous beyond the one marginal support but continuous beyond the other three sides of the panel, the curve marked "Panel with 1 edge discontinuous" is used. For panels having more marginal or discontinuous edges, the appropriate curve is used. Thus, an isolated panel built monolithically with the supports would be designed in accordance with the

top curve of the figure, since all four edges would be discontinuous. In using this diagram, it makes no difference whether the discontinuous edge is on the short or long side. Also, it is immaterial whether a panel with two discontinuous edges is a corner panel or whether the discontinuous edges are opposite each other.

The positive moment at the middle of the middle strip of the long span is computed by using the coefficient for a short span

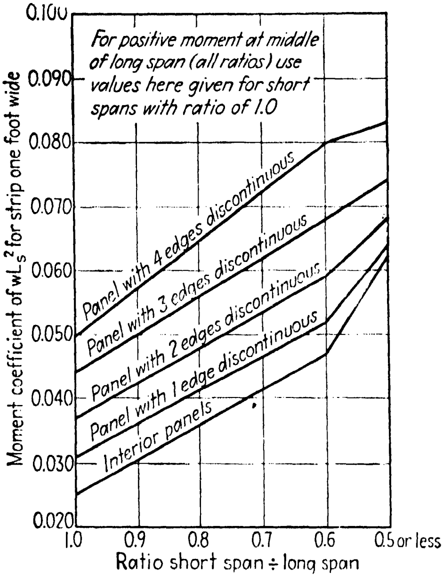


FIG. 21.—Positive moments at middle of middle strip, short span.

with ratio of 1.0. The proper coefficient is multiplied by wL_s^2, in which L_s is the short span, as previously defined.

Negative Moments at Continuous Edges. The design for negative moment across the panel edges where the slab is continuous into adjacent panels follows the same method as that for positive moments. The negative moment at such a continuous edge for the middle strip of the short span is computed from the coefficients shown in Fig. 22. The curves in Fig. 22 apply whether the discontinuous edge of the panel is adjacent to or opposite a continuous edge, the controlling factor being only the number of discontinuous edges in the panel.

The negative-moment coefficient at a continuous edge of the middle strip of the long span is the same for all span ratios and is

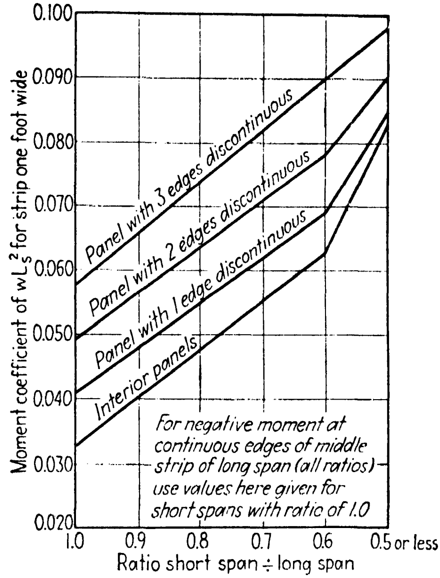


FIG. 22.—Negative moments at continuous edges in middle strip, short span.

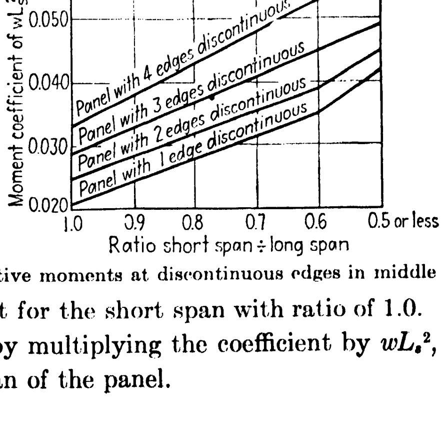


FIG. 23.—Negative moments at discontinuous edges in middle strip, short span.

taken as that for the short span with ratio of 1.0. The moment is obtained by multiplying the coefficient by wL_s^2, in which L_s is the short span of the panel.

Negative Moment at a Discontinuous Edge. The negative-moment coefficients at the edge of a panel terminating in and built monolithically with a wall or beam are shown in Fig. 23. These coefficients are for the negative moment at discontinuous edges of the middle strips of the short span.

If the middle strip under consideration is in the long span, the coefficient is selected for a span ratio of 1.0, using the proper curve for the given continuity conditions. The moment is obtained by multiplying the coefficient by wL_s^2, in which L_s is the short span of the panel.

Moments in Outer Strips. The moments in outer strips are taken as two-thirds of the moments for middle strips of corresponding span and end conditions.

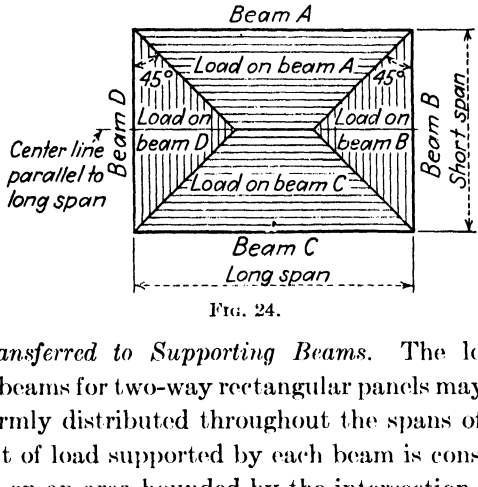


FIG. 24.

Load Transferred to Supporting Beams. The loads on the supporting beams for two-way rectangular panels may be assumed to be uniformly distributed throughout the spans of the beams. The amount of load supported by each beam is considered to be all the load on an area bounded by the intersection of 45-degree lines from the corners of the panel with the center line of the panel parallel to the long side, as shown in Fig. 24. The moments and shears in the beams are computed in the usual way, due consideration being given to the conditions of restraint at the supports of the beams.

Illustrative Problem. Design the slab in Problem I, Art. 64, in accordance with the method outlined above, using allowable unit stresses as given in that problem and assuming that the panel is an interior panel of a group. Assuming a 4-in. slab,

the total load is 350 lb. per sq. ft. The span ratio is 0.9. Computed moments for strips 1 ft. wide are as follows:

Middle strip, short span, positive moment (Fig. 21),

$$M = 0.0305 \times 350 \times 9^2 \times 12 = 10{,}400 \text{ in.-lb.}$$

Middle strip, short span, negative moment (Fig. 22),

$$M = 0.040 \times 350 \times 9^2 \times 12 = 13{,}600 \text{ in.-lb.}$$

Middle strip, long span, positive moment (Fig. 21),

$$M = 0.025 \times 350 \times 9^2 \times 12 = 8500 \text{ in.-lb.}$$

Middle strip, long span, negative moment (Fig. 22),

$$M = 0.033 \times 350 \times 9^2 \times 12 = 11{,}200 \text{ in.-lb.}$$

The critical moment to be used in determining the slab thickness is, therefore, 13,600 in.-lb.

In the short span,

$$d = \sqrt{\frac{13{,}600}{173 \times 12}} = 2.56 \text{ in.}$$

The thinnest slab that is permitted in most building codes is 4 in., and, with 1 in. of insulation below the center of the short bars (the lower row), the d furnished is 3 in., which is adequate.* Assuming $\frac{3}{8}$-in. bars, the d furnished is $2\frac{5}{8}$ in.

In the short span, middle strip, at the center of the span,

$$A_s = \frac{10{,}400}{18{,}000 \times 0.867 \times 3} = 0.22 \text{ sq. in. per ft.}$$

This is furnished by $\frac{3}{8}$-in. bars 6 in. on centers.

In the short span, middle strip, at the support,

$$A_s = \frac{13{,}600}{18{,}000 \times 0.867 \times 3} = 0.29 \text{ sq. in. per ft.}$$

If every third bar (as selected above) continues straight through the support, and if the other bars are bent up and continued into

* Computations for the effective depths required in the long span are unnecessary in this case. These values for both positive and negative moments are less than 2.56 in. The furnished values are $2\frac{5}{8}$ in. at the positive-moment section and 3 in. at the negative-moment section.

the adjacent span to the point of inflection, the area furnished at the support (including the bars which are bent from the adjacent span) is $\frac{4}{3} \times 0.22 = 0.29$ sq. in.

In the short span, outer strip, the required steel area at the center is $\frac{2}{3} \times 0.22 = 0.15$ sq. in. per ft., which is furnished by $\frac{3}{8}$-in. round bars, 9 in. on centers.

In the short span, outer strip, the required negative steel area at the support is $\frac{2}{3} \times 0.29 = 0.19$ sq. in. per ft. Bending two bars in three from each side as proposed above for the middle strip, the area furnished is $\frac{4}{3} \times 0.15 = 0.20$ sq. in. per ft.

In the long span, middle strip, at the center,

$$A_s = \frac{8500}{18{,}000 \times 0.867 \times 2.62} = 0.21 \text{ sq. in. per ft.}$$

In the long span, middle strip, at the supports (note—these bars can be brought to within 1 in. of the top of the slab),

$$A_s = \frac{11{,}200}{18{,}000 \times 0.867 \times 3} = 0.24 \text{ sq. in. per ft.}$$

The same steel spacings and arrangement are required as in the short span.

In the long span, outer strip, the required steel area at the center is $\frac{2}{3} \times 0.21 = 0.14$ sq. in. per ft., and the area required at the supports is $\frac{2}{3} \times 0.24 = 0.16$ sq. in. per ft. The same spacing and arrangement will be used as in the short span, furnishing 0.15 sq. in. per ft. at the center and $\frac{4}{3} \times 0.15 = 0.20$ sq. in. at the supports.

A comparison with the results obtained in Problem I, Art. 64, shows a saving of about 30 per cent in steel and 6 per cent in concrete in favor of the Joint Committee method. Additional economy in concrete would be obtained where slabs exceed the 4-in. minimum thickness.

Diagonal Tension, Shear, and Bond

67. Stresses in a Concrete Beam. The preceding articles contain an outline of the methods of calculating the maximum fiber stresses in the concrete and steel of a reinforced concrete

beam, and of so proportioning the amounts of steel and concrete that the working strength of any part of the beam in flexure is not exceeded.

There are other internal stresses existing in a concrete beam which, if not properly cared for, may in themselves cause failure of the beam. These stresses are: (1) shearing stresses, or those tending to make one plane of concrete, either vertical or horizontal, slide along an adjacent plane; (2) diagonal tension stresses, or those which cause cracks in the concrete along inclined planes near points of maximum shear; and (3) in reinforced beams, bond stresses, or those tending to cause the steel to pull away from the concrete when under stress and thus destroy the unity of the beam.

68. Shearing Stresses. If a pile of boards is used to support a load, the boards being free to slip on each other, it is noticeable that the ends overlap even when the boards are of equal length (see Fig. 25). Slipping has occurred along the surfaces of contact. If, however, they are glued together, piled as before, the slipping is prevented, but the *tendency to slip* still exists and is known as the shearing stress in surfaces parallel to the neutral axis. These shearing stresses exist in beams of any material as long as the two sides of the surface considered form a continuous substance.

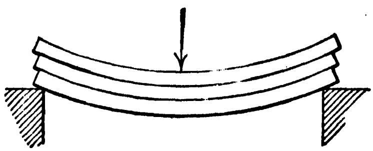


FIG. 25.

In addition to the horizontal shearing stresses described above, there are vertical stresses of the same nature, *i.e.*, a tendency for one side of the beam to slide upward past the other side. These two kinds of shearing stresses are of the same intensity per unit of area at any point in the beam.

69. Intensity of Shearing Stress in a Plain Concrete Beam. As stated in Art. 44, the value of the unit shear v in a plain concrete beam (or any homogeneous beam) is represented by the equation

$$v = \frac{QV}{Ib} \tag{9}$$

The value of the unit shear as represented by this equation becomes zero at the top and bottom surfaces of the beam, and a maximum at the neutral axis. The distribution for a rectangular section is shown in Fig. 26. The maximum stress equals $1\frac{1}{2}$ times the average, or $\frac{3}{2}\left(\frac{V}{ba}\right)$ (see Art. 44). Between the neutral axis and the extreme surfaces the value of the unit shear varies as the ordinates of a parabola. The absolute maximum value of v occurs in the section at which the total external vertical shear is a maximum.

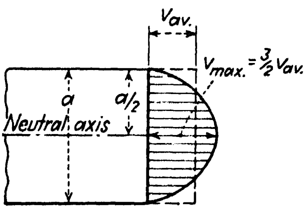


FIG. 26.

70. Intensity of Shearing Stress in a Reinforced Concrete Beam. In a non-homogeneous beam the unit shearing stresses vary in a very different manner from that described above.

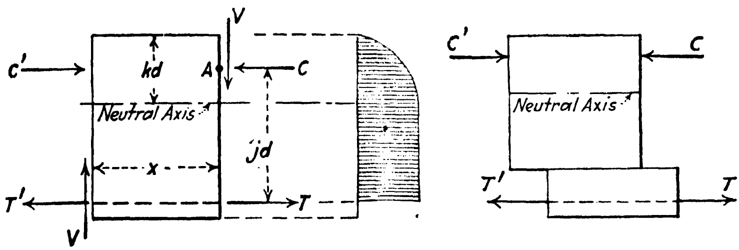


FIG. 27.

To derive an equation which expresses the variation of the unit shear at any section of a reinforced-concrete beam, consider a short length of the beam as a free body (Fig. 27a). The forces acting on this element are those of compression C and C', of tension T and T', and of total vertical shear V. Hence in Fig. 27a, which represents a length so short that no part of the external vertical load need be considered (the total vertical shear on the left equals that on the right of the element), $T - T'$ is equal to the total shearing stress, or tendency for the lower portion to slide along the upper, on any plane between the steel and the neutral axis, assuming that the concrete will take none of the tension.

This is illustrated in Fig. 27*b*. If the element is assumed on the left half of the span, and the beam is subjected to a uniform load, the fiber stresses on the right side of the element are greater than those on the left. The lower portion tends to slide toward the right and the upper portion toward the left, as shown in Fig. 27*b*. The force producing this horizontal sliding is equal to the difference in the forces acting on each part of the section, *i.e.*, $T - T'$ for the lower part and $C - C'$ for the upper. $C - C'$ must equal $T - T'$, and the shearing strength of any two consecutive horizontal planes between the neutral axis and the tension steel must be sufficient to transmit the effect of one set of these forces to the other, so as to prevent the movement indicated in Fig. 27*b*.

The unit horizontal shear on any such plane as described above is

$$v = \frac{T - T'}{bx}$$

bx being the area of the surface under consideration. The external forces acting on this portion of the beam must be in equilibrium, hence the summation of elements about any point such as A on the line of action of the compressive forces must equal zero, or

$$(T - T')jd = Vx$$

Therefore

$$T - T' = \frac{Vx}{jd}$$

Substituting this value of $T - T'$ in the above equation for v,

$$v = \frac{V}{bjd} \qquad (10)$$

This represents the value of the unit horizontal shearing stress along any plane between the steel and the neutral axis, and, as shown in Fig. 27*a*, it is also the maximum unit shear in the section. The amount of this shear per linear inch of beam for the full width b is

$$v_1 = \frac{V}{jd} \qquad (11)$$

The value of j for working loads varies between narrow limits, and this variation causes but insignificant differences in values of v. For this reason it is satisfactory to use the value of $j = 7/8$ in all computations involving shear and bond. This is an average value for beams in ordinary construction.

Above the neutral axis the shear follows the parabolic law as in the plain concrete beam (see Fig. 27*a*).

Illustrative Problem. A reinforced concrete beam (Fig. 28) has a span of 15 ft.-0 in. and supports a concentrated load of 15,000 lb. placed 5 ft.-0 in. from the left support, in addition to a uniform live load of 600 lb. per lin. ft. over the whole span. The overall cross-section of the beam is $12 \times 22\frac{1}{2}$ in., and the reinforcement consists of four $\frac{7}{8}$-in. round bars in one row, the center of which is $2\frac{1}{2}$ in. above the lower surface of the beam. The value of f'_c is 2500 p.s.i. Determine the maximum unit shearing stress in sections at the left and right supports, and in sections just to the left and right of the concentrated load.

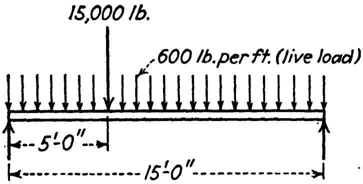


FIG. 28.

The dead weight of the beam is 280 lb. per ft., and the total uniform load is 600 + 280 = 880 lb. per ft. At the left support,

$$V = \tfrac{2}{3} \times 15{,}000 + \tfrac{15}{2} \times 880 = 16{,}600 \text{ lb.}$$

$$v = \frac{16{,}600}{12 \times \frac{7}{8} \times 20} = 79 \text{ p.s.i.}$$

At the right support,

$$V = \tfrac{1}{3} \times 15{,}000 + \tfrac{15}{2} \times 880 = 11{,}600 \text{ lb.}$$

$$v = \frac{11{,}600}{12 \times \frac{7}{8} \times 20} = 55 \text{ p.s.i.}$$

Just to the left of the concentrated load,

$$V = 16{,}600 - 5 \times 880 = 12{,}200 \text{ lb.}$$

$$v = \frac{12{,}200}{12 \times \frac{7}{8} \times 20} = 58 \text{ p.s.i.}$$

Just to the right of the concentrated load,

$$V = 16{,}600 - 5 \times 880 - 15{,}000 = -2800 \text{ lb.}$$

$$v = \frac{2800}{12 \times \frac{7}{8} \times 20} = 13 \text{ p.s.i.}$$

The practical importance of the above values is indicated in Problem I, Art. 76.

71. Inclined Tension Stresses (Diagonal Tension). Assume an infinitely small cube to be removed from a beam at any section along the neutral axis. Two pairs of shearing forces, horizontal and vertical, must be considered. These forces form two couples acting as shown in Fig. 29. Since the prism has been assumed at the neutral axis, the flexural stresses of tension and compression are zero. Therefore, the shearing stresses vbx develop inclined stresses of tension in the direction MN, and compression along the line PQ, each equal to $\frac{2vbx}{\sqrt{2}}$. Since the length of each

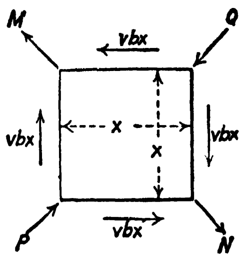


FIG. 29.

diagonal of the prism is $x\sqrt{2}$, the intensity of these inclined stresses, *i.e.*, the amount per unit area, equals $\frac{2vbx}{\sqrt{2}}$ divided by $bx\sqrt{2}$, or v. It follows that at any point along the neutral plane there exist tensile and compressive forces inclined at 45 degrees with the horizontal, and that the value of these forces per unit of area equals the unit shear at that point.

Since at all points above and below the neutral axis there exist, in addition to the horizontal and vertical shearing forces, horizontal fiber stresses of tension and compression, the values of the inclined tensile and compressive forces at such points are found by combining the fiber stresses with the shearing stresses.

Treatises on mechanics prove that the intensity of the inclined stress t at any point in a beam is represented by the equation

$$t = \frac{1}{2}f \pm \sqrt{\frac{1}{4}f^2 + v^2} \tag{12}$$

and the direction of this stress by the equation

$$\tan 2\alpha = \frac{2v}{f}$$

where f = fiber stress per unit of area.

v = intensity of vertical or horizontal shearing stress at the point.

α = angle made by the stress t with the horizontal.

In the equation for the angle of inclination of the inclined stress, two values of α, differing by 90 degrees, will satisfy, showing that the maximum compressive stress and the maximum tensile stress at any point make an angle of 90 degrees with each other.

On account of the comparatively large compressive strength of concrete, the inclined compressive stresses as determined above may be neglected; failure, if any, occurs because of the cracking of the concrete due to tensile stresses in excess of its strength.

For ordinary beams of homogeneous materials, such as beams of steel or timber, a determination of the normal, or flexural stresses, and the shearing stresses described above, gives sufficient information for purposes of design. In concrete beams, both plain and reinforced, the inclined stresses of maximum tension induced by the shearing and bending stresses are often fully as important as the maximum fiber stresses, and it is necessary to make some provision for them. This is because of the very low strength of concrete in tension. Hence the necessity for the further investigation of such stresses.

72. Diagonal Tension in Plain Concrete Beams. Examination of the above equations shows that at the center of a homogeneous beam, where the moment is a maximum, the direction of the lines of maximum tension is horizontal for the entire depth of the beam. As the end of the beam is approached, the shear becomes large and the bending moment small (assuming a simply supported beam). Here the effect of the shear on the diagonal tension is great, while the horizontal fiber stress has little weight in determining the inclination and amount of the inclined tensile

stress near the support. At the end of the beam, where the horizontal tension is zero, the diagonal tension stresses are inclined at practically 45 degrees throughout the entire depth of the beam.

Figure 30 illustrates the variation in direction of the maximum inclined tension stresses in a homogeneous rectangular beam. As seen above, the exact direction at any point depends upon the relation between shear and bending moment at the point. The short wavy lines represent the probable cracks which may form with large shearing stresses. Lines of maximum compression run at right angles to those of maximum tension.

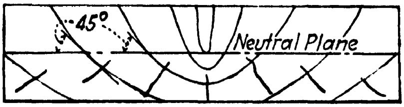


FIG. 30.

73. Diagonal Tension in Reinforced Concrete Beams. In reinforced beams, owing to the concentration of the tension in the steel, the direction of maximum tension at various depths is somewhat different from that in plain or homogeneous beams. Large shearing stresses exist immediately above the steel, and the maximum tensile stresses become considerably inclined at that plane, the exact direction depending upon the relation between the shear and horizontal fiber stress.

Figure 31 represents the general direction of the inclined tensile stresses in a uniformly loaded beam, the wavy lines representing the probable planes of rupture. The diagonal cracks near the bottom are approximately vertical at the center and become more and more inclined as the end of the beam is approached.

In beams with tension reinforcement the value of t as expressed by equation (12) is indeterminate, since the value of f, the horizontal fiber stress in the concrete, is variable. This is due to the fact that as the loading increases, the concrete cracks more and more, and the amount of tensile stress carried by it therefore decreases. The excess is picked up by the steel, and immediate failure prevented, whereas in a plain beam,

increasing the loading after the concrete commences to crack results in immediate failure of the beam. Therefore the exact amounts of the inclined tension stresses are unknown. It is seen, however, from a study of equation (12), that the vertical shearing stresses furnish a means of comparing or measuring the diagonal tension stresses. It must be remembered that *the vertical shearing stress is not the numerical equivalent of the diagonal tension stress* nor is there any definite relation between them.

By limiting the allowable unit shearing stress to a value which has been found by actual tests to be low enough to insure against failure by diagonal tension, it may be considered that the danger of such failure has been eliminated. This limit of the allowable shearing stress is considerably below the safe working stress of

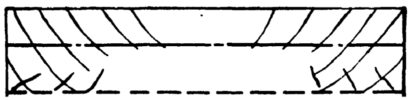


FIG. 31.

concrete in direct shear because of the fact that, when the shear in a beam is still low, the diagonal tension may be excessive. Failure occurs not by direct shear, but by the cracking of the concrete along inclined planes.

An examination of equation (12) shows that diagonal tension at any point varies with both the shear and horizontal tension in the concrete. In order to reduce the danger of failure by diagonal tension, large shearing stresses should be avoided and the horizontal tension in the concrete kept as small as possible. This latter may be accomplished by furnishing an excess of longitudinal steel at points of heavy shear, thus reducing the horizontal deformation and consequently the tension in the concrete.

74. Working Unit Shearing Stresses. When relatively large shearing stresses exist (the shear being a measure of the indeterminate diagonal tension) it becomes necessary to provide some form of web reinforcement. The Joint Code specifies $0.02f'_c$ as the safe limit of shearing stress for beams without web reinforce-

ment, and $0.06f'_c$ for beams in which adequate provisions have been made to care for the inclined stresses. If the longitudinal bars are adequately anchored by means of hooks at both ends or by some other satisfactory method, somewhat higher stresses are allowed, *i.e.*, $0.03f'_c$ for the concrete when no web reinforcement is provided and $0.12f'_c$ for beams with web reinforcement. This latter value is permitted only in certain cases where extreme reinforcement is made in the design, where definitely specified anchorage is provided, and where careful placing of the reinforcement is assured. Values much in excess of $0.06f'_c$ are normally not used in conservative practice.

75. Types of Web Reinforcement. A study of Figs. 30 and 31 shows that the most efficient web reinforcement consists of an arrangement of steel as shown in Fig. 32, the inclined portions being either a continuation of the horizontal bars or additional bars rigidly connected at their lower ends to the horizontal bars.

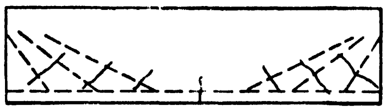


FIG. 32.

Such an arrangement is, however, not practical. Slight variations between the inclination of the reinforcing bars and the lines of maximum tension have but little effect on the efficiency of the system; hence in practice the most commonly used methods of arranging diagonal tension reinforcement are divided into three groups: (1) vertical bars or stirrups, attached to or looped about horizontal bars; (2) separate inclined bars or stirrups making an angle of 30 degrees or more with the horizontal, and secured to the horizontal bars in such a way as to prevent slipping; (3) longitudinal bars bent so that the axis of the inclined portion of each bar makes an angle of 15 degrees or more with the axis of the longitudinal portion of the bar. All inclined web reinforcement bars in a given beam are placed at the same angle with the horizontal; in the majority of cases a 45-degree angle is used. A combination of vertical stirrups and bent bars properly arranged gives the most practical and effective protection against diagonal tension failure (see Fig. 107). Separate inclined bars are seldom used.

When separate members, either vertical or inclined, are used as diagonal tension reinforcement, care must be taken to see that they are properly connected to the longitudinal steel so that slipping is prevented. When web reinforcement comes into action as the principal tension web resistance, the bond stresses between the longitudinal bars and the concrete are not distributed so uniformly along the bars as they otherwise would be, but tend to be concentrated at and near stirrups, and at and near the points where bars are bent up. When stirrups are not rigidly attached to the longitudinal bars, and the proportioning of bars and stirrups is such that local slip of the bars occurs at stirrups, the effectiveness of the stirrups is impaired, though their presence still gives an element of resistance to diagonal tension failure. It is on the tension side of a beam that diagonal tension develops in a critical way, so proper connection should always be made between stirrups or other web reinforcement and the longitudinal tension reinforcement. Where negative moment exists, as is the case near the supports in a continuous beam, web reinforcement to be effective must be looped over, or wrapped around, or be connected with the longitudinal tension reinforcing bars at the top of the beam in the same way as is necessary at the bottom of the beam at sections where the bending moment is positive. Requirements of the Joint Code with regard to the anchorage of web reinforcement bars are given in Art. 83. In all cases, stirrups should be carried as close to the upper and lower surfaces as fire-proofing requirements will permit.

76. Region Where No Web Reinforcement Is Required. As mentioned in Art. 74, web reinforcement is not required in regions where the unit shear is less than a given percentage of the ultimate compressive strength of the concrete; that is to say, the concrete is assumed capable of withstanding all of the diagonal tension as measured by a unit shear of that amount.

In a uniformly loaded beam, the distance from the support beyond which stirrups are not required is determined as follows: Let x_1 = the distance to be found, v_c the unit shear x_1 ft. from the support, and V_c the total shear at that point.

$$V_c = \frac{wl}{2} - wx_1 \qquad \text{and} \qquad v_c = \frac{V_c}{bjd}$$

By substituting the value of V_c from the former in the latter equation, and solving for x_1, this becomes

$$x_1 = \frac{l}{2} - \frac{v_c bjd}{w} \tag{13}$$

For beams with unsymmetrical or concentrated loads, the points where web reinforcement may be discontinued may be located by constructing the shear diagram for the beam and noting the point or points at which the unit shear is less than the given proportion (see Art. 74) of the compressive strength of the concrete.

Illustrative Problems. I. Consider the beam of the problem in Art. 70. Since for that beam $f'_c = 2500$ p.s.i., the allowable unit shear without web reinforcement, assuming that the main reinforcement is not anchored at the ends, is $0.02 \times 2500 = 50$ p.s.i. Determine the regions over which web reinforcement is required.

Since the unit shear at any section between the left support and the concentrated load (Fig. 28) is greater than 50 p.s.i., web reinforcement is required throughout this distance. The unit shear at the right support is 55 p.s.i., and this shear decreases uniformly to 13 p.s.i. at a section just to the right of the concentrated load. Hence web reinforcement is required from the right support to the section at which the unit shear is equal to 50 p.s.i. This section can be located by a simple proportion, as follows: Let x_1 be the required distance from the right support. Then

$$\frac{x_1}{10} = \frac{55 - 50}{55 - 13}$$

and

$$x_1 = 1.2 \text{ ft.}$$

A more general method of computing x_1 is as follows: The unit shear v at any section x ft. from the right support is

$$v = \frac{V}{bjd} = \frac{11{,}600 - 880x}{12 \times \frac{7}{8} \times 20}$$

At section x_1 the unit shear is 50 p.s.i. Hence,

$$50 = \frac{11{,}600 - 880x_1}{12 \times \frac{7}{8} \times 20}$$

from which $x_1 = 1.2$ ft., as above. Web reinforcement is required therefore for a distance of 1.2 ft. from the right support, and 5.0 ft. from the left support.

II. A reinforced concrete beam has a span of 18 ft.-0 in. and is to sustain a uniform live load of 1000 lb. per lin. ft. The reinforcement consists of three ¾-in. round bars placed 18 in. below the upper surface and 2½ in. above the lower surface of the beam. The width of the beam is 8 in. and a 2500-lb. concrete is to be used in the construction. Determine the regions over which web reinforcement is required, assuming that the longitudinal bars are not specially anchored.

The weight of the beam is 170 lb. per ft., and the total load is therefore $1000 + 170 = 1170$ lb. per ft. The unit shear at the supports, equation (10), is 83 p.s.i.; since this exceeds the allowable value for beams without web reinforcement, such reinforcement is required at the supports and for a distance x_1 ft. from the supports. Since the beam supports *only* a uniform load, equation (13) can be used in obtaining the value of x_1. Hence,

$$x_1 = \frac{18}{2} - \frac{50 \times 8 \times \frac{7}{8} \times 18}{1170} = 3.6 \text{ ft.}$$

77. Distribution of Diagonal Tension. Tests show that, in beams with web reinforcement, both the steel and the concrete resist the diagonal tension. The Joint Code specifies that the concrete shall be assumed to resist a total shear equivalent to a unit shear of $0.02f'_c$ p.s.i. for beams in which the longitudinal bars are not specially anchored, and $0.03f'_c$ p.s.i. for beams in which the longitudinal bars are adequately anchored, and that the remainder of the shear shall be resisted by the web reinforcement.

78. Spacing and Size of Vertical Stirrups. In Art 70 (equation 11) it was shown that the horizontal shear per linear inch of beam on any plane below the neutral axis is equal to $\frac{V}{jd}$. Hence,

for a length of beam s, the amount of horizontal shear may be represented by the equation $\frac{Vs}{jd}$, in which V is the average total vertical shear over the length s (at B in Fig. 33). Since the horizontal shear equals the vertical shear, it follows that this equation also represents the vertical component of the diagonal tensile stress in a length s. The horizontal component is resisted by the horizontal steel, while the vertical component is resisted by the concrete and web reinforcement.

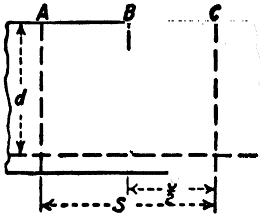


FIG. 33.

If vertical stirrups of an effective area A_v are employed, and if the allowable unit stress in the stirrups is f_v, then the safe strength of each stirrup is A_vf_v. The area A_v is the total area of all of the vertical legs in one stirrup; thus, if a U-shaped stirrup is used, A_v is equal to two times the area of the bar of which the stirrup is formed.

For the method of distribution as recommended in Art. 77, the vertical component of the diagonal tension to be resisted by the stirrups over a distance s is $\frac{(V - V_c)s}{jd}$, in which V_c is the amount of total shear that is assigned to the concrete as determined from equation (10), $v_c = \frac{V_c}{bjd}$.

Let $V - V_c = V'$; then

$$A_vf_v = \frac{V's}{jd}$$

and the required spacing of stirrups is

$$s = \frac{A_vf_vjd}{V'} \tag{14}$$

The Joint Code recommends that the longitudinal spacing of vertical stirrups shall not exceed $0.5d$, except that in beams where the unit shearing stress exceeds $0.06f'_c$ the limit is $0.25d$. The specification is worded so as to apply to both vertical stirrups

and inclined bars; it is given and explained in Art. 79. A spacing of less than 4 in. is generally undesirable. Tests indicate that the most effective results are obtained when $s = \frac{1}{3}d$. It is usually satisfactory to select a certain size of stirrup and to calculate the spacing at different points along the beam; if the computed spacings are unsatisfactory, another size of stirrup may be assumed and the corresponding spacings determined.

Common sizes of stirrup bars are ¼-, ⅜-, and ½-in. round. The diameter varies with the depth of the beam; ¼-in. bars are generally satisfactory for beams less than 10 in. deep, ½-in. bars for beams 36 in. deep, and ⅜-in. bars for intermediate depths.

Where web reinforcement is provided by means of vertical stirrups and is required over a comparatively short distance, it is good practice to space the stirrups uniformly over the entire distance, the spacing being calculated for the point of greatest shear (minimum spacing). If the web reinforcement is required over a long distance, if the shear varies materially throughout this distance, it is more economical to compute the spacings required at several sections and to place the stirrups accordingly, in groups of varying spacings.

Illustrative Problem. Consider again the beam which was described in the problem in Art. 70 and for which the regions over which web reinforcement is required were determined in Problem I, Art. 76. What are the required spacings of ¼-in. round vertical U-stirrups at the left and right supports? Assume $f_v = 18{,}000$ p.s.i. as specified in the Joint Code for structural-grade steel.

The total shear that can be resisted by the concrete $= V_c = 50 \times 12 \times \frac{7}{8} \times 20 = 10{,}500$ lb. At the left support the total shear is 16,600 lb., and, from equation (14),

$$s = \frac{2 \times 0.0491 \times 18{,}000 \times \frac{7}{8} \times 20}{16{,}600 - 10{,}500} = 5.1 \text{ in.}$$

At the right support the total shear is 11,600 lb., and

$$s = \frac{2 \times 0.0491 \times 18{,}000 \times \frac{7}{8} \times 20}{11{,}600 - 10{,}500} = 28.1$$

According to the Joint Code, the maximum allowable spacing is $0.50 \times 20 = 10$ in.

In detailing the beam, two stirrups will be placed near the right support, one about 5 in. from the edge of the support, and another 10 in. from the first stirrup. This carries the web reinforcement beyond the section where such reinforcement is no longer required (1.2 ft. from the support, as determined in Problem I, Art. 74). At the left end, a 5-in. spacing could be used between the left support and the concentrated load, the first stirrup being placed about 2 in. from the edge of the support; but a more economical arrangement could be made there by computing the required spacing just to the left of the concentrated load and varying the spacing from the left support to that point. For example, just to the left of the concentrated load the total shear is 12,200 lb. (see problem in Art. 70), and

$$s = \frac{2 \times 0.0491 \times 18{,}000 \times \frac{7}{8} \times 20}{12{,}200 - 10{,}500} = 18.2 \text{ in.}$$

Since the required spacing varies uniformly from 5.1 in. at the left support to 18.2 in. at the concentrated load, the maximum allowable spacing being 10 in. in accordance with the Code recommendation, by simple proportion the following selection

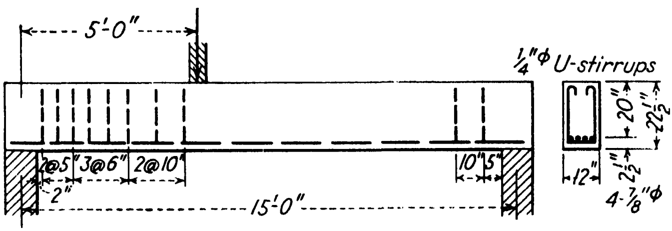


FIG. 34.

of spacings may be made: 2 at 5 in., 3 at 6 in., and 2 at 10 in. If a constant spacing of 5 in. had been maintained, 12 stirrups would have been required, instead of 8 as used above. The placing of the stirrups is shown in Fig. 34.

79. Spacing of Bent-up Bars or Inclined Stirrups. When web reinforcement consists of inclined bars, whether these bars are part of the longitudinal steel or are separate inclined stirrups,

more of the probable planes of rupture are crossed by a given length of bar than is the case when vertical bars are used. Inclined steel is, therefore, more effective in resisting diagonal tension stresses than vertical steel of the same amount. In Fig. 35, the diagonal force DE representing the diagonal tension over a distance s is the sum of the components of the horizontal force AB and the vertical force BC in the direction of DE. The components normal to the direction of DE are compressive forces and are absorbed by the concrete. It is assumed that the horizontal force AB is prevented from causing failure in the concrete by the longitudinal steel in the beam. The force BC or its component BE must be otherwise provided for. If this force is taken care of by vertical stirrups or bars, the amount to be resisted is represented by BC. When resolved into components, BE and CE, the amount to be resisted by inclined bars or stirrups is BE, which is BC (sin α), where α is the angle of inclination of the bar. BC represents the total external shear over the distance s, and the amount of stress to be resisted by the inclined bars is Vs (sin α). The maximum spacing of inclined bars is calculated in the same manner as for vertical stirrups, V' (sin α) being substituted for V' in equation (14), giving as a result

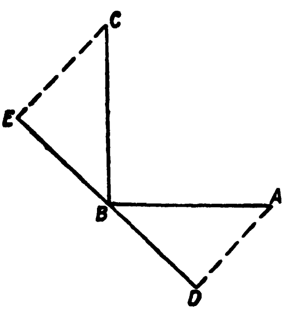


FIG. 35.

$$s = \frac{A_v f_v jd}{V' \sin \alpha} \tag{15}$$

For the special case where α is 45 degrees, this equation may be reduced to

$$s = \frac{A_v f_v jd}{0.7V'} \tag{15a}$$

The distance s, measured in the direction of the axis of the beam between two successive inclined stirrups or bent bars, is limited by the Joint Code to an arbitrary maximum value which

is defined as follows: "Where web reinforcement is required it shall be so spaced that every 45-degree line (representing a potential crack) extending from the mid-depth of the beam to the longitudinal tension bars shall be crossed by at least one line of web reinforcement, except that, if a shearing unit stress in excess of $0.06f'_c$ is used, every such line shall be crossed by at least two such lines of web reinforcement." The interpretation of this specification, for beams in which v is less than $0.06f'_c$, is illustrated in Fig. 36. When bars are bent up at a 45-degree

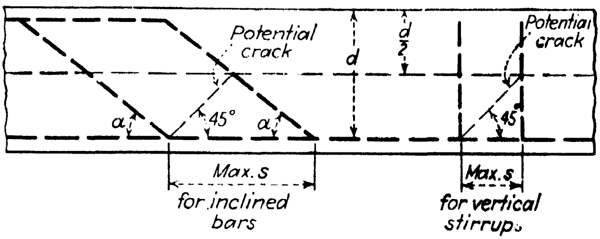


FIG. 36.

angle, from Fig. 36 it is obvious that the maximum spacing is equal to d when v is less than $0.06f'_c$, and $\frac{d}{2}$ when v is greater than $0.06f'_c$.

While separate inclined stirrups are more efficient theoretically than vertical stirrups, this advantage is counteracted somewhat because of the difficulty in fastening them to the tension steel and of assuring their correct position after the concrete is poured. For this reason, separate inclined stirrups are seldom used. This difficulty obviously does not exist in the case of bent-up longitudinal bars, since the inclined portion is merely a continuation of the horizontal portion of the bar.

80. Arrangement of Bent-up Bars. The points where horizontal bars may be bent up are governed by the amount of steel required to care for the horizontal fiber stresses caused by the bending moment at different sections along the beam. Since, in a simple beam, the bending moment decreases toward the ends, the total tension in the steel decreases in the same ratio. Enough steel must always remain at the bottom to care for the

bending stresses and for bond stresses; the remainder may be bent up to aid in resisting the diagonal tension. According to the Joint Code, not more than one-half of the steel may be so bent. The bars which are not bent shall extend into the support a distance of 10 or more bar diameters, or, if this distance is not available, the bars should be extended into the support as far as possible and terminated in standard hooks.

In a continuous beam, bending of the bars must be done in such a way as to satisfy both positive- and negative-moment stress requirements. These are discussed in Art. 104. According to the Joint Code at least one-quarter of the positive-moment steel area must extend along the same face of the beam into the support a distance of 10 or more bar diameters. At the outer ends of freely supported end spans of continuous beams, the requirements for simple beams, as given in the preceding paragraph, shall apply.

The location of the points of bending may be determined graphically as follows: Plot the bending-moment diagram for the given loads. Since the amount of tensile steel required at any section of the beam is proportional to the bending moment, the maximum ordinate of the bending-moment diagram may also be made to represent the total area of steel reinforcement. Assuming that all of the reinforcing bars are of the same area and will be stressed equally at the point of maximum moment, divide the maximum ordinate into the same number of equal parts as there are bars crossing the section of maximum moment. Draw a horizontal line through each point of division. The intersection of any one of these horizontal lines with the bending-moment curve locates a point beyond which all of the bars in excess of the number represented by the line may be bent up. The Code requires that the theoretical points must be exceeded by at least 12 bar diameters. The moment diagram for a uniformly loaded beam, when the condition of the supports is such as to give values of maximum moments equal to either $\frac{wl^2}{8}$ or $\frac{wl^2}{12}$, is shown in Diagram 1, Appendix *D*. Percentages of steel are

given in place of numbers, as explained above, so as to take care of those cases in which the bars are not all of the same size.

The bars which are bent should be selected so that the symmetry of the remaining bars about the vertical axis of the cross-section is not destroyed. As a rule, at least two bars are bent up together from corresponding points on either side of the beam. Sometimes, however, it becomes necessary to depart from this procedure, as when three bars are to be bent. Then either first one is bent, and then two, or all three are bent at the same point. In either case the odd bar is bent from the middle of the section of the beam if possible.

When the bending is done at two or more points, the distance between the points of bending, and if possible between the point of bending of the bar nearest the support and the edge of the support, should not exceed the limiting values given in Art. 79. The bent bars can then be assumed to take care of all of the diagonal tension between the point at which the first bar is bent up and the support, in accordance with the discussion in Art. 79, provided that the individual distances between points of bending do not exceed that computed from equation (15). If the distance from the point at which the bars nearest to the support are bent, and the support, exceeds either of these limitations, vertical stirrups will be required adjacent to the support, to provide for the diagonal tension over the excess distance. The spacing of these stirrups is computed as explained in Art. 78.

81. Bond Stresses. When steel bars are placed in a beam, there must be sufficient bond between the steel and the concrete to prevent the bars from pulling out when stressed. This bond stress, or tendency for the steel to slide out of the concrete, per square inch of bar surface, may be determined as follows:

If, in the short section of beam discussed in Art. 70, moments about the point A (Fig. 37) are taken,

$$(T - T')jd = Vx$$

and

$$\frac{T - T'}{x} = \frac{V}{jd}$$

But $T - T'$ represents the force which tends to pull the bars out of the concrete in a length x. Hence $\frac{T - T'}{x}$ represents this force per unit length of beam. Therefore $\frac{V}{jd}$ is the bond stress per unit of length between the two materials. If u is the bond stress per unit area of exposed steel surface, and Σ_0 the total perimeter of steel,

$$u = \frac{V}{\Sigma_0 jd} \tag{16}$$

This equation applies to the steel in tension only.

In order to prevent the pulling out of the bars when stressed, the value of the unit bond stress as computed by equation (16) should not exceed a safe working limit which has been established from a study of the results of tests on beams in which such failure has occurred (see Art. 82). Where high bond resistance is required, end anchorage, consisting of hooks bent through an angle of 180 degrees, may properly be used. It must be remembered, however, that adequate bond strength throughout the length of the bar is preferable to such anchorage.

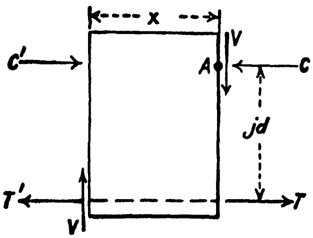


FIG. 37.

Illustrative Problem. In the beam described in the problem in Art. 70, determine the maximum unit bond stress, assuming that all of the bars continue in the bottom of the beam for the full length of the beam.

The maximum shear occurs at the left support (16,600 lb.); hence the maximum unit bond stress exists at that support. From equation (16),

$$u = \frac{16{,}600}{4 \times 2.749 \times 7/8 \times 20} = 86 \text{ p.s.i.}$$

82. Working Unit Bond Stresses. The Joint Code recommends that the unit bond stress between concrete and plain reinforcing bars in beams and slabs shall not exceed $0.04f'_c$, and

that the bond stress on approved deformed bars shall not exceed $0.05f'_c$, unless special anchorage is provided at the ends of the bars.

Where special anchorage is provided, the above values may be increased 50 per cent. Special anchorage in simple beams may consist of a standard hook at each end of each bar. Special anchorage in cantilever beams may consist of a standard hook at the free end of each bar. A standard hook (see Fig. 38) shall have a complete semicircular turn with a radius of bend on the axis of the bar of not less than three and not more than six bar diameters, plus an extension of at least four bar diameters at the free end of the bar. Special anchorage in continuous beams may be secured by bending the bar or bars across the web at an angle of not less than 15 degrees with the longitudinal portion of the bars and making them continuous with the negative or positive reinforcement. Special anchorage in continuous beams may also consist of: a standard hook at the end of the bar or bars in a region of compression, where these bars do not comply with the preceding requirement. Hooks shall not be permitted in the tension portion of a continuous beam, except at the freely supported ends.

The requirements for footings are discussed in Chap. VII.

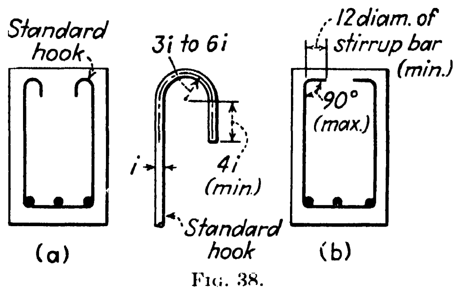


Fig. 38.

83. Anchorage of Web Reinforcement. Vertical U-stirrups, or vertical multiple-loop stirrups, are anchored at the loop ends by bending around the longitudinal reinforcement, as shown in Fig. 38. At the cut ends, anchorage is provided by a standard hook as shown in Fig. 38*a* or by bending the end of the stirrup bar

through an angle of at least 90 degrees around a longitudinal reinforcing bar not less in diameter than the stirrup bar and projecting the stirrup bar at least 12 diameters past the bend, as shown in Fig. 38*b*. In case there is sufficient depth in the compression area of the beam to permit sufficient length of embedment to develop in bond the full stress in the stirrup bar, at a unit bond stress of not exceeding $0.04f'_c$ for plain bars or $0.05f'_c$ for deformed bars, the hooks or bends at the cut ends of the stirrup bars may be omitted. The required length of embedment is computed from the equation in Art. 43. In computing the length available for embedment in the compression area, the middle of the effective depth d of the beam may be considered to be the dividing line between the tension and compression areas. Inclined stirrups are anchored in the same manner as for vertical stirrups; but, in addition, they must be welded or otherwise rigidly attached to the longitudinal reinforcing bars.

84. Typical Web Reinforcement Problem. A complete design of a reinforced concrete beam includes not only the computation of the concrete section and steel area required to resist the bending stresses but also the determination of bond and shearing stresses and adequate provision for taking care of such stresses. The method of providing for diagonal tension in a beam by bending up some of the horizontal steel at points of heavy shear may best be illustrated by making a complete design of a simply supported beam. The connection between the preceding discussions and this problem should be noted carefully.

Design a simply supported rectangular reinforced concrete beam with a span of 16 ft.-0 in., to support a uniform live load of 500 lb. per lin. ft., and three concentrated loads of 16,000 lb. each, placed at the quarter points of the span. A 3000-lb. concrete is assumed. Allowable stresses as specified in the Joint Code will be used; intermediate-grade steel will be assumed.

Design for Bending Stresses. Assume that the beam will weigh 400 lb. per lin. ft. Then w = 900 lb. per ft. The total moment is a maximun at the center line of the span, and this is equal to the sum of the uniform-load and concentrated-load moments.

M (uniform load) $= \frac{1}{8} \times 900 \times 16^2 \times 12 = 345{,}000$ in.-lb.

M (concentrated load) $= (24{,}000 \times 8 - 16{,}000 \times 4)12 =$ 1,536,000 in.-lb.

M (total, maximum) $= 1{,}881{,}000$ in.-lb.

From Table 6, $K = 197$ and $j = 0.875$

$$bd^2 = \frac{1{,}881{,}000}{197} = 9560 \text{ in.}^3$$

Let $b = 14$ in.; then d (required) $= 26.2$ in. Select $d = 26\frac{1}{2}$ in. Assuming that two rows of steel will be necessary, allowing $2\frac{1}{2}$ in. insulation below the center of the lower row, and 2 in. center to center of rows, the total height of beam is 30 in., and the weight per foot 440 lb.

Revised M (uniform load) $= {}^{940}\!/_{900} \times 345{,}000 = 360{,}000$ in.-lb.

Revised M (total, maximum) $= 1{,}896{,}000$ in.-lb.

Revised bd^2 (required) $= 9630$ in.^{3}

Select $b = 14$ in. and $d = 26\frac{1}{2}$ in., as before.

$$A_s = \frac{1{,}896{,}000}{20{,}000 \times 0.875 \times 26.5} = 4.08 \text{ sq. in.}$$

Four $\frac{3}{4}$-in. round bars and four $\frac{7}{8}$-in. round bars will be used; the four larger bars will be placed in the lower row, and the four smaller bars in the upper row, as shown in Fig. 39.

Design for Diagonal Tension Stresses. The bars which can be bent up and used to provide for the diagonal tension stresses are those which are not needed to furnish sufficient surface for the development of the bond stresses at the support.

The maximum end shear $= 24{,}000 + (8 \times 940) = 31{,}560$ lb. Since the allowable maximum unit bond stress for deformed bars without special anchorage, according to the recommendations in Art. 82, is 0.05×3000, or 150 p.s.i., the total perimeter of bars required at the support, from equation (16), is,

$$\Sigma_0 = \frac{31{,}560}{150 \times \frac{7}{8} \times 26.5} = 9.07 \text{ in.}$$

The four $\frac{7}{8}$-in. bars are required, thus leaving the four $\frac{3}{4}$-in.

bars to be bent up to reinforce the beam against diagonal tension.

At the support, $v = \dfrac{31{,}560}{14 \times \frac{7}{8} \times 26.5} = 97$ p.s.i.

At the left of the first concentrated load, $V = 31{,}560 - (4 \times 940) = 27{,}800$ lb., and

$$v = \frac{27{,}800}{14 \times \frac{7}{8} \times 26.5} = 86 \text{ p.s.i.}$$

At the right of the first concentrated load the unit shear is less than the allowable value of 60 p.s.i. and no web reinforcement is required beyond the load.

$$V_c = 60 \times 14 \times \tfrac{7}{8} \times 26.5 = 19{,}500 \text{ lb.}$$

The shaded portions of the shear diagram, Fig. 39, represent the amount of shear at any section that must be resisted by the web reinforcement.

In order to preserve the symmetry of the reinforcement at all points, the $\frac{3}{4}$-in. bars will be bent up in pairs and at 45 degrees with the horizontal. The maximum distance, measured from the point of bending, over which each pair of bars can provide for diagonal tension without overstressing the bars, is, from equation (15*a*),

$$s = \frac{2 \times 0.4418 \times 20{,}000 \times \frac{7}{8} \times 26.5}{0.7(31{,}560 - 19{,}500)} = 48 \text{ in.}$$

Since this is computed for the point of maximum shear and since it is greater than the arbitrary maximum permitted by the specification as given in Art. 79, *i.e.*, s (maximum) $= d = 26.5$ in., the latter value will govern the final selection of the points of bending and no further investigation of the diagonal tension stresses in the bars will be necessary.

Investigation must now be made to determine whether these bars may be bent up at the proper points to care for all of the diagonal tension, and still leave enough steel at the bottom at all sections to care for the flexual stresses. Part (*c*) of Fig. 39 shows the bending-moment diagram plotted to scale. This is constructed by plotting the bending-moment curves for uniform

load and for the concentrated loads on the same coordinate axis and then adding the two graphically.

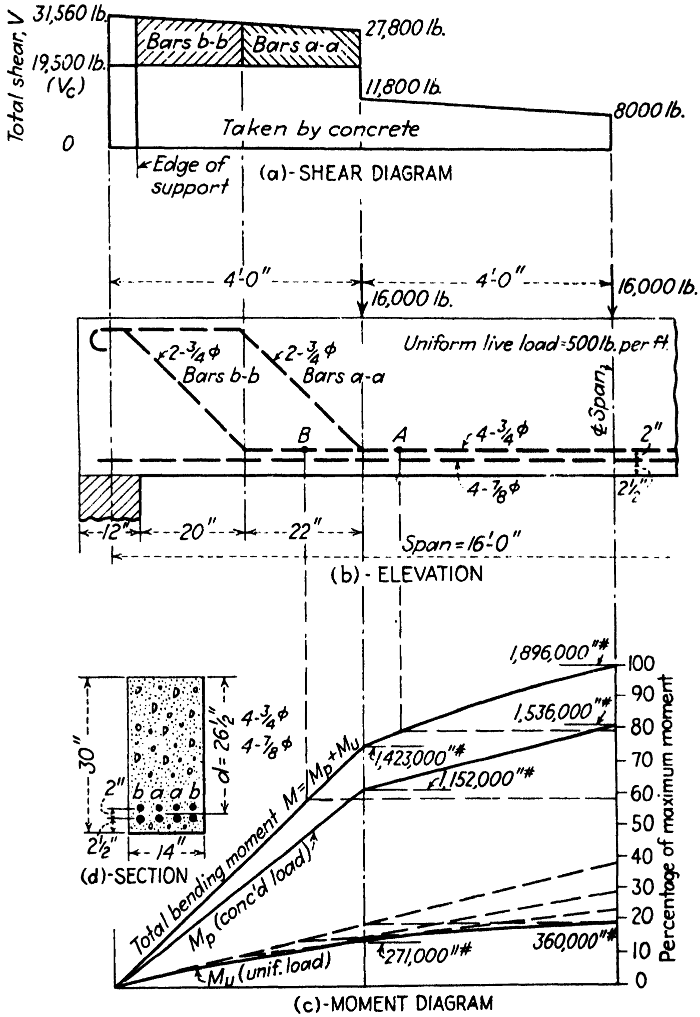


FIG. 39.

In determining the points at which the bars may be bent up and still leave sufficient tensile resistance at the bottom of the beam, the difference in sizes of the bars must be considered. One pair of ¾-in. bars is equivalent to 0.21 of the steel. When

the first pair is bent, there is left 0.79 of the total steel. When the second pair is bent, 0.58 of the total remains at the bottom of the beam.

Point A, vertically above the point of intersection of a horizontal line through the 0.79 ordinate of the moment diagram and the total-bending-moment curve, represents a point to the left of which two ¾-in. bars may be bent up. The remaining six bars are sufficient properly to provide for the tensile stress due to bending at any section between point A and the support. Similarly, at point B a total of 0.42 of the steel, or four ¾-in. bars, may be bent.

The first pair of bars will be bent at the concentrated load, and the next pair 22 in. to the left of this point. The remaining distance to the edge of the 12-in. support is 20 in. Since the amount of shear to be resisted by the web reinforcement is practically constant throughout the entire distance, being only slightly greater as the support is approached, the above arrangement assures practically equal stresses in each pair of bent bars; the slightly smaller distance (20 in.) from the point of bending of bars *b-b* to the edge of the support, as compared with the distance (22 in.) between the points of bending of bars *a-a* and *b-b*, tends to equalize the total amount of diagonal tension, as measured by the shear, that is to be resisted by each pair of bars.

The bent bars must have a length above the middle of the depth of the beam equal to $\frac{20{,}000}{4 \times 150} \times \frac{3}{4} = 25$ in. (see Art. 43). Since this length cannot be furnished in the case of bars *b-b*, the deficiency must be offset by hooking these bars, as shown in Fig. 39. As an additional safeguard, bars *a-a* will be continued to the end of the beam, and hooked. The straight bars should be continued into the support a distance of $10 \times \frac{7}{8} = 9$ in., which can be done if the support is 12 in. wide, as assumed, so that no hooks are required at the ends of these bars.

ADDITIONAL PROBLEMS

1. A simply supported rectangular beam, with a span of 16 ft.-0 in., has an overall cross-section of 12 × 25 in. and is reinforced with two ¾-in. and two ⅞-in. round bars in one row, the center of which is 2½ in. above the

lower surface of the beam. The beam supports a concentrated load of 16,000 lb. at a point 4 ft.-0 in. from the left support and a uniform live load of 690 lb. per ft. over the entire span. The bars are not anchored at the ends, and all four bars continue near the bottom of the beam for the full span. A 2000-lb. concrete is to be used in the beam, with reinforcement of intermediate-grade steel. Compute the unit shear at the left and right supports and at sections on either side of the concentrated load.

2. For the beam of Problem 1, compute the maximum unit bond stress and compare with the allowable bond stress.

3. For the beam of Problem 1, determine the regions over which web reinforcement is required.

4. For the beam of Problem 1, compute the required spacing of ¼-in. round U-stirrups at the left support, at a section 3 ft. from the left support, and at the right support.

5. A simply supported rectangular beam 10 in. wide has a span of 8 ft.-0 in. and supports a uniform live load of 6000 lb. per lin. ft. If f'_c = 3000 p.s.i. and f_s = 20,000 p.s.i., determine: (*a*) the depth required to provide adequately for moment and shear; (*b*) the number of 1-in. round bars to provide for moment and bond, assuming that the bars are all straight and without hooks.

6. For the beam of Problem 5, determine: (*a*) the distance from each support over which web reinforcement is required; (*b*) the required size and spacing of vertical U-stirrups at the support.

7. For the beam of Problem 4, page 65, determine the regions over which web reinforcement is required and the spacing of ¼-in. round U-stirrups at the right support. Is this spacing satisfactory, or should the size of the stirrups be changed? If any change is necessary, what size and spacing should be used? Assume that all bars are straight for the full length of the beam and that there are no hooks on the ends of these bars.

T-Beams

85. Types of T-beams. When a reinforced concrete floor slab is constructed as a monolith with the supporting beam,

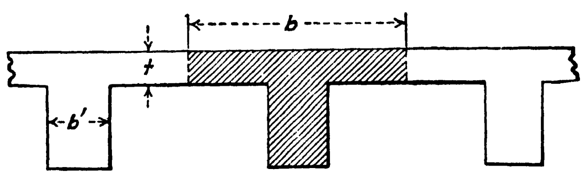


Fig. 40.

and the slab and the beam are thoroughly tied together by means of stirrups and bent-up bars, part of the slab may be assumed to

assist the upper part of the beam in resisting compressive stresses. These two acting together constitute what is known as a T-beam (Fig. 40). The slab is called the flange, and the portion of the beam beneath the slab is called the web or stem.

The exact width of slab that can be assumed as resisting the compressive forces is a variable. Tests have shown that it is dependent principally upon the relative thickness of the slab and upon the span of the beam. The Joint Code recommends that the effective width[1] of slab shall be determined as follows:

(*a*) It shall not exceed one-fourth of the span length of the beam.

(*b*) Its overhanging width on either side of the web shall not exceed eight times the thickness of the slab.

(*c*) In any case the flange width must not be greater than the distance center to center of adjacent beams.

Another form of T-beam, which is of infrequent occurrence, is one which does not form a part of a floor system, the flange being provided merely to furnish sufficient area in compression. Since the concrete in the lower part of the beam is assumed as taking no tension, its only purpose is to bind the tensile steel and the compressive concrete together. This involves mainly shearing stresses; all of the rectangular section is not required in large beams, and so a saving in concrete results when the T-form is used. It is, however, usually more satisfactory to use a rectangular beam with compressive reinforcement to care for cases requiring an excessive amount of concrete rather than to resort to the T-section. A saving in cost of forms, and certain evident structural advantages of the rectangular beam will, in general, counteract the saving in concrete in the T-beam.

The neutral axis of a T-beam may lie either in the flange or in the web, depending upon the relation between the thickness of the flange, the depth of the beam, and the amount of steel. When the neutral axis is in the flange, *i.e.*, when kd is less than t

[1] For beams having a flange on one side only, the effective overhanging width shall not exceed one-twelfth of the span length of the beam, nor six times the thickness of the slab, nor one-half the clear distance to the next beam.

the equations derived in Art. 47 for rectangular beams must be used, the width of the beam being equal to the effective width b of the flange. The reason for this is shown with the aid of Fig. 41a, which represents a beam, T-shaped in cross-section. The neutral axis is assumed above the bottom of the flange. The compressive area is represented by the shaded portion of the figure. If the additional concrete, indicated by the areas (1) and (2), had been added when the beam was poured, the physical

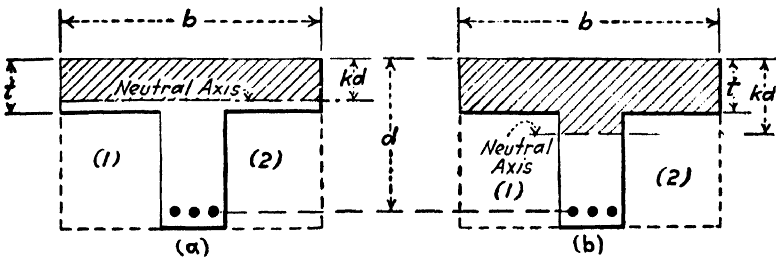


FIG. 41.

cross-section would be rectangular in shape, with a width equal to b. No bending strength would be added by the addition of this extra concrete, because the areas (1) and (2) are in the tension portion of the cross-section, and, as stated before, the tensile strength of the concrete is disregarded in all flexure formulas. The original T-shaped beam and the revised rectangular-shaped beam are equal in flexural strength, and the rectangular beam equations for flexure apply.

When the neutral axis is in the web, *i.e.*, when kd is greater than t, the rectangular beam equations no longer apply. For, in Fig. 41b, if the extra concrete represented by the areas (1) and (2) were added to the original T-shaped beam, the resulting rectangular-shaped beam would actually be stronger in flexure than the original beam, because some of the added concrete [those portions of areas (1) and (2) which are above the neutral axis] would be in compression. The application of the rectangular beam equations to this condition would therefore be incorrect in theory. The proper equations for use in this case are derived in the following article in a manner similar to that used in the

derivation of the rectangular beam equations, the difference in compression areas being taken into consideration.

When the neutral axis is at the bottom of the flange, *i.e.*, when $kd = t$, then by comparison of Figs. 41*a* and 41*b* it is obvious that both the rectangular beam equations and the T-beam equations will give the same results; *i.e.*, the values of k, j, M_c, M_s, etc., obtained by the one set of equations, will be the same as the corresponding values obtained by the other set of equations.

86. Flexure Formulas (Neutral Axis in the Web). Figure 42 represents an element of a T-beam. The amount of compression in the web, represented by the area *qrst* in the cross-section, is usually small in comparison with that in the flange, and hence it is neglected in the derivation of equations for ordinary design.

From the assumption that deformations vary as the distance from the neutral axis,

$$\frac{AA'}{BB'} = \frac{kd}{d - kd} = \frac{k}{1 - k} \tag{a}$$

Since $E = \dfrac{\text{unit stress}}{\text{unit deformation}}$, assuming that AA' and BB' represent the deformations of a unit length of the beam at the extreme compression surface and the plane of the reinforcement, respectively,

$$AA' = \frac{f_c}{E_c} \qquad \text{and} \qquad BB' = \frac{f_s}{E_s}$$

Hence,

$$\frac{AA'}{BB'} = \frac{nf_c}{f_s} = \frac{n}{r} \tag{b}$$

Equating (*a*) and (*b*) and solving for k,

$$k = \frac{n}{n + r} \tag{17}$$

This gives an expression for the value of k when n and r are known. This equation can be used only in the design of an isolated T-beam, *i.e.*, one not a part of a floor system, since in such a problem just enough flange width will be provided to bring the

unit stress in the concrete to its maximum allowable value simultaneously with that in the steel—the ratio r is known. In a T-beam which is part of a floor system already designed, the compressive area is so large (b is taken as one-fourth the span or $16t + b'$) that when f_s is a maximum, f_c is only a relatively small value—the ratio r is not known.

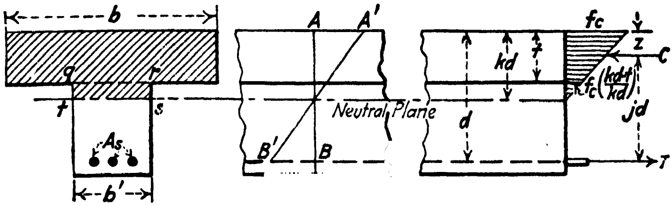


FIG. 42.

The total tension $= A_s f_s$.

The total compression is represented by a trapezoid whose parallel sides are f_c and $f_c \times \frac{kd - t}{kd}$, the amount of compression being, therefore,

$$\frac{f_c + f_c \times \frac{kd - t}{kd}}{2} \times t \times b = f_c \times \frac{2kd - t}{2kd} \times bt$$

For equilibrium, the total tension must equal the total compression; hence,

$$A_s f_s = pbdf_s = f_c \times \frac{2kd - t}{2kd} \times bt \qquad (c)$$

As in the derivation of rectangular beam equations, the relation between the unit stresses in steel and concrete is given by the equation

$$f_c = \frac{f_s k}{n(1 - k)} \qquad (18)$$

Substituting from equation (18) in (c) to eliminate unit stresses,

$$k = \frac{np + \frac{1}{2}\left(\frac{t}{d}\right)^2}{np + \left(\frac{t}{d}\right)} \qquad (19)$$

The distance of the center of compression (center of gravity of the trapezoid) from the upper face of the beam is

$$z = \frac{3kd - 2t}{2kd - t} \cdot \frac{t}{3} \tag{20}$$

and the lever arm of the couple formed by the tensile and compressive forces is

$$jd = d - z \tag{21}$$

From equations (19), (20), and (21),

$$j = \frac{6 - 6\left(\frac{t}{d}\right) + 2\left(\frac{t}{d}\right)^2 + \left(\frac{t}{d}\right)^3 \cdot \left(\frac{1}{2pn}\right)}{6 - 3\left(\frac{t}{d}\right)} \tag{22}$$

The resisting moments of the steel and concrete are equal to the product of the lever arm jd of the internal stress couple and the total tension and compression, respectively; hence,

$$M_s = A_s f_s jd \tag{23}$$

and

$$M_c = f_c\left(1 - \frac{t}{2kd}\right)bt \times jd \tag{24}$$

Approximate equations for resisting moments can be developed as follows: Since the center of gravity of the compression stress trapezoid is always above the middle of the slab, the lever arm jd of the resisting couple is never less than $d - \frac{1}{2}t$. The average unit compressive stress

$$\frac{f_c + f_c \cdot \dfrac{kd - t}{kd}}{2} = f_c\left(1 - \frac{t}{2kd}\right)$$

is never so small as $\frac{1}{2}f_c$ except when the neutral axis is at the bottom of the slab, in which case rectangular beam equations apply. Equations (23) and (24) may then be approximated by substituting these limiting values for jd and $f_c\left(1 - \frac{t}{2kd}\right)$,

respectively. Then

$$M_s = A_s f_s(d - \tfrac{1}{2}t) \text{ (approximate)} \tag{25}$$

and

$$M_c = \tfrac{1}{2} f_c bt(d - \tfrac{1}{2}t) \text{ (approximate)} \tag{26}$$

In the design of a continuous T-beam at the support, a slightly larger amount of steel will be required there than at the center. This is due to the fact that the value of j at the support will in all cases be less than at the center. Since the tension steel at the support of such beams is usually provided by bending up from each side one-half of the steel furnished at the center, a slight excess at this latter point is often of advantage. The use of equations (25) and (26) *in design* is therefore justified for all practical purposes. *They must not be used in review problems.*

87. Shearing Strength of T-beams. Owing to the relatively large width of flange, it is safe to say that the compressive strength of the beam will seldom govern the design. Since the only stress that will be imposed upon the concrete below the neutral axis is that of shear (the concrete is assumed incapable of resisting tensile stresses), it follows that the effective cross-section of the stem of the beam, $b'd$, need merely be large enough to keep the horizontal shear below its allowable value.

Since

$$v = \frac{V}{b'jd}$$

the amount of web area required equals

$$b'd = \frac{V}{vj} \tag{27}$$

As previously stated, a value of $j = \frac{7}{8}$ may be used in equation (27). Figure 27*b* explains the use of b' instead of b in equation (27).

In closely spaced long beams with light loads, it is possible that the compressive strength of the beam may govern. In such cases equation (26) may be used to get an approximate value of d; the review of the assumed section will then determine if any revision is necessary.

88. Ratio of Depth of Beam to Breadth of Stem. Numerous formulas have been devised to determine the economical depth of a T-beam, but very often the available head room is limited and the results of these formulas would exceed the limitation. The use of economical depth formulas for shallow beams such as are encountered in ordinary building construction involves, for this reason, an unnecessary computation, practical considerations in most cases governing the design.

A study of numerous successful designs shows that for ordinary beams a ratio between b' and d of one-half to one-third gives satisfactory results. For very large and deep beams a ratio of one-fourth is permissible. In modern building construction, the shallower and wider beam is to be preferred in order to obtain the maximum head room and minimum light obstruction.

The Joint Code recommends that beams in which the T-form is used only for the purpose of providing additional compressive area of concrete shall have a flange thickness not less than one-half the width of web and a total flange width not more than four times the web thickness. This applies only to isolated T-beams.

89. Diagrams for Review of T-beams. Since values of k and j, as expressed in equations (19) and (22), are dependent only upon the ratio $\frac{t}{d}$ and the product pn, both of which would be known in reviewing a T-beam, values of k and j may be taken from Diagram 2, Appendix D, the curves of which are based on these equations and drawn with t/d and pn as variables. Points which fall below the broken inclined line terminating the curves indicate that the neutral axis is in the flange (kd is less than t) and the formulas for rectangular beams (or Table 7, Appendix D) must be used. Since the value of p is required in order to enter Diagram 2, this diagram cannot be used in design.

90. Types of T-beam Problems. There are three main types of problems that may be encountered in practice.

1. To find the moment of resistance or fiber stresses.

The values of k and j may be obtained from equations (19) and (22) or from Diagram 2, the values of the fiber stresses from equations (23) and (18), or the resisting moments from equations

(23) and (24), the smaller of the latter being the resisting moment of the beam. Since the resisting moment of the steel will usually govern in T-beams whose flange is a part of a floor system, it is generally quicker to compute the value of M_s from equation (23), and then substitute the limiting value of f_s in equation (18) to determine the simultaneous value of f_c. If this is less than the allowable, the assumption that the steel governs is correct for the case in question. If it is greater than the allowable, determine from equation (18) the value of f_s that corresponds to the maximum permissible value of f_c, and use this value of f_s in equation (23). The result will be the moment in the steel when the concrete is stressed just to its limit, and hence it is the true resisting moment of the beam. (If kd is less than t, the neutral axis is in the flange; formulas for rectangular beams should then be used, the width being equal to the flange width b of the T-beam.)

2. To design a T-beam in which the flange is a portion of a floor slab already designed.

Compute, from equation (27) of Art. 87, the cross-section $b'd$ required, and select the width of stem and depth of beam with reference to the most satisfactory shape of beam, spacing of bars, etc. Usually d should be taken as from two to three times b'. The amount of steel may then be determined from equation (25).

In order to compute the steel area more accurately, determine the value of j from equation (22) or from Diagram 2, using for p the value corresponding to the approximate steel area. Equation (23) will then give the true steel area that is required. A slight variation between the values of p as determined by the approximate method and the true method will cause only a slight difference in equation (22), so further substituting is unnecessary. The value of k should be computed from equation (19) or taken from Diagram 2 to ascertain whether the neutral axis is in the stem or flange. Equation (2) for rectangular beams would give the same information.

3. To design a T-beam whose flange is not a part of a floor system.

In designing a beam of this type, determine from equation (27) of Art. 87 the shearing area required and select the values of b' and d. From equations (17), (20), and (21), the values of k and j may be determined. Equations (23) and (24) will then give the area of steel and breadth of flange required.

In all work on T-beams where flexural stresses are concerned, that is, in the determination of k, j, fiber stresses, resisting moments, and area of steel, the value of b is the width of flange. In the determination of the shearing area required, the value of b' is the width of stem. The reasons for each should be obvious from a study of the foregoing articles.

91. *Illustrative Problems.* I. A floor slab 4 in. thick is supported by reinforced concrete beams 9 ft.-0 in. center to center

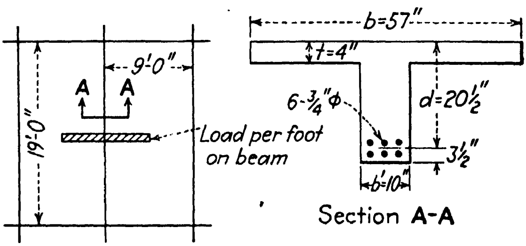


FIG. 43.

(Fig. 43) which together with the slab act as T-beams. The beams are continuous and their span is 19 ft.-0 in. The slab supports a live load of 175 lb. per sq. ft. The cross-section of each beam below the slab is 10 × 20 in.; the reinforcement consists of six ¾-in. round bars in two rows, 2 in. center to center vertically, the center of the lower row being 2½ in. above the lower surface of the beam. Assume $n = 15$. Determine f_s and f_c at the center of the span.

Weight of slab $= \frac{4}{12} \times 150 = 50$ lb. per sq. ft.

Total load on slab $= 225$ lb. per sq. ft.

Load from slab on each beam $= 9 \times 225 = 2025$ lb. per ft.

$$\text{Weight of beam below slab} = \frac{10 \times 20}{144} \times 150 = 208 \text{ lb. per ft.}$$

Total load on beam $= 2233$ lb. per ft.

$M = \frac{1}{12} \times 2233 \times 19^2 \times 12 = 805{,}000$ in.-lb.

The breadth of flange cannot exceed $\frac{1}{4} \times 19 \times 12 = 57$ in., or $(16 \times 4) + 10 = 74$ in. Hence, $b = 57$ in.

$$p = \frac{2.65}{57 \times 20.5} = 0.0023 \qquad pn = 0.0345 \qquad \frac{t}{d} = \frac{4}{20.5} = 0.195$$

From Diagram 2, $k = 0.233$ and $j = 0.926$. Hence,

$$f_s = \frac{805{,}000}{2.65 \times 0.926 \times 20.5} = 16{,}000 \text{ p.s.i.}$$

$$f_c = \frac{16{,}000 \times 0.233}{(1 - 0.233) \times 15} = 325 \text{ p.s.i.}$$

II. Determine the positive resisting moment of the beam whose dimensions and reinforcement are given in the preceding example, assuming a 2000-lb. concrete and an allowable steel stress of 18,000 p.s.i.

If the strength of the beam were governed by that of the steel, the resisting moment would be as follows:

$$M_s = 2.65 \times 18{,}000 \times 0.926 \times 20.5 = 908{,}000 \text{ in.-lb.}$$

The corresponding stress in the concrete is

$$f_c = \frac{18{,}000 \times 0.233}{15(1 - 0.233)} = 365 \text{ p.s.i.}$$

The steel, therefore, governs as assumed, since, when it is stressed to its limit, 18,000 p.s.i., the concrete unit stress is much less than the allowable stress of $0.40 \times 2000 = 800$ p.s.i.

III. Using the specifications of the Joint Code for a 2500-lb. concrete and assuming intermediate-grade reinforcement with an allowable unit stress of 20,000 p.s.i., determine the cross-section of the web below the slab and the sectional area of steel required for a continuous T-beam supporting a 5-in. floor slab which sustains a live load of 125 lb. per sq. ft. Distance center to center of adjacent beams, 11 ft.-0 in. Span of beams, 23 ft.-0 in. (Fig. 44).

Weight of slab $= \frac{5}{12} \times 150 = 62$ lb. per sq. ft.

Total load on slab $= 187$ lb. per sq. ft.

Load on beam from slab = 11 × 187 = 2060 lb. per ft.

Assume weight of stem = 200 lb. per ft.

Total load on beam = 2260 lb. per ft.

Maximum shear, $V = 2260 \times 23/2 = 26{,}000$ lb.

Assuming that web reinforcement is to be provided, the allowable unit shearing stress is 0.06 × 2500 = 150 p.s.i.

$$b'd = \frac{26{,}000}{7/8 \times 150} = 198 \text{ sq. in.}$$

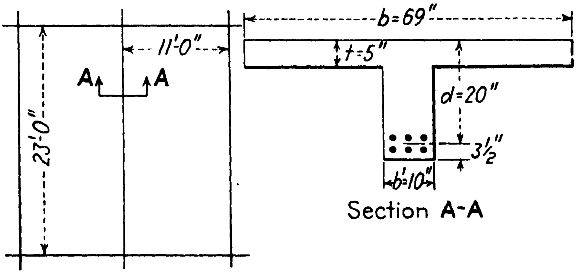


FIG. 44.

Select $b' = 10$ in. and $d = 20$ in.

Since two rows of bars will undoubtedly be necessary, the total height of beam must be $20 + 3\frac{1}{2} = 23\frac{1}{2}$ in., and the cross-section below the slab is $10 \times 18\frac{1}{2}$ in.

$$\text{Weight of stem} = \frac{10 \times 18.5}{144} \times 150 = 192 \text{ lb. per ft., approxi-}$$

mately as assumed.

$$M = 1/12 \times 2260 \times 23^2 \times 12 = 1{,}200{,}000 \text{ in.-lb.}$$

$$A_s = \frac{1{,}200{,}000}{20{,}000(20 - 2.5)} = 3.42 \text{ sq. in.}$$

This is furnished by six 7/8-in. round bars, the area of which is 3.61 sq. in.

For all practical purposes of design this approximate area of steel would be satisfactory. If the true area were to be determined, the procedure would be as follows:

$$\frac{t}{d} = \frac{5}{20} = 0.250$$

The flange width b is limited in this case to $\frac{1}{4}$ span or $\frac{1}{4}(23 \times 12) = 69$ in. Assuming that the true area of steel will be equal to the approximate value just found and that the six $\frac{7}{8}$-in. bars will be used,

$$p = \frac{3.61}{69 \times 20} = 0.0026 \qquad pn = 0.031$$

Diagram 2 shows that the neutral axis is in the flange, and so the value of $j = 0.927$ is taken from Table 7. The revised required steel area is then

$$A_s = \frac{1,200,000}{20,000 \times 0.927 \times 20} = 3.23 \text{ sq. in.}$$

The assumed bars are satisfactory.

Table 7 gives $k = 0.219$. Therefore, $kd = 4.38$ in. which checks the assumed location of the neutral axis relative to the flange. The maximum unit concrete stress is, by equation (24), well below the allowable value of 1000 p.s.i.

IV. Design a simply supported, isolated T-beam with a span of 30 ft.-0 in. which must support a live load of 3000 lb. per lin. ft. Use working stresses as follows: $f_c = 650$ p.s.i., $f_s = 16,000$ p.s.i., $v = 120$ p.s.i., $n = 15$.

Assume the weight of beam = 950 lb. per lin. ft.

Total load to be carried = 3950 lb. per lin. ft.

$$M = \tfrac{1}{8} \times 3950 \times 30^2 \times 12 = 5,330,000 \text{ in.-lb.}$$
$$V = 3950 \times \tfrac{30}{2} = 59,300 \text{ lb.}$$
$$b'd \text{ (required)} = \frac{59,300}{120 \times \frac{7}{8}} = 565 \text{ sq. in.}$$

Since b' should preferably be from $\frac{1}{2}$ to $\frac{1}{3}$ d, the values selected will be $b' = 16$ in. and $d = 36$ in. These are selected in preference to any other possible combination that falls within the limits stated above, in order to keep b' in even inches and to secure as wide a beam as possible to allow for convenient placing of the reinforcement.

The thickness of the flange is usually made $\frac{1}{3}$ d. Hence, t will be taken as 12 in.

$$\frac{t}{d} = \frac{12}{36} = 0.333$$

$$k = \frac{15}{15 + \frac{16,000}{650}} = 0.379 \qquad \text{and} \qquad kd = 0.379 \times 36 = 13.6 \text{ in.}$$

Therefore, the neutral axis is in the stem and the T-beam formulas apply.

From equations (20) and (21), $z = 4.42$ in. and $j = 0.877$.

$$A_s = \frac{5,330,000}{16,000 \times 0.877 \times 36} = 10.5 \text{ sq. in.}$$

The width of flange is determined from equation (24), all other quantities of which are known; the required width is 38.6 in. A width of 40 in. will be used.

Seven 1¼-in. square bars will be selected and placed in two rows. The total height of beam is 39½ in., and the weight per foot 960 lb. The error in the assumed weight is only ¼ of 1 per cent of the total load, and the design is, therefore, satisfactory.

ADDITIONAL PROBLEMS

1. If the beams of Problem 1, page 132 (see also Fig. 43) each support a concentrated load of 5000 lb. at the mid-span in addition to the load from the slab as specified in that problem, determine the revised cross-section required and the area of steel that is necessary to provide for the positive bending moment. Assume $f'_c = 2000$ p.s.i. and $f_s = 18,000$ p.s.i.

2. For the T-beam shown in Fig. 40, assume $b = 48$ in., $b' = 10$ in., $t = 4$ in., the depth below the flange 18 in., and the reinforcement four ⅞-in. round bars in one row, the center of which is 2½ in. above the bottom of the beam. Determine the resisting moment of the beam, with allowable unit stresses of 650 p.s.i. and 16,000 p.s.i. for the concrete and steel, respectively, and with $n = 15$.

3. If the span of the beams in Problem 2 is 16 ft.-0 in., the spacing of beams 7 ft.-0 in., if they are of one span only, and if they rest freely on brick walls at the ends, what uniform live load may be placed on the slab without overstressing the beam in bending?

4. If the allowable unit shearing stress for the beam of Problem 3 is 120 p.s.i. (assuming that adequate web reinforcement will be used), what uniform live load may be placed on the slab without overstressing the beam in shear?

5. If the thickness of the flange of the beam in Problem 2 were increased to 5 in., the overall height of the beam remaining the same, what would be the resisting moment of the beam?

92. Analysis of T-beams by the Principle of the Transformed Section. The determination of the required concrete section of a T-beam which forms a part of a floor system is based upon the shearing requirements, as previously explained. After the concrete dimensions have been thus established, the approximate required steel area can be computed from equation (25) so readily that a more complicated analysis is unwarranted. After the approximate steel area is obtained, the beam can be reviewed to determine the maximum unit stress in the steel; if this is much less than the allowable, a reduced steel area can be assumed and the investigation repeated. The use of the transformed section in the analysis of T-beams is, therefore, restricted preferably to review problems, involving the determination of the maximum unit stresses for a given load, or the determination of resisting moments.

If the neutral axis is located in the flange, the method of procedure as outlined in Art. 54 for rectangular beams can be followed, using for b the effective width of the flange. The following analysis applies to the analysis of T-beams in which the neutral axis is in the stem, the condition that makes the beam a T-beam in theory as well as in physical form. By comparing the slab thickness with the total depth of the beam, an experienced designer can, in most cases, predict which of the two conditions is apt to exist, but if this prediction cannot be made, the rectangular beam analysis can be carried out and discarded if the resulting value of x is greater than the slab thickness.

The transformed section of a previously designed T-beam, in which the neutral axis is below the flange, is shown in Fig. 45. The distance x from the top of the beam to the neutral axis can be obtained by taking moments of the shaded areas in Fig. 45a about the neutral axis, breaking up the compression area into the parts (1), (2), and (3) for convenience. The total compression C can then be computed in terms of the extreme fiber stress f_c by first considering the compression area to consist of the

rectangles (1), (2), (3), (4), and (5), and later deducting the stress on the imaginary areas (4) and (5). The location of the resultant force C can be computed by taking moments of the stresses on the areas (1), (2) and (3) about the top of the beam, using the same expedient as above to allow for the small part of area (2) which is below the flange. The lever arm of the internal stress couple, *i.e.*, the distance between forces T and C, Fig. 45*b*, can then be obtained by subtracting the distance z, between C and the top of the beam, from the effective depth d. The actual value of the total compression C, which is also equal to the total tension T, can be computed by equating the moment of the internal stress couple and the external bending moment. The unit stress

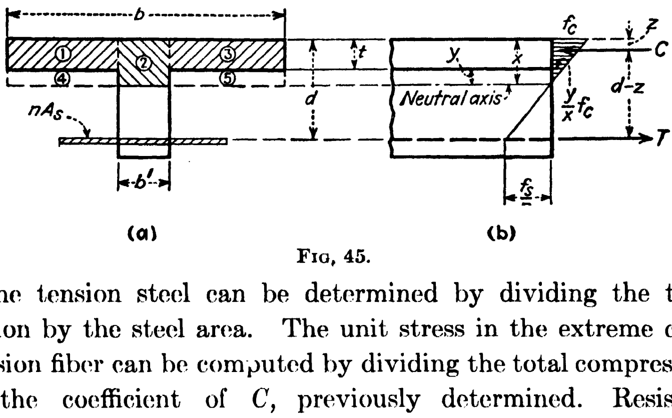


FIG. 45.

in the tension steel can be determined by dividing the total tension by the steel area. The unit stress in the extreme compression fiber can be computed by dividing the total compression by the coefficient of C, previously determined. Resisting moments can be computed with certain obvious modifications to the above processes. The following examples will serve to illustrate the application of the principles to specific problems.

93. *Illustrative Problems.* I. Determine the values of f_s and f_c at the center of the beam described in Problem I of Art. 91, using the principles of the transformed section.

The maximum moment in each beam, as computed in Art. 91, is 805,000 in.-lb. The effective flange width is 57 in., and $nA_s = 15 \times 2.65 = 39.75$ sq. in. The transformed section, assuming the neutral axis to be below the bottom of the flange, and considering that the equivalent area of concrete, nA_s, is concentrated at the center of gravity of the two rows of bars, is shown in Fig. 46*a*. The neutral axis is located by equating the moment

of the shaded compression area about the neutral axis to that of the shaded effective tension area, as follows:

$$10x \times \frac{x}{2} + (57 - 10) \times 4 \times \left(x - \frac{4}{2}\right) = 39.75(20.5 - x)$$

$$x = 4.74 \text{ in.}$$

The total compression C, in terms of the extreme fiber stress f_c, is obtained as outlined in Art. 90, as follows:

$$C = \tfrac{1}{2}f_c \times 57 \times 4.74 - \tfrac{1}{2} \times 0.16f_c \times 47 \times 0.74$$
$$= 135.1f_c - 2.8f_c = 132.3f_c$$

The center of gravity of the compressive stress is obtained by taking moments about the extreme compression fiber, as explained

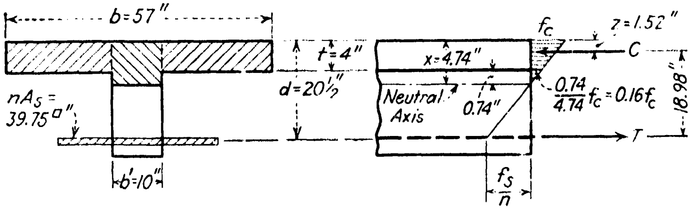


FIG. 46.

in Art. 92; the resulting equation from which the distance z from the top of the beam to the center of compression can be computed, is as follows:

$$135.1f_c \times \frac{4.74}{3} - 2.8f_c\left(4 + \frac{0.74}{3}\right) = 132.3f_c \times z$$

$$z = 1.52 \text{ in.}$$

The lever arm of the internal stress couple is, therefore, $20.5 - 1.52 = 18.98$ in., and

$$C = T = \frac{805{,}000}{18.98} = 42{,}400 \text{ lb.}$$

The maximum unit stresses are, therefore,

$$f_c = \frac{42{,}400}{132.3} = 321 \text{ p.s.i.}$$

$$f_s = \frac{42{,}400}{2.65} = 16{,}000 \text{ p.s.i.}$$

These stresses agree very closely with those obtained in Problem 1 of Art. 91. The very slight difference is due to the fact that, in Art. 91, the small amount of compression in the stem below the slab was neglected, whereas in the above solution, this compression was included in the computations. If the compression in the stem had been neglected in the above solution, the two results would theoretically be the same. The fact that the two results are so nearly alike can be taken as a demonstration of the statement made in Art. 86, to the effect that the compression in the stem below the slab can be disregarded without material error. When the transformed section is used in the solution it is just as easy to include the compression in the stem as it is to neglect it, whereas the derivations of the equations in Art. 86 would be complicated unduly if the stem compression were not neglected.

II. If the allowable unit stresses in the beam in Problem I were 18,000 and 800 p.s.i. for the tension in the steel and the compression in the concrete, respectively, what would be the maximum moment that could safely be resisted by the beam?

The location of the neutral axis was determined in Problem I, where $x = 4.74$ in. The lever arm of the stress couple was also computed in Problem 1, the value being 18.98 in. The maximum tension that can be resisted by the steel is $18,000 \times 2.65 = 47,700$ lb., and the maximum compression that can be resisted by the concrete (see Problem I) is $132.3f_c$ or $800 \times 132.3 = 105,800$ lb. The strength of the beam is therefore governed by the strength of the steel (as was to be expected in view of earlier discussions), and the resisting moment is

$$M = 47,700 \times 18.98 = 907,000 \text{ in.-lb.}$$

This agrees closely with the corresponding value in Problem II of Art. 91. The slight difference is again accounted for by the fact that, in the above solution, the compression in the stem below the slab was not neglected, as was the case in Art. 91. This extra compression causes a slight decrease in the lever arm of the stress couple, and therefore a corresponding decrease in the effectiveness of the steel.

Rectangular Beams Reinforced for Compression

94. Use of Beams Reinforced for Compression. A beam is reinforced for compression when its size is limited by structural conditions or architectural limitations. The moment in excess of the carrying capacity of the concrete is provided for by placing steel in the compressive portion of the beam. While the effectiveness of steel in compression has been questioned, numerous tests indicate that the steel assumes its proportion of the stress. In order to furnish adequate lateral support for the compression steel, ties similar to those used in columns (see Art. 111) should be placed in all doubly reinforced beams. These ties serve to prevent buckling of the compression steel, and also to resist diagonal tension stresses.

A common example of a rectangular beam reinforced for compression occurs at the supports of a continuous T-beam, *i.e.*, a floor beam or girder in monolithic beam-and-girder-floor construction. On account of its importance, this is considered separately in Art. 102.

95. Formulas for Design. The notation used in the following derivations is as follows:

M_1 = moment that can be developed by the limited cross-section of concrete without compression reinforcement.

M_2 = moment in excess of the compressive strength of the concrete and which must be developed by compression reinforcement.

M = total moment to be developed by the beam $= M_1 + M_2$.

A_{s_1} = area of tensile steel required to develop the moment M_1.

A_{s_2} = area of additional tensile steel necessary to develop the moment M_2.

A_s = total tensile steel area $= A_{s_1} + A_{s_2}$.

A'_s = total area of compressive steel.

f_s = unit stress in tensile steel.

f'_s = unit stress in compressive steel.

Assuming that a rectangular beam, which will be called upon to resist a bending moment M, is limited in size to an effective cross-section bd, the resisting moment M_1 of the concrete being

less than the moment M, compressive steel of an amount A'_s will be required in order to keep the unit stress in the concrete within the allowable limit.

The moment M_1 depends upon the concrete and equals (see Fig. 47 and Art. 47)

$$M_1 = \tfrac{1}{2}f_c kjbd^2 = Kbd^2 \tag{28}$$

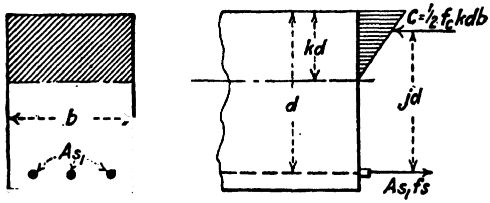


FIG. 47.

Since the resisting moment of the tensile steel $= A_{s_1}f_s jd$, the area of steel required to provide sufficient tensile resistance fully to develop the strength of the concrete is

$$A_{s_1} = \frac{M_1}{f_s jd} \tag{29}$$

The values of k, j, and K are computed from the same equations as for rectangular beams with only tensile reinforcement (Art. 47).

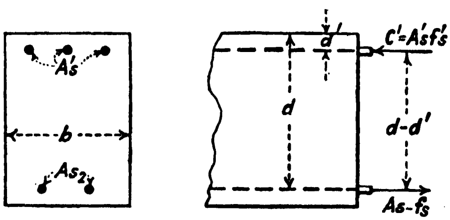


FIG. 48.

A moment of an amount $= M_2 = M - M_1$ still remains to be provided for by the necessary amount of compressive steel and additional tensile steel A_{s_2}. The stresses in these two quantities of steel form a couple, the lever arm of which is $d - d'$ (Fig. 48). The resisting moment of the couple is, therefore, $A_{s_2}f_s(d - d')$,

and this must be sufficient to develop the moment M_2; hence

$$M_2 = A_{s_2} f_s (d - d')$$

from which

$$A_{s_2} = \frac{M_2}{f_s(d - d')} \tag{30}$$

The total tensile steel then equals

$$A_s = A_{s_1} + A_{s_2} \tag{31}$$

Since the beam must be in equilibrium

$$A'_s f'_s = A_{s_2} f_s \tag{a}$$

From the assumption that unit stresses vary as the distance from the neutral axis

$$\frac{f_s}{f'_s} = \frac{d - kd}{kd - d'}$$

from which

$$f'_s = f_s \cdot \frac{k - \left(\frac{d'}{d}\right)}{1 - k} \tag{b}$$

Substituting in equation (a) the value of f'_s from equation (b), in order to eliminate the unit stresses,

$$A'_s f_s \cdot \frac{k - \left(\frac{d'}{d}\right)}{1 - k} = A_{s_2} f_s$$

from which

$$A'_s = A_{s_2} \cdot \frac{1 - k}{k - \left(\frac{d'}{d}\right)} \tag{32}$$

The position of the neutral axis has not been changed by the addition of the compressive steel, since just enough tensile steel was added to counterbalance it. Therefore, the value of k remains constant throughout the design. On account of the commercial sizes of reinforcing bars, as soon as the bars are

selected there will, in all probability, be a slight difference in both tensile steel and compressive steel from the theoretical. Hence the balance in stresses indicated above no longer exists, and in reviewing the beam, the values of k and j must be obtained from equations other than the above. Such equations are derived in the following article.

The preceding derivation and that in the following article ignore the fact that some of the concrete in the compression area is replaced by the compression steel. The resulting theoretical compression resistance is therefore greater than that which actually exists. To consider the area of the replaced concrete in the derivations would complicate the derivations materially, and since the relative error is small, the formulas as given are generally accepted.

In order to develop fully the resistance of the compression steel, these bars must be supported laterally by stirrups or ties in much the same way that the longitudinal reinforcing bars in a column are supported. The Joint Code requires that compression steel be held by means of ties or stirrups not less than $\frac{1}{4}$ in. in diameter which shall be spaced not farther apart than 16 bar diameters or 48 tie diameters over the distance where the compression steel is required.

96. Formulas for Review. The equations (for k, j, and the resisting moment) to be used in reviewing a rectangular beam with compressive reinforcement are derived in a manner similar to that used in deriving the corresponding equations for rectangular beams with tensile reinforcement only.

The total tension in the steel $= T = A_s f_s = pbdf_s$.

The total compression in steel and concrete $= C' + C$; the former (see Fig. 49) is equal to $A'_s f_s'$, and the latter is equal to $\frac{1}{2} f_c kbd$.

Hence,

$$C' + C = A'_s f'_s + \tfrac{1}{2} f_c kbd = p'bdf'_s + \tfrac{1}{2} f_c kbd$$
$$= bd(\tfrac{1}{2} f_c k + p'f'_s)$$

in which p is the ratio of tensile steel and p' the ratio of compressive steel in terms of the effective cross-section bd.

For equilibrium, the total tension must equal the total compression; hence,

$$pf_sbd = bd(\tfrac{1}{2}f_ck + p'f'_s)$$

or

$$pf_s = \tfrac{1}{2}f_ck + p'f'_s \tag{c}$$

From Fig. 49 it is seen that, since deformations vary as the distance from the neutral axis, if AA' represents the deformation

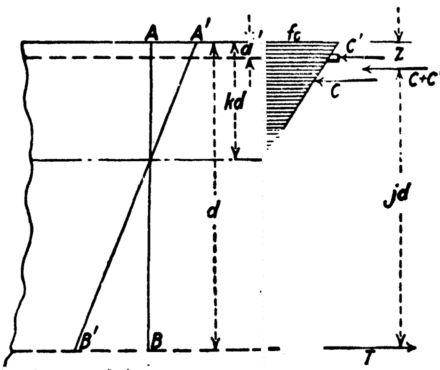


FIG. 49.

of the extreme compression fiber and BB' the elongation of the tension steel,

$$\frac{AA'}{BB'} = \frac{kd}{d - kd} \tag{d}$$

If AA' and BB' are further assumed to be the deformations for a unit length of beam,

$$AA' = \frac{f_c}{E_c} \quad \text{and} \quad BB' = \frac{f_s}{E_s}.$$

Hence,

$$\frac{AA'}{BB'} = \frac{nf_c}{f_s} \tag{e}$$

Equating (d) and (e) and solving for f_c,

$$f_c = \frac{f_sk}{n(1 - k)} \tag{33}$$

From equation (*b*) of Art. 95

$$f'_s = \frac{f_s\left(k - \frac{d'}{d}\right)}{1 - k} \qquad (34)$$

Substituting in equation (*c*) the value of f_c from equation (33) and that of f'_s from equation (34),

$$pf_s = \frac{f_s k^2}{2n(1 - k)} + \frac{p'f_s\left(k - \frac{d'}{d}\right)}{1 - k}$$

from which

$$k = \sqrt{2n\left(p + p'\frac{d'}{d}\right) + n^2(p + p')^2} - n(p + p') \qquad (35)$$

By taking the summation of moments of the compressive forces about the top of the beam, the position of the center of gravity of these forces may be determined. The resulting equation is

$$z = \frac{\frac{1}{3}kdC + d'C'}{C + C'} = \frac{\frac{1}{3}kd + d'\frac{C'}{C}}{1 + \frac{C'}{C}} \qquad (f)$$

Since C', the total compressive stress in the steel $= p'bdf'_s$ and C, the total compressive stress in the concrete $= \frac{bdf_c k}{2}$,

$$\frac{C'}{C} = \frac{2p'f'_s}{f_c k}$$

Substituting from equations (33) and (34) the values of f_c and f'_s

$$\frac{C'}{C} = \frac{2p'f_s\left(k - \frac{d'}{d}\right)}{(1 - k)\cdot\frac{f_s k^2}{n(1 - k)}} = \frac{2p'n\left(k - \frac{d'}{d}\right)}{k^2}$$

Substituting this value of $\frac{C'}{C}$ in equation (*f*)

$$z = \frac{\frac{1}{3}k^3d + 2p'nd'\left(k - \frac{d'}{d}\right)}{k^2 + 2p'n\left(k - \frac{d'}{d}\right)}$$

From Fig. 49, $jd = d - z$, or $j = 1 - \frac{z}{d}$, and therefore

$$j = \frac{k^2 - \frac{1}{3}k^3 + 2p'n\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right)}{k^2 + 2p'n\left(k - \frac{d'}{d}\right)} \tag{36}$$

The resisting moment of the tension steel is found by taking moments about the center of gravity of the compressive forces, from which

$$M_s = A_sf_sjd \tag{37}$$

The resisting moment of the compression forces could be found by taking moments about the center of the tensile steel, but the resulting equation would be extremely cumbersome. It is simpler to use equation (37) in conjunction with equation (33) as explained in Art. 90, Problem I, and in the third paragraph of Art. 98, to determine the relative strength of the compression forces, as compared with the tension forces, from which relation the true resisting moment of the beam can be computed.

97. Diagrams for Review of Beams Reinforced for Compression. In order to simplify the computations for the review of beams of this type, Diagrams 3 to 6 (Appendix *D*) have been constructed from which the values of k and j as represented by equations (35) and (36) may readily be found. Since these two quantities depend upon the relation between pn, $p'n$, and $\frac{d'}{d}$, the curves have been drawn with these quantities as variables. For intermediate values of $\frac{d'}{d}$, interpolation is necessary to find the true values of k and j.

98. Types of Problems. In designing a beam of limited cross-section which is called upon to support a greater load than the compressive strength of the concrete permits, the rational process is to solve equations (28) to (32) in order. The amounts of tensile and compressive steel are then known, so that proper selection of bars can be made.

In reviewing a beam with compressive reinforcement to determine the existing unit stresses under a given load, the values of k and j may be obtained from equations (35) and (36) or from Diagrams 3 to 6, the value of f_s from equation (37), f_c from equation (33), and f'_s from equation (34).

If the safe resisting moment is required, the maximum allowable values of f_s and f_c having been given, values of k and j should first be obtained from equations (35) and (36) or from Diagrams 3 to 6. Then to determine whether the strength of the concrete or that of the tensile steel governs, the value of f_s corresponding to the maximum allowable value of f_c should be obtained from equation (33). If this is greater than the allowable, the safe working limit of the strength of the steel will be exceeded, provided the full strength of the concrete is developed, *i.e.*, the steel governs the strength of the beam. Hence the true resisting moment of the beam will be found from equation (37), f_s being taken as the specified limit. If f_s determined as above is less than the allowable, the full strength of the steel cannot be developed without overstressing the concrete. Hence the concrete governs and the safe resisting moment of the beam may be computed from equation (37), the value of f_s being that just computed from equation (33)—the value that results when the concrete is stressed to its maximum.

99. *Illustrative Problems.* I. A simply supported, reinforced concrete beam having a span of 20 ft.-0 in. is limited in cross-section to 8 × 18 in. The beam sustains a live load of 580 lb. per lin. ft. $f_s = 18{,}000$, $f_c = 1000$, and $n = 12$. Using 2½ in. of insulation measured from the center of the bars, determine the area of steel required for tension (A_s) and for compression (A'_s).

$$\text{Weight of beam} = \frac{8 \times 18}{144} \times 150 = 150 \text{ lb. per lin. ft.}$$

Total load carried by beam = 730 lb. per lin. ft.

$$M = \tfrac{1}{8} \times 730 \times 20^2 \times 12 = 438{,}000 \text{ in.-lb.}$$

From Table 6, $K = 173$, $k = 0.400$, $j = 0.867$.
Assuming that only one row of tensile steel will be required,

$$M_1 = 173 \times 8 \times (15.5)^2 = 333{,}000 \text{ in.-lb.}$$
$$M_2 = 438{,}000 - 333{,}000 = 105{,}000 \text{ in.-lb.}$$
$$A_{s_1} = \frac{333{,}000}{18{,}000 \times 0.867 \times 15.5} = 1.38 \text{ sq. in.}$$
$$A_{s_2} = \frac{105{,}000}{18{,}000(15.5 - 2.5)} = 0.45 \text{ sq. in.}$$
$$A_s = 1.38 + 0.45 = 1.83 \text{ sq. in.}$$
$$A'_s = 0.45 \times \frac{1 - 0.400}{0.400 - \dfrac{2.5}{15.5}} = 1.13 \text{ sq. in.}$$

Three $\tfrac{7}{8}$-in. round bars in tension and two $\tfrac{7}{8}$-in. round bars in compression are selected, each set placed in one row as assumed.

II. A simply supported concrete beam whose span is 21 ft.-0 in. has a cross-section of 8 × 19 in. and is reinforced as follows: for tension, four $\tfrac{7}{8}$-in. round bars, and for compression, four $\tfrac{7}{8}$-in. round bars, each set in two rows, the center of the row nearest the surface being 2 in. from the surface, and the vertical distance center to center of rows being 2 in.; $n = 15$. Determine the values of the unit stresses in the tensile steel, compressive steel, and concrete, if the beam sustains a live load of 600 lb. per lin. ft.

$$\text{Weight of beam} = \frac{8 \times 19}{144} \times 150 = 160 \text{ lb. per lin. ft.}$$

Total load carried by beam = 760 lb. per lin. ft.

$$M = \tfrac{1}{8} \times 760 \times 21^2 \times 12 = 505{,}000 \text{ in.-lb.}$$

$$pn = p'n = \frac{2.41}{8 \times 16} \times 15 = 0.282 \quad \text{and} \quad \frac{d'}{d} = \frac{3}{16} = 0.19$$

From Diagrams 5 and 6, interpolating for $\dfrac{d'}{d} = 0.19$, $k = 0.432$, and $j = 0.840$,

$$f_s = \frac{505{,}000}{2.41 \times 0.840 \times 16} = 15{,}500 \text{ p.s.i.}$$

$$f_c = \frac{15{,}500 \times 0.432}{15(1 - 0.432)} = 790 \text{ p.s.i.}$$

$$f'_s = \frac{15{,}500(0.432 - \frac{3}{16})}{1 - 0.432} = 6670 \text{ p.s.i.}$$

III. A reinforced concrete beam has a cross-section of 12 × 30 in. and is reinforced as follows: for tension, eight ⅞-in. round bars in two rows, 2 in. center to center, the center of the lower row being 2½ in. above the lower surface of the beam, and for compression, four ⅞-in. round bars in one row, the center of which is 2½ in. below the upper surface of the beam; $f_s = 18{,}000$, $f_c = 800$, and $n = 15$. What is the safe resisting moment of the beam?

$$\frac{d'}{d} = \frac{2.5}{26.5} = 0.09$$

$$pn = \frac{8 \times 0.6013}{12 \times 26.5} \times 15 = 0.2265$$

$$p'n = \tfrac{1}{2}pn = 0.1133$$

From Diagrams 3 and 4, $j = 0.872$ and $k = 0.429$.

When f_c is a maximum, the corresponding value of f_s is, from equation (33),

$$\frac{800 \times 15(1 - 0.429)}{0.429} = 16{,}000 \text{ p.s.i.}$$

Therefore, the strength of the beam depends upon the concrete; the safe maximum resisting moment occurs when the tension steel is stressed to 16,000 p.s.i. and equals:

$$M = 8 \times 0.6013 \times 16{,}000 \times 0.872 \times 26.5 = 1{,}780{,}000 \text{ in.-lb.}$$

ADDITIONAL PROBLEMS

1. Assume that the simply supported beam in Fig. 28 is limited in size to an overall cross-section of 10 × 21 in. What reinforcement must be used, if $f_s = 18{,}000$ p.s.i., $f_c = 1000$ p.s.i., and $n = 12$? Assume that the tension steel will be placed in one row, the compression steel will be placed in one row, and the distance from the center of each row to the nearest surface of the beam is 2½ in.

2. A simply supported rectangular beam with a span of 20 ft.-0 in. has an overall cross-section of 8 × 23 in. It is reinforced for compression with two ⅞-in. round bars, in one row, the center of which is 2½ in. from the upper surface of the beam, and for tension with four ⅞-in. round bars in two rows, 2 in. center to center, the center of the lower row being 2½ in. above the lower surface of the beam. What is the resisting moment of the beam, if $f_c = 1000$ p.s.i., $f_s = 20{,}000$ p.s.i., and $n = 12$?

3. If the beam in Problem 2 supports a single concentrated load at the mid-span, what is the maximum safe value of this load?

4. A simply supported beam 10 × 22½ in. in cross-section has a span of 19 ft.-0 in. and is reinforced for tension with six ¾-in. round bars in two rows 2 in. center to center, the center of the lower row being 2½ in. above the lower surface of the beam. The beam also has compression reinforcement consisting of three ¾-in. round bars with their centers 2 in. from the upper surface of the beam. If $f_s = 18{,}000$ p.s.i., $f_c = 800$ p.s.i., and $n = 15$, what uniform live load per foot can the beam sustain?

100. Analysis of Beams Reinforced for Compression, by the Principle of the Transformed Section. The cross-section of a beam reinforced for compression can be transformed into a homogeneous section by assuming that the compression steel and the tension steel are replaced by narrow strips of concrete, the areas of which are obtained as explained in Art. 54. Thus, if A_s is the area of the tension steel, it is assumed to be replaced by an area of concrete equal to nA_s, in the same horizontal plane as the steel. Similarly, the compression steel area A'_s is assumed to be replaced by an area of concrete equal to nA'_s in the same horizontal plane as the steel. Since the hole that is left by the removal of the compression steel is assumed to be filled with concrete, the actual area of the projecting wings which replace the compression steel in the transformed section is $(n - 1)A'_s$. The use of $(n - 1)$ in place of n as a factor of A'_s does not complicate the subsequent analysis in any way. The resulting transformed section, or equivalent homogeneous section, of the beam reinforced for compression shown in Fig. 50*a* is represented, therefore, by the shaded areas in Fig. 50*b*.

If the problem is one of review, all of the dimensions of the transformed section are known. The distance x from the neutral axis to the extreme compression fiber can be obtained by equating the moments of the areas above and below the axis, about the

axis, as follows:

$$\tfrac{1}{2}bx^2 + (n - 1)A'_s(x - d') = nA_s(d - x)$$

When the known values of b, $A_s A'_s$, n, d, and d' are substituted in this equation, the unknown value of x can be computed. The total compression C_1 in the rectangular portion of the transformed section (Fig. 40c) is equal to $\tfrac{1}{2}f_c bx$. The total compression C_2 in the projecting wings is $(n - 1)\ A'_s \times \frac{f'_s}{n}$, and since $\frac{f'_s}{n}$ is equal to $f_c\left(\frac{x - d'}{x}\right)$, the value of C_2 is equal to $(n - 1)A'_s f_c\left(\frac{x - d'}{x}\right)$. The total compression C is equal to $C_1 + C_2$, and the lever arm of

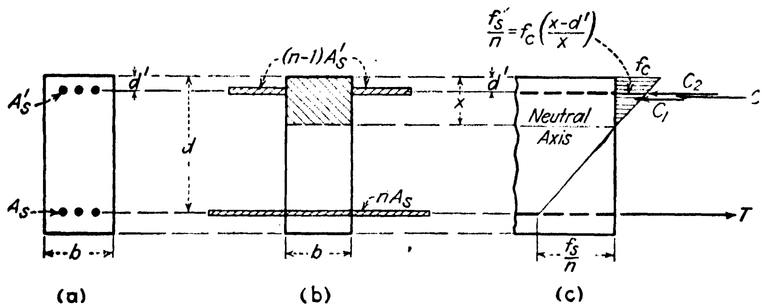


FIG. 50.

the internal stress couple can be obtained by equating the sum of the moments of C_1 and C_2 about the plane of the tension steel to the amount of C about the same plane. Resisting moments or unit stresses can then be computed in the manner explained in Arts. 54 and 92.

If the problem is one of design, the dimensions of the cross-section of the beam and the location of the steel are known. Considering the beam before the compression steel is added, the location of the neutral axis can be determined from the following assumptions: (1) that unit stresses vary directly as the distance from the neutral axis, (2) that the extreme fiber stress in compression is f_c, and (3) that the fiber stress on the equivalent concrete in the plane of the tension steel is $\frac{f_s}{n}$. The resisting moment

M_1 of the beam before the compression steel is added can then be computed, and the area of tension steel A_{s_1} required to resist this same moment can be determined as in Art. 54. The remainder of the moment $M_2 = M - M_1$ must be resisted by the equivalent concrete in the projections above the neutral axis and by additional equivalent concrete in the plane of the tension steel. The lever arm of the couple formed by the stresses in these areas is equal to $d - d'$. The required areas can be computed readily from the previously explained principles (see Arts. 54, 55, 92, and 93).

Illustrative Problem. Solve Problem II of Art. 99, by means of the principles of the transformed section. This problem is one of review, in which maximum unit stresses are required. The ratio $n = 15$.

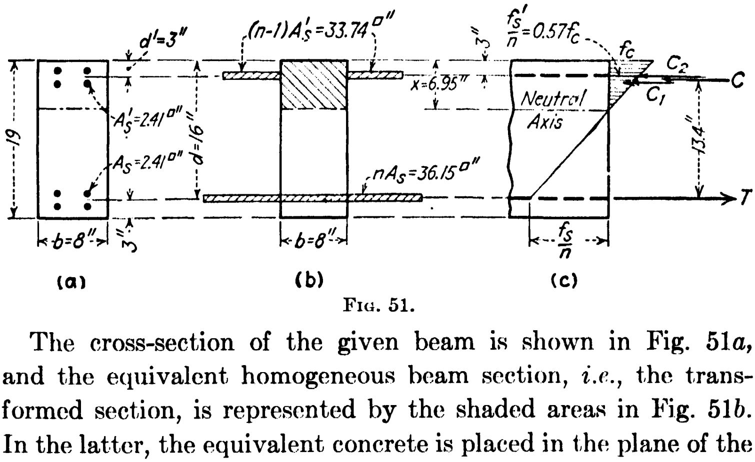


FIG. 51.

The cross-section of the given beam is shown in Fig. 51*a*, and the equivalent homogeneous beam section, *i.e.*, the transformed section, is represented by the shaded areas in Fig. 51*b*. In the latter, the equivalent concrete is placed in the plane of the center of gravity of the two rows of bars in each group, which is in accordance with previous assumptions. The distance x from the compression surface to the neutral axis is obtained from the equation

$$\tfrac{1}{2} \times 8x^2 + 33.74(x - 3) = 36.15(16 - x)$$
$$x = 6.95 \text{ in.}$$

The total compression C_2 in the projecting wings (see Fig. 51*c*) is equal to $33.74 \times \frac{f'_s}{n}$, and since $\frac{f'_s}{n} = f_c\left(\frac{6.95 - 3}{6.95}\right) = 0.57f_c$,

$C_2 = 19.23f_c$. The total compression C_1 in the rectangular portion of the transformed section is $\frac{1}{2}f_c \times 8 \times 6.95 = 27.80f_c$. The total compression C is equal to $19.23f_c + 27.80f_c = 47.03f_c$.

The sum of the moments of C_1 and C_2 about the plane of the tension steel is

$$19.23f_c \times 13 + 27.80f_c\left(16 - \frac{6.95}{3}\right) = 630.3f_c$$

The lever arm of the stress couple is therefore equal to

$$\frac{630.3f_c}{47.03f_c} = 13.4 \text{ in.}$$

The maximum moment in the beam is 505,000 in.-lb. so that the maximum total tension T and the maximum total compression C are each equal to $\frac{505{,}000}{13.4} = 37{,}700$ lb. The maximum unit stresses are, therefore,

$$f_c = \frac{37{,}700}{47.03} = 800 \text{ p.s.i.}$$

$$f'_s = 15 \times 0.57 \times 800 = 6840 \text{ p.s.i.}$$

$$f_s = \frac{37{,}700}{2.41} = 15{,}640 \text{ p.s.i.}$$

These stresses are all slightly greater than those obtained in Problem II, Art. 99. This is accounted for by the fact that, in the transformed-section theory, allowance has been made for the area of concrete occupied by the compression bars, whereas in the regular formulas no such allowance has been made. The effective compressive resisting area in the transformed theory is therefore less than that assumed in the analysis by formulas, and the compression unit stresses are consequently larger.

101. Analysis Proposed in Standard Building Code (1940). Tests have shown that under load the compressive steel is much more highly stressed than is indicated by equation (34). Accordingly, the Standard Building Code Committee of the American Concrete Institute is proposing (1940) the following:

"(a) Compression steel in beams, girders, or slabs shall be anchored by ties or stirrups not less than ¼ inch in diameter spaced not farther apart

than 16 bar diameters or 48 tie diameters. Such stirrups or ties shall be used throughout the distance where the compressive steel is required.

"(b) The effectiveness of compressive reinforcement in resisting bending may be taken at twice the value indicated from the calculations assuming a straight-line relation between stress and strain and the modular ratio . . . , but in no case shall the unit stress in compression be taken greater than the allowable unit stress in tension."

Except for very shallow members with considerable concrete protection for the compressive steel, the last phrase of (b) will usually govern.

For design purposes therefore $A_s(= A_{s_1} + A_{s_2})$ is determined as in Art. 95, but since the stress in the compressive steel may be taken as equal to that in the tensile steel, $A'_s = A_{s_2}$.

For review purposes, assuming $f'_s = f_s$,

$$k = \sqrt{2n(p - p') + n^2(p - p')^2} - n(p - p') \qquad (a)$$

and

$$j = \frac{k^2 - \frac{k}{3} + 2p'n(1 - k)\left(1 - \frac{d'}{d}\right)}{k^2 + 2p'n(1 - k)} \qquad (b)$$

Equation (a) is similar to equation (2) page 56. Therefore k may be obtained from Table 7, using $p - p'$ for p.

Values of j may be obtained from Diagram 21, plotted from equation (b).

Illustrative Problems. I. A reinforced concrete beam is limited in cross-section to 10 × 20 in. and withstands a bending moment of 750,000 in.-lb. The beam is to be constructed of 2500-lb. concrete and $f_s = 18{,}000$ p.s.i. The reinforcement is to be protected by 2 in. of concrete measured from the center of the bars. Determine the required steel areas.

From Table 6, $K = 173$ and

$$M_1 = 173 \times 10 \times 18^2 = 560{,}500 \text{ in.-lb.}$$
$$M_2 = 189{,}500 \text{ in.-lb.}$$
$$A_{s_1} = \frac{560{,}500}{18{,}000 \times 0.867 \times 18} = 2.00 \text{ sq. in.}$$
$$A_{s_2} = A'_s = \frac{189{,}500}{18{,}000(18 - 2)} = 0.66 \text{ sq. in.}$$

For tension, three 1-in. square bars are selected ($A_s = 3.00$ sq. in.) For compression, two ¾-in. round bars are selected ($A'_s = 0.88$ sq. in.).

II. As an illustration of the application of this method, the beam as designed in Problem I is reviewed as follows:

$$p = \frac{3.00}{10 \times 18} = 0.0167 \qquad p' = \frac{0.88}{10 \times 18} = 0.0049$$
$$p - p' = 0.0118 \qquad k = 0.410 \text{ (Table 7)}$$
$$pn = 0.200 \qquad p'n = 0.059 \qquad \frac{d'}{d} = 0.11$$

From Diagram 21, $j = 0.878$

$$f_s = \frac{750{,}000}{3.00 \times 0.878 \times 18} = 15{,}800 \text{ p.s.i.}$$

$$f_c = \frac{15{,}800 \times 0.410}{12(1 - 0.410)} = 920 \text{ p.s.i.}$$

The resisting moment developed by the concrete is

$$\tfrac{1}{2}f_ckjbd^2 = \frac{920 \times 0.410 \times 0.878 \times 10 \times 18^2}{2} = 536{,}000 \text{ in.-lb.}$$

The compression steel must therefore develop 214,000 in.-lb. and the unit stress in that steel is $\dfrac{214{,}000}{18 \times 0.878} = 15{,}400$ p.s.i.

Design of Continuous T-beams

102. Investigation at the Support. The required cross-section of a continuous T-beam, and the required steel area at the center of the beam are determined as explained in Art. 90, Problem II. At the supports, the bending moment is negative so that the upper surface becomes the tensile surface, while the lower portion of the section of the beam is in compression. Since in reinforced concrete design the steel is assumed to resist all of the tensile forces, sufficient steel must be placed near the upper surface of the beam over the support to develop the negative moment at that point. Where the moment at the support is assumed numerically equal to the moment at mid-span, the tensile steel required near the upper surface over the support is approximately equal to that required in the lower section of the beam at the center of the span to develop the positive bending moment. The length of the lever arm of the resisting couple jd is usually somewhat smaller at the support than at mid-span, in which case the amount of tensile steel required over the support is slightly greater than that required at the mid-span.

The usual method of furnishing the required tension steel at the support is to bend up about one-half of the bars from each adjacent span and to extend them far enough across the support to insure proper development of the negative moment. Such bars are usually continued slightly (about 12 bar diameters)

beyond the assumed location of the point of inflection. In a uniformly loaded beam, this point is generally considered to be at distances from the support varying from one-fifth to one-third of the span, depending upon the condition of continuity and loading. More bars must be bent up or additional bars must be placed in the upper portion over the support if the allowable unit stresses are exceeded.

Since the tension side is uppermost, the flange of the T-beam can no longer be considered effective in resisting stress. Hence, the form of beam becomes rectangular at the support, the width being equal to the width of the stem. On account of the small compressive area of concrete (now below the neutral axis) a failure by compression would probably occur if steel were not added in the compressive area to assist the concrete. Since not all of the horizontal bars are bent up over the support, the remaining bars may be brought straight through and extended far enough into the adjacent panel to develop their full strength in bond, and thus furnish the added compressive resistance required. The Joint Code requires that at least one-fourth of the positive-moment steel shall extend along the same face of the beam into the support a distance of 10 or more bar diameters.

In determining the length necessary to develop adequate bond strength of the straight bars, it will be sufficient to consider only the maximum stress in the bars and to furnish a length from the center of the support to the end of the bar properly to transmit this stress to the concrete.

Where one-half of the longitudinal reinforcement which is furnished at the center of the span is bent up from each of two adjacent beams, and the remainder of the reinforcement is continued straight through the support far enough to develop its compressive strength in bond, the amounts of tensile and compressive reinforcement are equal. If less compressive reinforcement is required, the bars from the adjacent beams need be carried beyond the support only far enough to develop a lap splice. With such an arrangement the compressive reinforcement is equal in amount to one-half of the tensile reinforcement. Other arrangements can be made to suit any individual case,

For example, if there are seven bars at the center, four bars can be bent up and continued over the support, leaving three from each side at the bottom of the section; the tension steel at the support would then consist of eight bars, and the compression steel six bars. In any case, the effective section of the beam at the support is an inverted rectangular beam reinforced for compression, and it may be analyzed according to the principles of Art. 96.

Since the negative moment decreases very rapidly and only a short section is under maximum stress, a higher compressive stress is allowed in the concrete at the support than at mid-span. The Joint Code specifies an allowable unit compression stress of $0.45f'_c$ at the supports of continuous beams, as compared with $0.40f'_c$ at the center of the span of such beams.*

Illustrative Problem. The cross-section of a fully continuous T-beam below the 4-in. slab which forms the flange of the beam is 8 × 19 in. The span is 20 ft.-0 in., and the distance between adjacent beams is 10 ft.-0 in. At the support the reinforcement near the top of the beam consists of four ⅞-in. round bars in two rows, 2 in. center to center, the center of the upper row being 2 in. from the top surface of the slab, and the reinforcement near the bottom of the beam consists of two ⅞-in. round bars with their centers 2 in. from the lower surface. If the slab supports a uniform live load of 150 lb. per sq. ft., what are the values of f_c and f_s at the support of the beam? Assume $n = 15$.

The weight of the slab is 50 lb. per sq. ft., and the weight of the stem of the beam is 160 lb. per ft. The total load on the beam is $(150 + 50)10 + 160 = 2160$ lb. per lin. ft., and the maximum bending moment is $\frac{1}{12} \times 2160 \times 20^2 \times 12 = 864{,}000$ in.-lb.

$$np = 15 \times \frac{4 \times 0.6013}{8 \times 20} = 0.2255$$

$$np' = 15 \times \frac{2 \times 0.6013}{8 \times 20} = 0.1127$$

$$\frac{d'}{d} = \frac{2.0}{20} = 0.1$$

From Diagram 4, $k = 0.430$ and $j = 0.870$.

* See footnote, pg. 61.

$$864{,}000 = (4 \times 0.6013) \times f_s \times 0.870 \times 20$$

$$f_s = 20{,}600 \text{ p.s.i.}$$

$$f_c = \frac{20{,}600 \times 0.430}{15(1 - 0.430)} = 1030 \text{ p.s.i.}$$

ADDITIONAL PROBLEM

If the allowable unit stresses at the support of the beam in the preceding problem were $f_s = 20{,}000$ p.s.i. and $f_c = 900$ p.s.i., what safe uniform live load could be placed on the slab without overstressing the beam at the support? Assume $n = 15$.

103. Bond Stresses. The bond stress on the tension bars at the support may be computed in the same manner as for the tension bars at the end of a simply supported beam. A slight excess of the computed stress over the allowable is of no consequence, since the actual stress is undoubtedly less than the theoretical, due to the stiffening action of the bent-up portion of the bars, and the effective anchorage which is provided by the continuity of the bars.

104. Points at Which Bars May Be Bent Up. The points at which the horizontal bars may be bent up will depend upon the variation in positive bending moment along the beam. The location of these points may be determined as in Art. 80. It is also necessary to determine the points at which the upper bars may be bent down, *i.e.*, the points at which the inclined bars must intersect the uppermost portion of the beam in order to furnish sufficient reinforcement for the negative moment existing at those sections. It is safe to assume the curve of negative moment as a straight line from the support to the point of zero moment, and to determine the location of the bending points accordingly.

If the two sets of values, one for positive and one for negative moment, are such that the bending of the bars cannot be accomplished without exceeding the limitations imposed, a greater number of bars must be used at the center of the span, or additional bars placed over the support, and the design governed accordingly.

105. Diagonal Tension. It is usually desirable to provide for as much of the diagonal tension as possible with bent-up bars,

but on account of the restrictions placed upon the bending of the bars, it is necessary in most cases to add stirrups or some other form of web reinforcement fully to provide for the inclined stresses. The analysis is similar to that for simply supported beams, the spacing of the stirrups being computed for the regions over which they are required. In all computations relating to diagonal tension, shear, and bond, the average value of $j = 7/8$ may be used.

106. Design of a Typical Continuous Floor Beam. In order to illustrate the application of the principles outlined in Arts. 102 to 105, a typical continuous T-beam, which is a part of a reinforced concrete floor system of the beam-and-girder type, is designed in Art. 197. This problem should be studied at this time, so as to coordinate the various steps which are essential to the complete analysis of a beam of this type.

107. Beams of One Span with Ends Restrained. It is sometimes necessary that beams framing into columns or girders should be calculated as simple beams where a moment coefficient of $1/8\ wl^2$ is used. Some negative moment will actually exist at the supports due to the monolithic nature of the construction. It is good practice to provide a small amount of steel at the top over the supports to prevent the formation of cracks, the amount of steel being left to the judgment of the designer. Ordinarily, about one-half of the positive-moment bars are bent up near the support, to provide for probable negative-moment stresses. The bent bars should preferably be hooked into the support for additional bond resistance.

CHAPTER IV

COLUMNS

108. Types of Columns. Concrete compression members whose unsupported length is more than four times the least dimension of the cross-section are classified as columns. Such members should not be built without reinforcement of some type. In modern construction four types of reinforced concrete columns are used, namely,

1. Columns reinforced with longitudinal steel and closely spaced spirals.
2. Columns reinforced with longitudinal steel and lateral ties.
3. Composite columns: in which a structural-steel or cast-iron column is thoroughly encased in a concrete core of type 2.
4. Combination columns: in which a structural-steel column is wrapped with wire and encased in at least $2\frac{1}{2}$ in. of concrete over all metal except rivet heads.

Types 1 and 2 are more generally used, types 3 and 4 being economical with heavy construction loads or extremely heavy permanent loads.

Pipe columns, in which a steel pipe is filled with concrete are sometimes used.

109. Unsupported Length and Limiting Dimensions. The reinforced concrete column, as it is commonly used in ordinary construction, may be classified as a short column. In specifications it is usual to establish a ratio of length to diameter, or of length to least radius of gyration, above which the column can no longer be considered as a short column. Tests have shown that as long as the ratio of length to diameter is less than 20 or the ratio of length to least radius of gyration is less than 60, there is little variation in the actual strength of columns of the same cross-section for variations in length. In practice it is customary to specify a somewhat smaller ratio. The Joint Code limits a

short column to one whose unsupported length is not greater than 10 times the least lateral dimension.[1] This same code limits the minimum diameter to 12 in. for principal members or, in the case of rectangular columns, a minimum thickness of 10 in. and a minimum gross area of 120 sq. in. The difficulty of making uniform deposition of the concrete in a form of smaller dimensions is obvious, especially with several longitudinal bars with their spirals or ties in the form. In addition, a smaller column has very little reserve strength to withstand possible shocks not allowed for in the design, and if at any time damaged by fire, the loss in effective section is relatively large.

The unsupported length h of a column is the distance between those points at either end where lateral support is present in at least two directions, making an angle of 90 degrees or nearly 90 degrees with one another. Therefore, it follows that the unsupported length is:

1. In flat-slab construction, the clear distance between the floor and the underside of the capital.

2. In beam-and-slab construction, the clear distance between the floor and the underside of the shallowest beam framing into the column.

3. In floor construction with beams in one direction only, the clear distance between floor slabs.

4. In cases where the columns are supported between floors by struts or beams, the unsupported length may be considered decreased, provided these struts or beams meet the column at approximately the same elevation and make horizontal angles of approximately 90 degrees with one another.

5. When haunches are used on beams or struts, the unsupported length may be considered to be reduced by the depth of

[1] For long columns, the Joint Code gives the working load on columns whose length is greater than 10 times the least dimension as

$$P\left(1.3 - 0.03\frac{h}{d}\right)$$

in which P is the total safe load on a column of the same section where the h/a (or d) ratio is less than 10, where a or d is the least dimension of the column and h the unsupported length of the column.

the haunch, provided the haunch is as wide as the beam and at least half the width of the column.

110. Columns with Spiral and Longitudinal Reinforcement. Whenever a material is subjected to compression in one direction, there will be an expansion in the direction perpendicular to the compression axis. Where this expansion is resisted, lateral compressive stresses are developed, which tend to neutralize the effect of the longitudinal compressive stress, and thus increase resistance against failure. This is the principle involved in the use of spiral or hooped reinforcement (see Fig. 52). Within the limit of elasticity the hooped reinforcement is much less effective than longitudinal reinforcement. Such reinforcement, however, raises the ultimate strength of the column, because the hooping delays ultimate failure of the concrete. The concrete continues to compress and to expand laterally, thus increasing the tension in the bands, while final failure occurs upon the excessive stretching or breaking of the hooping. Thus a somewhat higher working stress may be employed on the concrete contained within such hooping than on a concrete not so confined. Tests show that about 1 per cent of closely spaced spiral hooping increases the resistance to ultimate failure sufficiently to allow a reasonable increase in the working stress in the concrete.

As long as the bond between the steel and the concrete is effective, the two materials will deform equally, and the intensities of the stresses will be proportional to their moduli of elasticity. That is, since the deformation λ_c of the concrete must be equal to the deformation of the steel λ_s,

$$\lambda_c = \frac{f_c}{E_c} = \lambda_s = \frac{f_s}{E_s}$$

Therefore

$$f_s = nf_c$$

Let

A_c = net area of concrete in this section.

A_s = area of longitudinal steel in this section.

A_g = overall or gross area of concrete section = $A_c + A_s$

$A_t = A_c + nA_s = A_g + (n - 1)A_s.$

p_g = steel ratio $\frac{A_s}{A_g}$.

f_s = unit compressive stress in steel.

f_c = unit compressive stress in concrete.

P = total strength of reinforced column.

Then

$$
\begin{aligned}
P = f_cA_c + f_sA_s &= f_c(A_g - p_gA_g) + f_cnp_gA_g \\
&= f_cA_g[1 + (n - 1)p_g] \\
&= f_c[A_g + (n - 1)A_s]
\end{aligned}
$$

In the above analysis the stress in the steel does not exceed nf_c. Tests conducted by the American Concrete Institute at Lehigh University and at the University of Illinois showed that the steel reinforcement actually was capable of withstanding much higher stresses without the bond between steel and concrete being destroyed. In general these tests showed that the strength of a reinforced concrete column is actually the sum of the strengths of the concrete and the longitudinal reinforcement, regardless of the ratio of the moduli of elasticity.

For this type of column, the Joint Code (1940) specifies for the safe axial load

$$P = 0.225f'_cA_g + f_sA_s \tag{38}$$

where f'_c is the ultimate strength of the concrete in pounds per square inch and f_s the working stress in the longitudinal reinforcement (16,000 p.s.i. for intermediate grade and 20,000 p.s.i. for hard grade).

The longitudinal reinforcement should consist of at least six bars of a minimum diameter of 5/8 in., and its effective cross-sectional area should be not less than 1 per cent or more than 8 per cent of the gross area of the column section. The ratio of spiral reinforcement p' must be not less than

$$p' = 0.45(R - 1)\frac{f'_c}{f'_s}$$

where p' = ratio of volume of spiral reinforcement to the volume of spiral core (out to out of spirals).

R = ratio of gross area of column to core area.

f'_s = useful limit stress of spiral reinforcement, 40,000 p.s.i. for hot rolled bars of intermediate grade, 50,000 p.s.i. for hard grade, and 60,000 p.s.i. for cold drawn wire.

The center-to center spacing of spirals shall not exceed one-sixth the core diameter, and the clear spacing between spirals shall not exceed 3 in. or be less than 1⅜ in. The spirals should be continuous and held firmly in place and true to line by at least three vertical spacer bars. The spiral bars or wire shall not be less than ¼ in. in diameter for columns up to 18 in. in diameter or less than ⅜ in. above 18 in. The thickness of concrete outside the spiral reinforcement shall be not less than 1½ in.

Where the reinforcing bars are spliced by lapping, the length of the lap should be 24 bar diameters for intermediate grade steel and 30 bar diameters for rail steel provided that the concrete has a strength of 3000 p.s.i. or more. With concrete of lesser strength the lengths given above should be increased one-third. The above values are for deformed bars; lap lengths of plain bars should be increased by 25 per cent. Where changes in the cross-section of a column occur, the offset of the bars should be made where there is lateral support, such as a column capital, floor slab, or metal ties or spirals. The slope of the inclined portion of the bars should not exceed 1 in 6, and the bars above and below should be parallel with the axis of the column.

111. Columns with Longitudinal Reinforcement and Lateral Ties. Tests show that columns without spirals develop lower stresses in both the concrete and the steel, and the Joint Code (1940) specifies the safe axial load for this type of column as 0.8 of that for a spirally reinforced column, or

$$P = 0.18f'_cA_g + 0.8A_sf_s \tag{39}$$

The longitudinal reinforcement should consist of at least four bars of a minimum diameter of ⅝ in., and its effective cross-sectional area should be not less than 1 per cent or more than

4 per cent of the gross area of the column section. There should be a clear distance between the longitudinal bars and the face of the column of $1\frac{1}{2}$ in. plus the diameter of the tie.

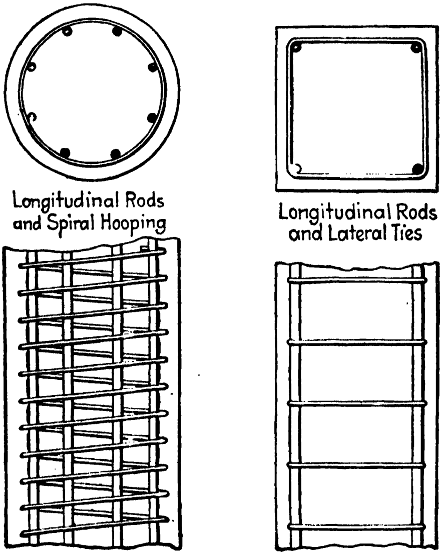


FIG. 52.

The longitudinal bars are held in alignment during construction by lateral ties as illustrated in Figs. 52 and 53. These ties should be made of wire at least $\frac{1}{4}$ in. in diameter,[1] and the verti-

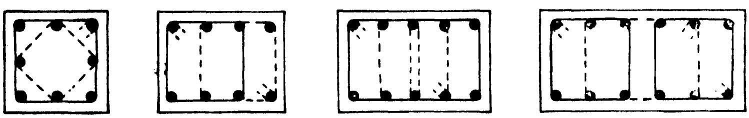


FIG. 53.

cal distance between ties or sets of ties should not exceed 16 bar diameters, 48 tie diameters, or the least dimension of the column. When the number of bars in a column exceeds four, the ties should be so detailed as to prevent the outward bending of every bar at

[1] There is no rational method of determining the size of wire that should be used for a lateral tie. A safe rule to follow is to use wire of such diameter that the area of its section is not less than 2 per cent of the section of the longitudinal reinforcement held in place by the tie.

each tie interval. The methods of accomplishing this are illustrated in Fig. 53.

112. Composite, Combination, and Pipe Columns. Though columns of these types are not strictly reinforced concrete columns, the addition of concrete does, under certain conditions, add to the strength of the column.

A composite column, consisting of a structural-steel or cast-iron column, whose cross-sectional area does not exceed 20 per cent of the gross area of the column, thoroughly encased in concrete, reinforced with both longitudinal and spiral reinforcement, may sustain a load

$$P = 0.225\, A_c f'_c + A_s f_s + f_r A_r$$

where A_c = net area of concrete section = $A_g - A_s - A_r$.

A_s = cross-sectional area of longitudinal bar reinforcement.

A_r = cross-sectional area of steel or cast-iron core.

f_r = permissible unit stress in metal core: 16,000 p.s.i. for a steel core; 10,000 p.s.i. for a cast-iron core.

A combination column consisting of a structural-steel column encased in concrete at least 2½ in. in thickness over all metal (except rivet heads) and reinforced by welded wire mesh wrapped completely around the steel column may sustain a load

$$P = A_r f'_r \left(1 + \frac{A_g}{100\, A_r}\right)$$

where f'_r = permissible unit stress for an unencased steel column and the remaining notation is the same as for a composite column.

A steel pipe filled with concrete may sustain a load

$$P = 0.225\, f'_c A_c + f'_r A_r$$

where f'_r = average unit stress in metal core

$$= \left(18{,}000 - 70\frac{h}{K}\right)F.$$

h = unsupported length of column.

K = least radius of gyration of metal-core section.

$$F = \frac{\text{yield point of pipe}}{45{,}000}.$$

113. Flexural Stresses in Columns. The previous articles have dealt with columns subject to direct axial load only. While there are many cases where this is the only type of load sustained by the column, there are many more cases where the maximum stress developed in the column is a combination of axial stress and bending.[1]

Bending moments are produced in columns (*a*) by reactions from eccentrically placed beams; (*b*) by the loads on brackets or cantilevers; (*c*) by the eccentricity of the columns themselves, a condition which often occurs in the wall columns of a building where the sections of the columns are changed at some floor levels while the exterior faces of the columns are kept in line throughout the height of the structure; (*d*) by the application of a direct horizontal force or of a force having a horizontal component; or (*e*) by the transfer from slabs or girders built monolithic with the columns of unbalanced moments due to the loads on the slabs or girders.

With conditions such as are described in (*a*), (*b*), and (*c*), the amount of moment produced in the column is easily determined, for the amount of load and the eccentricity of its center of application are known. A condition such as described in (*d*) is caused by the wind pressure on the walls of a building, but, on account of the massiveness and rigidity of the structure, it is not usual to calculate the wind stresses in the frame of any but high and narrow buildings of reinforced concrete. A moment caused by the direct application of any other type of horizontal force is not common, but in such cases the moment is usually directly determinate. With conditions such as described in (*e*), the column is a component part of a rigid frame made up of columns and slabs or columns and girders. The distribution of moments in rigid frames will be considered in Chap. VI.

114. Eccentric Loads on Columns. An eccentric load applied to a column at any point will produce a maximum moment at the point of application. The distribution of the moment to the column depends upon the height at which the load is applied, and the end conditions of the column. When the load is applied at

[1] See Chap. V.

the top or bottom of the column, the bending moment has its maximum possible value, and is equal to Px (Fig. 54). For other positions of the load the moment is less, the minimum moment being $\frac{Px}{2}$. The coefficient of Px for different end conditions and different positions of the load may be obtained from Fig. 54. The values of the extreme fiber stresses on either side of the column are obtained by the general method for combined bending and axial stress explained in Chap. V. The value of e is obtained

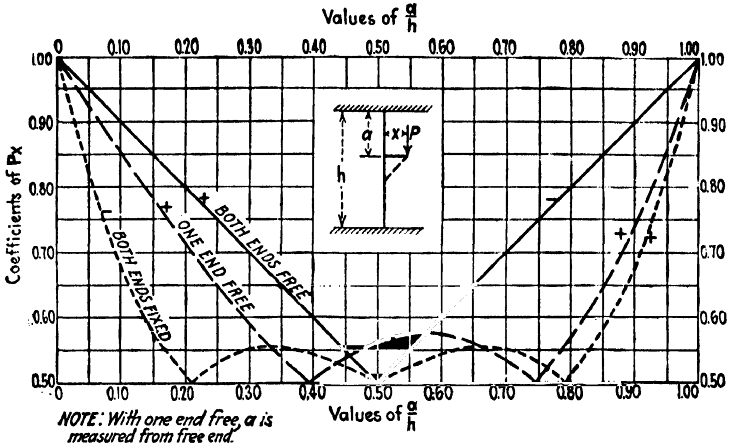


Fig. 54.

by dividing the moment M by the total load (not necessarily P alone) supported by the column at the point where the eccentric load is applied.

115. Working-unit Stresses in Columns Subject to Bending. The maximum permissible stress in the concrete may be increased when the combined effect of bending and axial stress is considered.[1] The maximum stress occurs only on one side of the column and rapidly decreases toward the axis of the column. Tests have shown that a much higher unit stress can be developed on the critical extreme fiber than when the stress is uniformly distributed over the cross-section.

[1] The unincreased unit stress must, of course, not be exceeded when the maximum axial load is sustained or when the total load is considered as an axial load.

The Joint Code (1940) allows a stress on the compressive side of the column of[1]

$$f_c = f_a\left(\frac{1 + \dfrac{ec}{R^2}}{1 + C\dfrac{ec}{R^2}}\right) \tag{40}$$

where f_a = average permissible stress on an equivalent axially loaded concrete column.

$= \dfrac{0.225 f'_c + f_s p_g}{1 + (n - 1)p_g}$ for spiral columns.

$= \dfrac{0.18 f'_c + 0.8 f_s p_g}{1 + (n - 1)p_g}$ for tied columns.

C = ratio of f_a to the permissible fiber stress for members in flexure ($f_a \div 0.45f'_c$).

c = distance from the gravity axis to the extreme fiber in compression.

e = eccentricity of the resultant load on the column, measured from the gravity axis.

R = least radius of gyration of the column section.

As the principal variable in the above equation is the eccentricity e, it involves but a slight error if the transformed steel area is neglected. Diagrams 7 to 12 are plotted on this basis, for tied columns with $\dfrac{e}{a}$ and p_g as arguments, and for spiral columns with $\dfrac{e}{d}$ and p_g as arguments, respectively.

116. Column Tables. Tables 8 and 9 give the safe concentric loads[2] on the concrete and the longitudinal bars of reinforced concrete columns. Values are given for the minimum amount of longitudinal steel. Loads for greater percentages of steel are proportional.

[1] If $D = \dfrac{t^2}{2R^2}$, where t = the side or diameter of the column, this equation may be written $f_c = f_a\left(\dfrac{t + De}{t + CDe}\right)$. Equating this to equation (41), in which $Ne = M$ and $I = A_tR^2$, $P = N\left(1 + \dfrac{CDe}{t}\right)$, indicating that a column subject to flexural stresses may be designed for an equivalent load P, as given in the last equation.

[2] Based on the Joint Code (1940).

Table 10 gives the area of section, weight per foot, and moment of inertia of circular and octagonal sections. It also gives these same functions for rectangular sections 1 in. in width.

Table 11 gives the moment of inertia of the longitudinal reinforcement where the bars composing it are arranged in the form of a circle. The circle is assumed to be 5 in. less in diameter than the diameter of the column. With 1-in. bars, ½-in. spirals, and 1½ in. of concrete outside the spirals, this is exactly true. The possible variation (as long as the minimum 1½ in. is used) either way is negligible. The values are given for a steel ratio p_g of 1 per cent. Other values are proportional.

Table 12 gives the moment of inertia of single bars about an axis at varying distances from the center of the bars. The values obtained from this table, multiplied by the number of bars and $n - 1$, may be used in conjunction with Table 10 in obtaining the moment of inertia of rectangular columns.

Table 13 gives the size and pitch of spirals for various column diameters which satisfy the provisions of the Joint Code given in Art. 110. These values are based on a thickness of concrete outside the spirals of 1½ in. A greater thickness of concrete requires slightly more spiral reinforcement.

117. *Illustrative Problems.* I. A circular column reinforced with longitudinal steel and spiral hooping has an unsupported length of 14 ft.-0 in. and sustains a direct axial load of 250,000 lb. The ultimate strength of the concrete is 3000 p.s.i. and hard grade steel reinforcement is to be used. Design the column.

From Table 8 the following selections are made:

Diam. of col.	Load carried by concrete	Load by steel	Weight of col. (Table 10)	Net load, kips	p_g	A_s sq. in.	Bars selected
20	212	63	5	270	0.010	3.14	8-¾ φ
19	191	63	4	250	0.011	3.12	8-¾ φ
18	172	82	4	250	0.016	4.08	10-¾ φ
17	153	100	3	250	0.022	4.99	9-⅞ φ
16	136	117	3	250	0.029	5.83	8-1 φ
15	119	134	3	250	0.038	6.72	9-1 φ
14	104	148	2	250	0.048	7.39	10-1 φ

All these selections satisfy the specifications of the Joint Code. The 20-in. column has the minimum amount of reinforcement allowed by the specifications; but since it requires no more steel in obtainable sizes than the 19-in. column, the latter is theoretically more economical. However, metal column forms are usually available only in diameters of even integral inches, so that the 20-in. column would usually be chosen.

If it were desirable to have the column as small as possible the 14-in. column could be selected.[1] Care must be taken when large percentages of steel are used that sufficient space is left between the bars. Usually about 2 per cent of longitudinal steel furnishes the most desirable column. For the 20-in. column, ⅜-in. spiral with a pitch of 2 in. (hot rolled) or 2¾ in. (cold drawn) is selected.

II. Select a column reinforced with longitudinal steel and lateral ties with an unsupported length of 14 ft.-0 in. to carry an axial load of 240,000 lb. The concrete strength is 3000 p.s.i., and intermediate grade steel is to be used. Furthermore, the column is to be as small as possible, and its least dimension cannot exceed 14 in.

This column is a long column since $\frac{h}{a} = 12$ and its safe load is $(1.3 - 0.03 \times 12) = 0.94$ times that of a short column. From Table 9 it appears that a 14×18 in. column will be required, which weighs (Table 10) $18.8 \times 14 \times 14 = 2700$ lb., or approximately 3 kips. The total load to be carried is 243 kips, and a short column must be selected which is capable of sustaining $243 \div 0.94 = 259$ kips. From Table 9, the concrete can sustain 136 kips, leaving 123 kips for the steel, which requires a percentage of $123 \div 32 = 3.8$. This is slightly under the maximum of 4 per cent and requires an A_s of $0.038 \times 14 \times 18 = 9.58$ sq.

[1] Such a column would, however, be a long column, since $\frac{h}{d} = 12$, and the safe load would be $(1.3 - 0.03 \times 12) = 0.94$ of that for a short column. $252 \div 0.94 = 268$. This leaves 164 kips to be carried by the reinforcing bars, requiring $164 \div 31 = 5.3$ per cent, or an A_s of $0.053 \times 154 = 8.16$ sq. in. This requires additional steel, or eleven 1-in. round bars.

in. This is furnished by ten 1-in. square bars. The bars must be held in place by ties. The arrangement in Fig. 55 shows three sets of ties. Applying the rule given in the footnote on page 166, each tie should have a sectional area of $0.02 \times 4.000 = 0.08$ sq. in. Wire $\frac{5}{16}$ in. in diameter furnishes approximately this

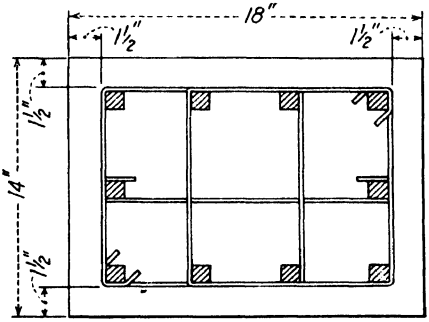


FIG. 55.

cross-section and is selected. Ties are placed 14 in. center to center according to the specifications given on page 166.

ADDITIONAL PROBLEMS

1. Design the column of Problem II as a square column using the minimum amount of reinforcement.

2. Design a column with an unsupported length of 20 ft.-0 in. to sustain a direct axial load of 300,000 lb. $f'_c = 2500$; $f_s = 16{,}000$.

(a) As a spirally reinforced column with $p_g = 0.02$.

(b) As a tied column with $p_g = 0.02$.

3. What safe load can be sustained by a spiral column 22 in. in diameter reinforced with twelve $\frac{7}{8}$-in. round bars?

(a) If $f'_c = 3750$, $f_s = 16{,}000$, and $h = 15$ ft.

(b) If $f'_c = 2000$, $f_s = 20{,}000$, and $h = 20$ ft.

4. What safe load can be supported by a tied column 12×16 in. reinforced with four 1-in. round bars?

(a) If $f'_c = 2500$, $f_s = 16{,}000$, and $h = 16$ ft.

(b) If $f'_c = 3000$, $f_s = 20{,}000$, and $h = 10$ ft.

CHAPTER V

BENDING AND AXIAL STRESS

118. General Theory. In the two preceding chapters, members in bending and members sustaining direct axial stress have been treated separately. In some structural members both types of stresses occur simultaneously and the combined stress due to bending and direct axial load becomes the critical working stress. The more usual cases are

1. A beam subject to inclined forces, or a beam acting as a strut between its supports.

2. A column sustaining in addition to its axial load a bending moment caused by one of the conditions mentioned in Art. 113.

3. An arch rib, where the arch thrust acts other than parallel to, and along the arch axis.

In all of these cases, the resultant stress is a combination of that produced by bending and that produced by the direct load acting along the axis of the member.

Let Fig. 56 represent a plain concrete section BC. The resultant of all the forces R is applied at distance e from the gravity axis of the member. If the resultant R were applied at the point O, the intensity of stress over the whole section BC, whose area is A, would be uniform and equal to $\frac{N}{A}$. Since, however, the resultant R is not applied at the center of the section O, it produces a moment M about the point O equal to Ne; that is, the force N applied at a distance e from the axis may be replaced by an equal force N applied at O and a couple whose moment is Ne. The intensity of the stress at the extreme fibers of the section produced by this moment is $M \times \frac{a}{2} \div I$, in which I is

the moment of inertia of the section about an axis O perpendicular to the plane of the paper. The total intensity of the compression at the edge B is then

$$f_c = \frac{N}{A} + \frac{Ma}{2I}$$

and at the edge C,

$$f_c = \frac{N}{A} - \frac{Ma}{2I}.$$

If the stress f_c is a negative quantity, it shows that the stress produced by the flexure is greater than that produced by the direct action of N, and the resultant stress at the edge C is tension.

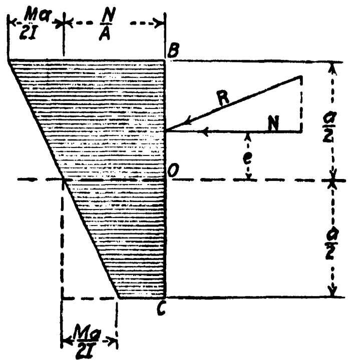


FIG. 56.

In a reinforced concrete member it is presupposed that the bond between the steel and the concrete remains intact under stress. Therefore, the steel in the compression side of a member subject to combined bending and axial stress can withstand a stress only sufficient to make it deform equally with the concrete, or n times the stress in the concrete. This steel might then be replaced by n times the amount of concrete at the same distance from the axis of the section. Such a section is known as the transformed section.[1]

The following additional notation will be used. The face of the member most highly stressed will be called the "compressive surface," and the opposite face, the "tension surface."

R = resultant of all forces on the section.

N = resultant of all forces acting normal to the section, *i.e.* the normal component of R.

e = eccentric distance of N.

M = bending moment = Ne.

A_s = area of steel near tension surface.

A'_s = area of steel near compressive surface.

$$p = \frac{A_s}{ba}, \qquad p' = \frac{A'_s}{ba}, \qquad p_0 = \frac{A_s + A'_s}{ba}$$

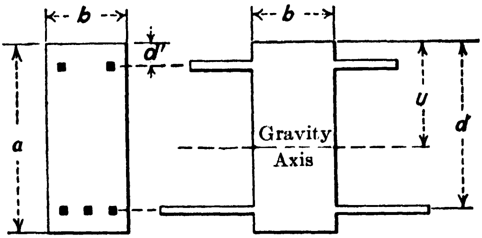
[1] See Art. 54.

u = distance from compressive surface to gravity axis of transformed section.

A_t = area of transformed section.

I_c = moment of inertia of concrete about gravity axis.

I_s = moment of inertia of steel about gravity axis.



Transformed Section

FIG. 57.

By referring to Fig. 57 it may be seen that

$$A_t = ba + (n - 1)(A_s + A'_s)$$

$$I = I_c + (n - 1)I_s$$

$$u = \frac{\frac{a}{2} + p(n - 1)d + p'(n - 1)d'}{1 + p(n - 1) + p'(n - 1)}$$

$$I_c = \frac{1}{3}b[u^3 + (a - u)^3]$$

Neglecting I_s about the gravity axis of the bars,

$$I_s = A_s(d - u)^2 + A'_s(u - d')^2$$

If the reinforcement is symmetrical, then $u = \frac{a}{2}$ and

$$I_c = \frac{1}{12}ba^3 \qquad \text{and} \qquad I_s = 2A'_s\left(\frac{a}{2} - d'\right)^2$$

If the eccentricity $\frac{e}{a}$ is within certain limits, then compression exists over the whole section. For greater eccentricities there will be tension over a part of the section. If it be assumed that the concrete takes no tension, the analyses for these two cases are quite different. The value of $\frac{e}{a}$ which results in zero stress on

the tension surface is dependent upon the relative amounts of steel and concrete, and the ratio of the moduli of elasticity of the two materials.

119. Case I. Rectangular Sections. Compression over the Whole Section, Fig. 58. *Only symmetrical reinforcement will be considered hereafter, and the total steel area will be referred to as*

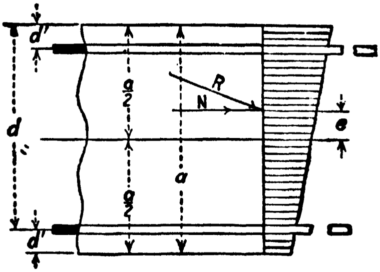


FIG. 58.

A_s. The maximum unit stress in the concrete may be computed as though the member were homogeneous, and is

$$f_c = \frac{N}{A_t} + \frac{Mc}{I} \tag{41}$$

where c = distance from the gravity axis to the extreme fiber in compression.

This equation may be written

$$f_c = \frac{N}{ba + (n - 1)A_s} + \frac{Mc}{I_c + (n - 1)I_s} \tag{42}$$

On the compressive side of the member the unit flexural stress in the plane of the reinforcement is $\frac{M(c - d')}{I}$, and on the other side $\frac{M(d - c)}{I}$. The maximum unit stress in the steel is then

$$f'_s = n\left(\frac{N}{A_t}\right) + \frac{nM(c - d')}{I}$$

which is less than nf_c and is therefore always within the limits of a reasonable value for f'_s, provided f_c has a safe value. Since $f_s = n\left(\frac{N}{A_t}\right) - \frac{nM(d - c)}{I}$, it will always be less than f'_s.

For cases where compression exists over the whole section, equation (41) furnishes an exact solution. For an approximate solution, equation (42) may be written

$$f_c = \frac{N}{ba + nA_s} + \frac{Ma}{2(I_c + nI_s)} \tag{43}$$

The error involved in the above equation is not large, and from this equation diagrams may be prepared that use the same arguments as diagrams for the solution of cases where tension exists over a portion of the section and where the tensile stress in the concrete is neglected.

Equation (43) may be written

$$f_c = \frac{N}{ba + p_o nba} + \frac{Ne\left(\frac{a}{2}\right)}{\frac{1}{12}ba^3 + p_o nba\left(\frac{a}{2} - d'\right)^2}$$

$$= \frac{N}{ba}\left[\frac{1}{1 + p_o n} + \frac{\frac{ea}{2}}{\frac{a^2}{12} + p_o n\left(\frac{a}{2} - d'\right)^2}\right]$$

For given values of $\frac{d'}{a}$ the above equation may be simplified further to

$$f_c = \frac{N}{ba}\left[\frac{1}{1 + p_o n} + \frac{e}{a} \cdot \frac{6}{1 + p_o n Z}\right] \tag{44}$$

where $Z = 3\left(1 - \frac{2d'}{a}\right)^2$.

By allowing the expression within the brackets in equation (44) to be known as K the lower portions of Diagrams 16 to 20 have been plotted from equation (44) for different values of $\frac{d'}{a}$. By entering these diagrams with $p_o n$ and $\frac{e}{a}$ as arguments, the value of K may be obtained for use in the equation

$$f_c = \frac{NK}{ba} \tag{45}$$

120. Case II. Rectangular Sections. Tension over Part of the Section, Fig. 59. When the second term of equation (41) is greater than the first, it indicates tension over part of the section. Unless this tension is so small that the concrete can take its proportionate part, the analysis of Case I is not applicable. With any appreciable tension on the tension surface of the member, it is usual to neglect the tension taken by the concrete and assume the full stress to be taken by the steel.

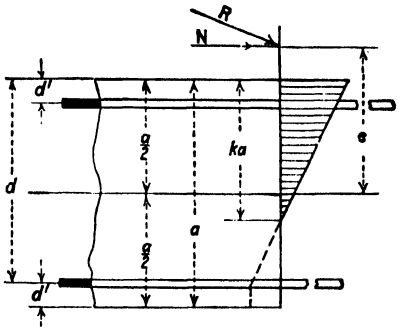


FIG. 59.

By reference to Fig. 59,

$$f'_s = nf_c\left(1 - \frac{d'}{ka}\right) \tag{46}$$

and

$$f_s = nf_c\left(\frac{d}{ka} - 1\right) \tag{47}$$

Since the total resultant stress $= N$,

$$\frac{1}{2}f_cbka + \frac{A_s}{2}f'_s - \frac{A_s}{2}f_s = N$$

Substituting the values of f'_s and f_s from equations (46) and (47),

$$N = \frac{f_cba}{2} \times \frac{k^2 + 2np_ak - np_a}{k} \tag{48}$$

or

$$f_c = \frac{N}{ba}\left[\frac{2k}{k^2 + 2np_ak - np_a}\right] \tag{49}$$

This equation is similar in form to equation (44), but the position of the neutral axis must be determined before it can be used.

Since the moment of the stresses about the gravity axis $= M$,

$$\frac{1}{2}f_c bka\left(\frac{a}{2} - \frac{ka}{3}\right) + \frac{f'_s A_s}{2}\left(\frac{a}{2} - d'\right) + \frac{f_s A_s}{2}\left(d - \frac{a}{2}\right) = M$$

and by eliminating f'_s and f_s as before,

$$\frac{M}{ba^2 f_c} = \frac{np_g(a - 2d')^2}{4ka^2} + \frac{k}{12}(3 - 2k) \qquad (50)$$

Since $M = Ne$, equation (48) may be multiplied by e and this value substituted for M in equation (50). The following equation results:

$$k^3 - 3\left(\frac{1}{2} - \frac{e}{a}\right)k^2 + 6np_g\frac{e}{a}k = 3np_g\left[\frac{e}{a} + \frac{(a - 2d')^2}{2a^2}\right] \qquad (51)$$

In equation (49) the expression within the brackets is designated as K, resulting in an equation exactly like equation (45). For constant values of $\frac{d'}{a}$, values of k and np_g may be substituted in equation (51) and values of $\frac{e}{a}$ obtained. Substituting these same values of k and np_g in equation (49) the corresponding values of K are obtained. The upper portions of Diagrams 16 to 20 have been plotted from such computations.

121. Circular Sections. Compression over the Whole Section. The reinforcement in a circular section is practically always symmetrical and, although it is in the form of bars, it may be considered to be in the form of a hollow cylinder whose average diameter is equal to the diameter of the circle on which the bars are placed, the cross-sectional area of the wall being equal to the area of the bars (see Fig. 60).

Following the same procedure as for rectangular sections, with c equal to r

$$f_c = \frac{N}{A + (n - 1)A_s} + \frac{Mr}{I_c + (n - 1)I_s} \qquad (52)$$

Considering the longitudinal bars replaced by a cylinder of

sectional area A_s, $I_s = A_s R^2 = \frac{A_s s^2}{8}$, where R is the radius of gyration of the equivalent cylinder, whose average diameter s is the diameter of the circle on which the bars are placed. Then the above equation becomes

$$f_c = \frac{N}{\pi r^2 + (n-1)A_s} + \frac{Mr}{\frac{\pi r^4}{4} + (n-1)A_s\frac{s^2}{8}} \qquad (53)$$

These equations give an exact solution where there is no tension on the section. Equation (52) can be used in conjunction with

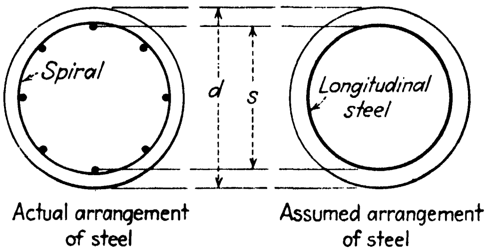


FIG. 60.

the values given in Tables 10 and 11, or equation (53) can be used by making the proper numerical substitutions.

122. Circular Sections. Tension over Part of the Section. A direct solution by equations similar to those used for rectangular sections cannot be made for tension over part of a circular section. The expression of relations becomes very complicated. However, by carrying the derivation partially to completion, values of some of the variables may be assumed, and from these other values determined. In such a manner Diagrams 13 to 15 have been plotted. In the lower portion of these diagrams where there is no tension on the section, equation (53) has been modified to

$$f_c = \frac{N}{\pi r^2}\left[\frac{1}{1+p_g n} + \frac{e}{r}\cdot\frac{1}{0.25 + 0.5pn\left(\frac{s}{d}\right)}\right]$$

in the same manner as equation (44) in order to complete the diagrams. When the diagrams are entered with values of $\frac{s}{d}$

and $p_g n$ as arguments, values of K are obtained for use in the equation

$$f_c = \frac{NK}{\pi r^2} \tag{54}$$

Since a solution by equation (52) is not laborious, the use of the diagrams where there is no tension on the section is not recommended.

123. *Illustrative Problems.* I. A column reinforced with longitudinal steel and lateral ties is to support a direct axial load of 200,000 lb., and in addition an eccentric load of 25,000 lb. on a bracket whose center is 10 ft.-0 in. above the base of the column. The center of bearing of the 25,000-lb. load is 8 in. from the outside face of the column. The unsupported height of the column is 17 ft.-0 in. The ultimate strength of the concrete is specified as 2500 lb. p.s.i., and intermediate grade steel is to be used. Design the column.

Solution a. From Table 9 a column 18 × 20 in. is selected for trial. This column with 2 per cent of steel will sustain 162 + 2 × 46 = 258 kips as a direct axial load as a short column. From Table 10, its weight is 20 × 18.8 = 378 lb. per ft., and the total load on the column is 200 + 25 + 17 × 0.378 = 231 kips. The value of $\frac{h}{d}$ is $\frac{17 \times 12}{18} = 11.3$ so that as a long column it can sustain 1.3 − 0.03 × 11.3 = 0.96 of the safe load of a short column. 258 × 0.96 = 248 kips, showing that the column is safe for direct load.

Assuming that the bracket is on the 18-in. face of the column, if the eccentric load were applied at either the top or the bottom of the column the moment would be 25,000 (10 + 8) = 450,000 in.-lb. Since, however, the load is applied 10 ft.-0 in. from the base of the column, from Fig. 54 (considering both ends fixed) the actual moment at the point of application is 0.54 × 450,000 = 243,000 in.-lb. Eight 1-in. square bars are selected for reinforcement. From Tables 10 and 12, the moment of inertia of the column section is 18 × 667 + 11 × 6 × 56 = 15,700 in.^{4}, and from equation (41).

$$f_c = \frac{7 \times 376 + 225{,}000}{360 + 11 \times 8} + \frac{243{,}000 \times 10}{15{,}700} = 508 + 155$$
$$= 663 \text{ p.s.i.}$$
$$e = \frac{M}{N} = \frac{243{,}000}{227{,}600} = 1.07 \qquad \text{and} \qquad \frac{e}{a} = 1.07 \div 20 = 0.054$$
$$p_g = 8.00 \div 360 = 0.0222$$

From Diagram 11, the allowable unit stress f_c is 665 p.s.i., and the column as selected is satisfactory.

Solution b (Approximate). The column size and the amount of reinforcement are assumed as in solution *a*. Considering that the eight bars are replaced by four bars of equal area symmetrically placed and whose moment of inertia about the short axis of the column is equal to that of the eight bars, $6 \times \frac{4A}{8} \times (7\frac{1}{2})^2 = 2 \times 2A \times s^2$, $s^2 = 42.19$, and $s = 6.50$ in. where $s = \frac{a}{2} - d'$ and A = the area of each of the four replacing bars. Therefore, $d' = 3.5$ in. and $\frac{d'}{a} = 0.175$, $np_g = 0.27$ and $\frac{e}{a} = 0.054$ as before. From Diagrams 18 and 19, $K = 1.05$ and $f_c = \frac{227{,}600}{360} \times 1.05 = 662$.

II. The column of Fig. 61, which is the same as the column designed in Problem I, supports a total load of 125,000 lb. This load is an eccentric load and produces a moment at the top of the column of 750,000 in.-lb. Compute the maximum value of f_c.

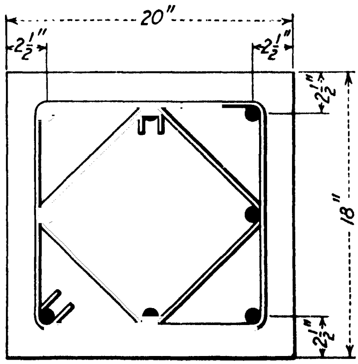


FIG. 61.

By comparison with Problem I, it is seen that the first term of equation (41) is decreased, while the second term is greatly increased, so that the latter becomes greater than the former, indicating tension over a part of the section.

$$e = \frac{M}{N} = \frac{750{,}000}{125{,}000} = 6.0 \text{ in.}$$

$$\frac{e}{a} = \frac{6.0}{20} = 0.30$$

$$\frac{d'}{a} = 0.175$$

$$np_g = 0.27$$

From Diagram 18, $K = 2.18$, and from Diagram 19, $K = 2.35$. For $\frac{d'}{a} = 0.175$, $K = 2.27$ and

$$f_c = \frac{125{,}000}{360} \times 2.27 = 788 \text{ p.s.i.}$$

From Diagram 11, the allowable stress f_c is 845 p.s.i.

NOTE: Where diagrams are used for the solution of problems similar to I and II, it should be noted that the value of K changes rapidly with tension over part of the section for changes in the value of $\frac{d'}{a}$. Therefore, in such cases interpolation between diagrams is necessary if a solution without appreciable error is desired.

III. A circular column reinforced with longitudinal steel of hard grade and spiral hooping sustains a total load of 265,000 lb. The load is so applied as to produce a bending moment of 500,000 in.-lb. at the top of the column. The ultimate strength of the concrete is 3000 p.s.i. Design the column.

Solution a. Assuming about 2 per cent of longitudinal steel, it appears from Table 8 that a column 18 in. in diameter will be required. The actual steel percentage required is $(265 - 172) \div 51 = 1.82$, and $A_s = 0.0182 \times 255 = 4.65$ sq. in. Eight ⅞-in. round bars furnish 4.81, an actual p_g of 0.0189, and are selected for trial. From Tables 10 and 11, the moment of inertia of the column section is $5153 + 1.89 \times 485 = 6070$ in.^{4}, and from equation (41) or equation (52)

$$f_c = \frac{265{,}000}{255 + 9 \times 4.81} + \frac{500{,}000 \times 9}{6070} = 890 + 740 = 1630 \text{ p.s.i.}$$

A glance at Diagram 8 shows that this stress is far greater than the allowable.

Assuming a 22-in. column and ten ⅞-in. round bars, $p_g = 0.0158$, $A = 380 + 9 \times 6.01 = 434$, $I = 11{,}499 + 1.58 \times 1236 = 13{,}450$, and

$$f_c = \frac{265{,}000}{434} + \frac{500{,}000 \times 11}{13{,}450} = 609 + 409 = 1018 \text{ p.s.i.}$$

With $\dfrac{e}{d} = \dfrac{500{,}000}{265{,}000 \times 22} = 0.085$, from Diagram 8, the allowable unit stress is 1020 p.s.i.

Solution b. Assuming the 22-in. column with ten ⅞-in. round bars and a spiral of ⅜ in. at 2 in. (Table 13),

$$s = 22 - 2(1\tfrac{1}{2} + \tfrac{3}{8}) - \tfrac{7}{8} = 17\tfrac{3}{8} \text{ in.,} \qquad \frac{s}{d} = 0.79,$$

$$np_g = 10 \times 0.0158 = 0.16$$

From Diagram 13, $K = 1.45$ and

$$f_c = \frac{265{,}000 \times 1.45}{\pi \times 11^2} = 1011 \text{ p.s.i.}$$

IV. If the bending moment in Problem III were 1,500,000 in.-lb., it is evident that there would be tension in the section. A solution entirely by diagrams is in order. Increasing the column to 26 in. and using twelve 1-in. square bars with a ½-in. spiral at 3 in. (Table 13),

$$s = 26 - 2(1\tfrac{1}{2} + \tfrac{1}{2}) - 1 = 21, \qquad \frac{s}{d} = 0.81, \qquad p_g = 12.00 \div 531$$

$$= 0.0226, \qquad np_g = 0.23, \qquad \frac{e}{d} = \frac{1{,}500{,}000}{265{,}000 \times 26} = 0.22$$

From Diagram 13, $K = 2.28$ and

$$f_c = \frac{265{,}000 \times 2.28}{531} = 1138 \text{ p.s.i.}$$

From Diagram 8, the allowable unit stress $f_c = 1160$ p.s.i.

ADDITIONAL PROBLEMS

1. In a unit section of an arch rib, the bending moment is 60,000 ft.-lb. and the thrust 75,000 lb. The depth of the section is 22 in. and the reinforcement is ⅞-in. round bars 5½ in. center to center, placed 2 in. from each surface. If $n = 12$, what is the maximum unit stress in the concrete?

2. A square column reinforced with longitudinal steel and lateral ties supports a load on a bracket of 100,000 lb., the center of bearing of the load being 4 in. from the face of the column and the top of the bracket being 8 ft. above the base of the column whose unsupported length is 14 ft.-0 in. In addition, it supports a direct axial load of 75,000 lb. Design the column for $f'_c = 3000$ and $f_s = 16,000$.

3. A circular column with spirals sustains a bending moment at its base of 50,000 ft.-lb. In addition, at the top of the column, whose unsupported length is 15 ft.-0 in., there is a direct axial load of 250,000 lb. Design the column for $f'_c = 2500$ and $f_s = 20,000$.

4. A circular column with spirals has a diameter of 20 in. and is reinforced with ten 1-in. round bars. It sustains a direct load of 400,000 lb. $f'_c = 3750$; $f_s = 16,000$. What bending moment can it sustain at the top, its unsupported length being 12 ft.-0 in.?

CHAPTER VI

STRESSES IN CONTINUOUS BEAMS AND BUILDING FRAMES

124. Introductory. The design of a reinforced concrete structure involves an analysis of stress distribution somewhat different from that required for a structure of steel or timber. In the last two types various elementary members are fabricated or cut separately and joined together in the structure by rivets, bolts, welds, or nails. Such joints do not establish complete continuity of a beam over a support (except where welded joints are used), and the junction of beams and columns is not necessarily of sufficient rigidity to transfer bending moments from the beams to the columns. In a reinforced concrete structure consisting of slabs, beams, and columns, as much of the concrete as is practical is poured in one continuous operation, and the whole structure is more or less of a monolith. The slabs and beams are, therefore, continuous from span to span and rigidly joined to the columns which support them. Even when these are conservatively designed as simple beams, negative moment occurs over the supports and must be provided for. It is, then, desirable to recognize the continuity of the slabs and beams in their design and in some cases to analyze the columns for the bending stresses transferred to them.

125. Moments in Continuous Beams. The calculation of moments, shears, and reactions for continuous beams is based on the theorem of three moments. Considering all supports on the same level, the two fundamental equations are (see Fig. 62): For uniform loads,

$$M_1l_1 + 2M_2(l_1 + l_2) + M_3l_2 = -\tfrac{1}{4}w_1l_1^3 - \tfrac{1}{4}w_2l_2^3 \qquad (55)$$

For concentrated loads,

$$M_1l_1 + 2M_2(l_1 + l_2) + M_3l_2 = -\Sigma P_1l_1^2(k_1 - k_1^3) - \Sigma P_2l_2^2(2k_2 - 3k_2^2 + k_2^3) \qquad (56)$$

By using the equation applicable to the particular case, the bending moments at all of the supports may be determined, the reactions computed, and finally, the bending moment at any section of the beam may be obtained. From equation (56), influence lines may be plotted for the moment at any section of the beam, and the loading determined which will produce the

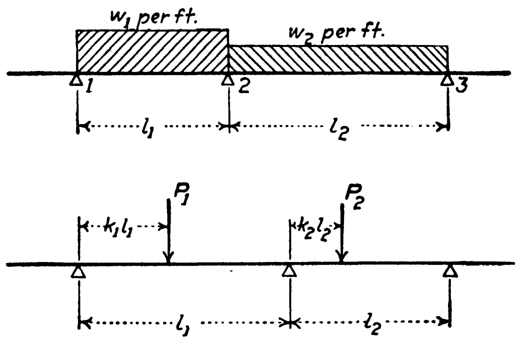


FIG. 62.

maximum moment in that section. In the case of uniform load and equal spans, the exact coefficients of wl^2 for the theoretical maximum moments are tabulated below.

Number of spans	Intermediate spans and supports				End spans and second support			
	At Center Positive Moment		At Support Negative Moment		At Center Positive Moment		At Support Negative Moment	
	Dead load	Live load	Dead load	Live load	Dead load	Live load	Dead load	Live load
Two					.070	.095	.125	.125
Three	.025	.075			.080	.100	.100	.117
Four	.036	.081	.071	.107	.071	.098	.107	.120
Five	.046	.086	.079	.111	.072	.099	.105	.120
Six	.043	.084	.086	.116	.072	.099	.106	.120
Seven	.044	.084	.085	.114	.072	.099	.106	.120

These coefficients are for freely supported beams. In a reinforced concrete structure the more or less fixed condition of the supports and their width tend to make the actual maximum

moments considerably less than those tabulated. Disregarding the case of the two-span beam, the maximum coefficients are:

For intermediate spans	
At the center	+0.086
At the support	−0.116
For end spans	
At the center	+0.100
At the support	−0.120

These coefficients are all for live load, while those for dead load are much smaller. Therefore, it is recommended that beams and slabs of approximately equal spans,[1] built to act integrally with columns, walls, or other restraining supports and assumed to carry uniformly distributed loads be designed for the following moments at critical sections:

Interior Spans:

Negative moment at interior supports except the first,

$$M = \frac{wl^2}{12}$$

Positive moment near centers of interior spans,

$$M = \frac{wl^2}{16}$$

End spans of continous beams or slabs, and beams or slabs of one span:

Where $\frac{I}{l}$ is less than twice the sum of the values of $\frac{I}{h}$ for the exterior columns above and below which are built into the beams:

Positive moment near center of span and negative moment at first interior support,

$$M = \frac{wl^2}{12}$$

[1] It does not require a great variation in length of spans to make the arbitrary adoption of these recommendations dangerous. For instance with three freely supported equally loaded spans of length l, nl, and l, where n is less than 0.84, no positive moment occurs in the span nl.

Negative moment at exterior supports,

$$M = \frac{wl^2}{12}$$

Where $\frac{I}{l}$ is equal to or greater than twice the sum of the values of $\frac{I}{h}$ for the exterior column above and below which are built into the beams:

Positive moment near center of span and negative moment at first interior support,

$$M = \frac{wl^2}{10}$$

Negative moment at exterior support,

$$M = \frac{wl^2}{16}$$

In the above, I represents the moment of inertia of the section (see Art. 136) and l and h, the effective span length of the beam, and the unsupported length of the column (see Art. 109), respectively.

In the case of a single concentrated load or symmetrical concentrated loads, such as often occur in beam-and-girder floor construction, it is usually satisfactory to compute the moment as for a simple beam and make a reduction of one-half, one-third, or one-fifth (whichever is applicable), due to the continuity of the construction.

For beams and slabs of two spans only, a coefficient of $\frac{1}{8}$ is recommended for center supports, and $\frac{1}{10}$ for the center of the span.

Beams and slabs of unequal spans or those sustaining unsymmetrical heavy concentrated loads should be analyzed more exactly, consideration being given to the actual conditions of restraint.

In many reinforced concrete structures, a considerable economy may be effected by making an exact analysis of the moments in

the beams and girders. The principal justification for using the more or less arbitrary coefficients given above is the uncertainty of the live load. The simultaneous application of full live load to all portions of the structure is rarely realized. Even the live load on one panel may vary from a maximum to zero. If all such possible (and in many instances probable) variations of the live load are considered, the maximum moment is seldom found to be much less than that determined by the use of the arbitrary coefficients.

126. Bending Moments in Columns. When the slabs or the beams and slabs of a floor system of reinforced concrete are built monolithic with the columns which support them and the floors above, a certain portion of the bending stresses in the floor is distributed to the columns. When a column is symmetrically loaded in all directions, the bending moments from the adjacent beams and slabs act equally and opposite to one another, so no bending results in the column. With unsymmetrical loading, however, a bending moment is produced in the column, and since the whole structure acts as one rigid frame, this moment produces proportional moments in other members of the frame. In members widely separated, the effect may be so small as to be negligible. Where the unsymmetrical load is a small part of the total load, the increase in the stress is slight. For example, in the upper floors of a building the bending stresses produced in the exterior columns are relatively large, while in the lower floors the bending stresses in the interior columns are comparatively small. Tests show[1] that reinforced concrete buildings act as rigid frames and that bending stresses in the columns are developed in sufficient magnitude to warrant their consideration in design.

The amount of moment transferred to the columns from the floor depends upon the relative stiffnesses of the members of the frame. The stiffness of a member depends upon its length and cross-section and is defined as the moment of inertia of the cross-section divided by the length. The analysis of rigid frames is

[1] See *Bulls.* 64, 84, and 107, Engineering Experiment Station, University of Illinois.

developed in the following articles. This analysis is based on the principles of moment area and slope deflection.[1]

127. The Principles of the Moment Area Method. The line AB of Fig. 63a represents a portion of the elastic curve of a member in flexure. An elementary length ds of the member is

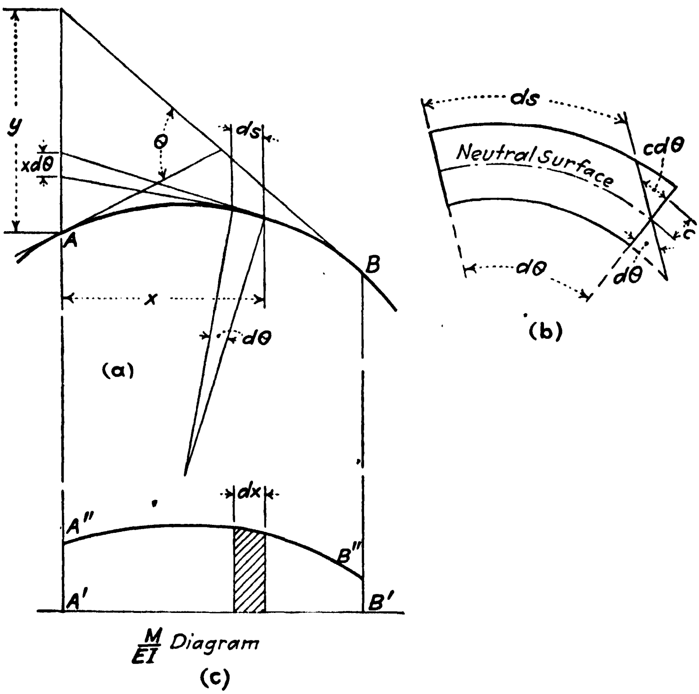


FIG. 63.

shown in Fig. 63b. The angle between the radii at the ends of ds is denoted by $d\theta$. The linear deformation of a fiber at a distance c from the neutral surface is $cd\theta$, and the unit deformation of the same fiber is $\frac{cd\theta}{ds}$. The unit stress in the fiber is

[1] See "Analysis of Statically Indeterminate Structures by the Slope Deflection Method," by W. M. Wilson, F. E. Richart, and Camillo Weiss, *Bull.* 108, Engineering Experiment Station, University of Illinois.

$f = \frac{Mc}{I}$, in which M is the resisting moment and I the moment of inertia of the section.

Since the modulus of elasticity is the ratio of unit stress to unit deformation,

$$E = \frac{Mc}{I} \div \frac{cd\theta}{ds}$$

from which

$$d\theta = \frac{M}{EI}ds$$

In a well-designed beam the curvature and slope are small, so that dx may be substituted for ds without appreciable error. Then,

$$d\theta = \frac{M}{EI}dx$$

In Fig. 63c an ordinate measured between the curve A'' B'' and the straight line $A'B'$ at any point between A' and B' represents, to some scale, the moment in the member AB at that point, divided by EI, or $A''B''B'A'$ is the $\frac{M}{EI}$ diagram for the member AB. The area of the diagram for the length dx is $\frac{M}{EI}dx$, and the area of the diagram $A''B''B'A'$ is $\int_A^B \frac{M}{EI}dx$. But $d\theta = \frac{M}{EI}dx$, and the angle between the tangents to the elastic curve at A and B is $\theta = \int_A^B d\theta = \int_A^B \frac{M}{EI}dx$. Hence:

The change in the slope of the elastic curve between any two points is equal to the area of the $\frac{M}{EI}$ *diagram for the portion of the member between those two points.*

In Fig. 63a the tangents at the extremities of the elementary length ds are extended until they intersect the vertical line through A. Since the angles are small, the intercept between these tangents is practically equal to $xd\theta$. The total vertical distance y is the algebraic sum of all the intercepts between the tangents to the curve between A and B. That is,

$$y = \int_A^B x d\theta$$

Substituting for $d\theta$, its value as previously determined,

$$y = \int_A^B \frac{M}{EI} x dx$$

In the $\frac{M}{EI}$ diagram of Fig. 63c, $\frac{M}{EI}dx$ is the area of the diagram for the length dx, and $\frac{M}{EI}dx$ times x is the moment of this area about the point A. The moment of the entire area of the $\frac{M}{EI}$ diagram between the points A and B may be expressed as

$$\int_A^B \frac{M}{EI} x dx$$

which is equal to the expression developed above for y. Hence:

The distance of any point on the elastic curve from a tangent to the curve at any other point measured in a direction normal to the initial position of the member is equal to the moment of the area of the $\frac{M}{EI}$ diagram, included between the two points, about the first point.

128. Slope Deflection. In computing moments and shears on isolated beams, the ends are usually considered as freely supported or as fixed. However, there is, even in isolated beams, often an intermediate condition. In frames in which the joints are rigid such an intermediate condition is commonly encountered. No matter what the degree of restraint, once the end moments are known the beam is fully determined. These end moments will depend upon (1) the loads on the beam (if any), (2) the rotation of the end tangents to the elastic curve from their original positions, and (3) the relative displacement of their supports. Condition (2) indicates a possible change in slope of the end tangents, while condition (3) indicates a deflection of one or more supports. Hence the term "slope deflection" has been applied to this method of analysis.

129. The Fundamental Slope-deflection Equations. The beam in Fig. 64 is a portion of a frame of which the joints 1 and 2 are rigid joints; *i.e.*, the joint may rotate, but the angles between the tangents to the members making the joint remain constant. In (*b*) the left support, 1, of the beam has been rotated by the action of forces outside of the span 1–2. This rotation causes bending in the beam and produces compression in the upper fiber at 1. In (*c*) the right support, 2, has also been rotated in the same direction by the action of forces outside the beam. This produces tension in the upper fiber at 2, and the rotation is small enough so that the upper fiber at 1 remains in compression. Since the stresses in the upper fiber at 1 and 2 are of opposite sign, there is evidently a point of inflection in the beam and the moment diagram for the beam will be similar to that shown in (*d*).

Since the angles of rotation are small, the distance at 2 from a tangent to the axis of the beam at 1 is $l\alpha_1$, where l is the span 1–2. Then by the principle of moment areas, $l\alpha_1$ is equal to the moment of the $\frac{M}{EI}$ diagram between 1 and 2 about 2. This moment is evidently equal to the algebraic sum of the moments of the shaded areas about 2, *i.e.*,

$$l\alpha_1 = \frac{1}{EI}\left(\frac{M_1 l}{2} \times \frac{2l}{3} - \frac{M_2 l}{2} \times \frac{l}{3}\right) \qquad \text{or} \qquad \alpha_1 = \frac{l}{6EI}(2M_1 - M_2)$$

Similarly,

$$\alpha_2 = \frac{l}{6EI}(2M_2 - M_1)$$

In (*e*) the joint 2 has been moved downward a vertical distance d from its original position, the horizontal distance between joints 1 and 2 remaining constant. This has increased the angles between the tangents to the member 1–2 at 1 and 2 and the horizontal by an amount $\frac{d}{l} = R$. If then $\theta_1 = \alpha_1 + R$ and $\theta_2 = \alpha_2 + R$,

$$\theta_1 = \frac{l}{6EI}(2M_1 - M_2) + R$$

and

$$\theta_2 = \frac{l}{6EI}(2M_2 - M_1) + R$$

from which

$$M_1 = \frac{2EI}{l}(2\theta_1 + \theta_2 - 3R) \tag{57}$$

and

$$M_2 = \frac{2EI}{l}(2\theta_2 + \theta_1 - 3R) \tag{58}$$

Equations (57) and (58) are the fundamental slope-deflection equations for the moments at the ends of a member sustaining no intermediate loads, in terms of the relative change in slope and displacement of its ends.

If a load is applied vertically downward, on the beam 1–2, it will tend to rotate the joint 1 in a clockwise direction and joint 2 in a counter-clockwise direction. However, to resist these rotations, moments are developed in the joints that tend to rotate the joints in the opposite directions. Since the determination of the total resisting moment developed in a member forming part of a rigid joint is the ultimate aim, the *resisting moment* developed by loads on the member must be added algebraically to the moments induced into the member by external forces. The rotation caused by this resisting moment at joint 1 is opposite to those hitherto considered, while at joint 2 it is in the same direction. From another point of view, the vertical load on the member 1–2 causes tension in the upper fibers at both 1 and 2. This is the reverse of the stress caused by the original rotation of joint 1 and the same as the original rotation of joint 2. Therefore, it is seen that the additional term of the equation for M_1 caused by a vertical load acting downward on the member 1–2 will be of opposite sign to those terms developed for forces outside of the member, while for M_2 the additional term will have the same sign. Another conception of the relative signs is as follows. If

before the application of the intermediate load the beam 1–2 is cut, the joint 1 will rotate in a clockwise direction. If after the intermediate load is applied the beam is cut to the left of the load, the joint 1 will tend to rotate in a counter-clockwise direction

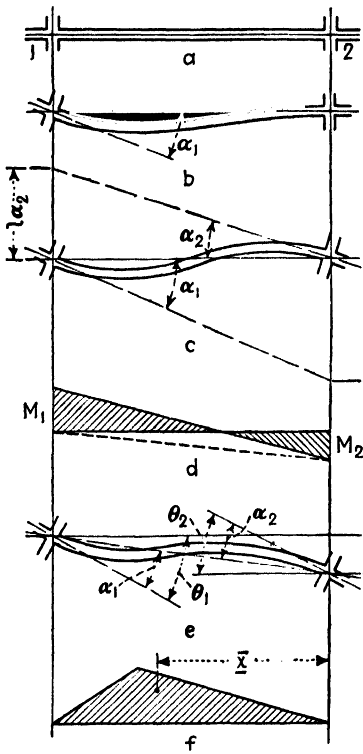


FIG. 64.

tion owing to the resisting moment developed at 1 by the application of the intermediate load.

The moment diagram for a single vertical load is shown in (*f*). Considering the distance at the center of gravity of its area F from support 2 as $\bar{x}$, and again applying the principle of moment areas,

$$l\theta_1 = \frac{l^2}{6EI}(2M_1 - M_2) + lR - \frac{F\bar{x}}{EI}$$

and

$$l\theta_2 = \frac{l^2}{6EI}(2M_2 - M_1) + lR - F\left(\frac{l - \bar{x}}{EI}\right)$$

from which

$$M_1 = \frac{2EI}{l}(2\theta_1 + \theta_2 - 3R) - \frac{2F'}{l^2}(3\bar{x} - l) \tag{59}$$

and

$$M_2 = \frac{2EI}{l}(2\theta_2 + \theta_1 - 3R) + \frac{2F}{l^2}(2l - 3\bar{x}) \tag{60}$$

Equations (59) and (60) are identical with Eqs. (57) and (58), except that each contains an additional term due to the effect of intermediate loads. This term is independent of the *slopes* and *deflections* of the member; and if θ_1, θ_2, and R are all zero, which is true of a fixed beam with stationary supports, this last term, which depends solely on the intermediate loads, is equal to the "fixed end moment" of such a beam.

Substituting K for $\frac{I}{l}$ and C for the last terms, Eqs. (59) and (60) are usually written as

$$M_1 = 2EK(2\theta_1 + \theta_2 - 3R) - C_1 \tag{61}$$
$$M_2 = 2EK(2\theta_2 + \theta_1 - 3R) + C_2 \tag{62}$$

In the foregoing derivation the following sign convention has been used:

(*a*) When the angular movement of the end tangents is clockwise, the angles measuring this movement are positive angles.

(*b*) The deflection of one end with respect to the other is measured normal to the original position of the member and is positive when measured in the same direction as positive angles.

(*c*) External end moments are positive when they tend to rotate the joint on which they act in a clockwise direction.

(*d*) If the load in a member, independent of the member, tends to rotate around the joint under consideration in a clockwise direction, the sign of the fixed end moment term is negative, since the resisting moment must act opposite to the loads.

Conversely, if the load tends to rotate counter-clockwise, the sign of the fixed end moment term is positive.

As an illustration of the sign convention, let AC in Fig. 65*a*, represent a beam fixed at the left end A and resting on a support at B, over which it continues, to be fixed again at C. The load P tends to rotate the support or joint A in a clockwise direction. Therefore the moment in the beam at A is negative. Conversely, the load tends to rotate the joint B in a counter-clockwise direction; hence, the moment at B in AB is positive.

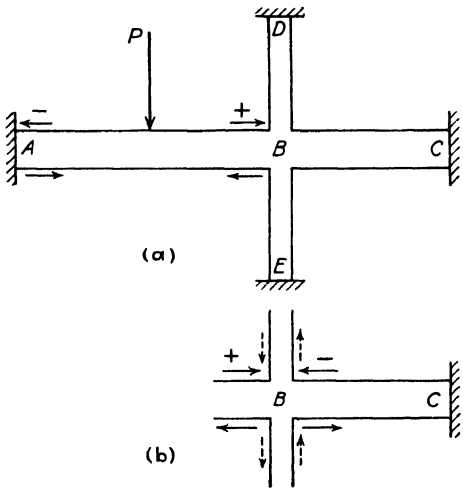


FIG. 65.

In Fig. 65*b* the right-hand portion of (*a*) is shown as a free body. The positive moment from the beam AB tends to rotate the joint B in a clockwise direction. This is resisted by an opposite moment in the beam BC which is therefore negative. If a downward vertical load were added on BC, the resisting moment at B would be increased, since such a load would tend to rotate the joint B in a clockwise direction and the additional moment [the C term of equation (61)] would be negative.

In the above it has been assumed that DB and EB are incapable of resisting the rotation of the joint B. If they were of sufficient stiffness, they would, in proportion to their stiffness, develop counter-clockwise or negative resisting moments in the

same manner as BC, while the resisting moment developed in the latter would be reduced accordingly.

130. Conditions of Restraint. The degree of restraint at one end of a member is frequently quite different from that at the other end. For example, a beam may be rigidly fixed at one end and hinged at the other, or it may be partially fixed at one end and rigidly fixed at the other. It is seen from equation (57)

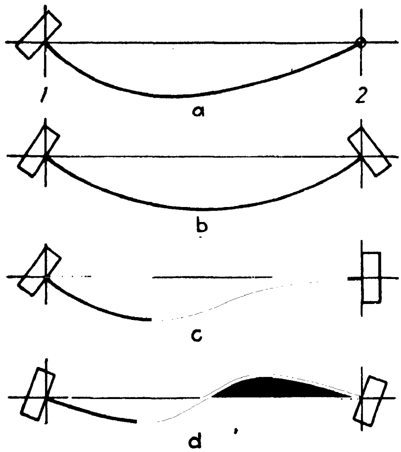


Fig. 66.

that the value of M_1 depends not only upon the condition of that end ($2\theta_1$) but also upon the condition at the other end (θ_2).

The four usual variations in end restraint are shown in Fig. 66. In a, with the right end hinged, $M_2 = 0$, and assuming for this and the following cases that R equals zero, equations (61) and (62) can be combined to eliminate[1] θ_2, and

$$M_1 = 3EK\theta_1 \tag{63}$$

[1] Combining equations (61) and (62), with the right end considered hinged,

$$2M_1 - M_2 = 2EK(3\theta_1 - 3R) - 2C_1 - C_2$$

and since $M_2 = 0$,

$$M_1 = 3EK(\theta_1 - R) - \left(C_1 + \frac{C_2}{2}\right) \tag{61a}$$

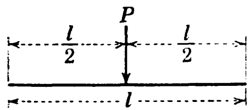
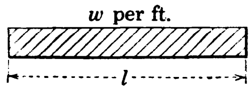
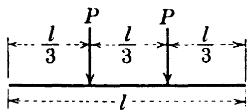
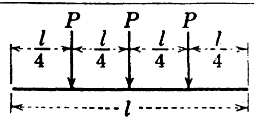
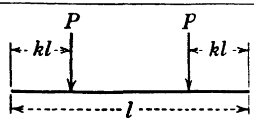
Similarly with the left end hinged,

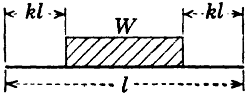
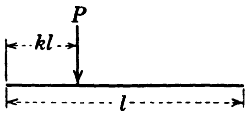
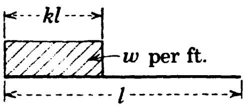
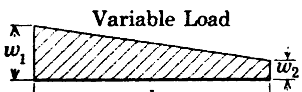
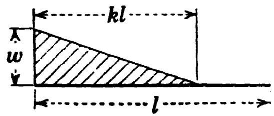
$$M_2 = 3EK(\theta_2 - R) + \left(C_2 + \frac{C_1}{2}\right) \tag{62a}$$

TABLE *A*.—MOMENTS FOR DIFFERENT CONDITIONS OF RESTRAINT

Case (see Fig. 57)	Relation between θ angles	Moment at left end in terms of θ_1; no load between ends	Moment at right end in terms of θ_1 and θ_2; no load between ends	Moment at left end in terms of θ_1; intermediate loads	Moment at right end in terms of θ_1 and θ_2; intermediate loads
a Right end hinged	$\theta_2 = -\frac{1}{2}\theta_1$	$M_1 = 3EK\theta_1$	$M_2 = 0$	$M_1 = 3EK\theta_1 - \left(C_1 + \frac{C_2}{2}\right)$	$M_2 = 0$
b	$\theta_2 = -\theta_1$	$M_1 = 2EK\theta_1$	$M_2 = -2EK\theta_1$ $M_2 = 2EK\theta_2$	$M_1 = 2EK\theta_1 - C_1$	$M_2 = -2EK\theta_1 + C_2$ $M_2 = 2EK\theta_2 + C_2$
c Right end fixed	$\theta_2 = 0$	$M_1 = 4EK\theta_1$	$M_2 = 2EK\theta_1$	$M_1 = 4EK\theta_1 - C_1$	$M_2 = 2EK\theta_1 + C_2$
d	$\theta_2 = \theta_1$	$M_1 = 6EK\theta_1$	$M_2 = 6EK\theta_1$ $M_2 = 6EK\theta_2$	$M_1 = 6EK\theta_1 - C_1$	$M_2 = 6EK\theta_1 + C_2$ $M_2 = 6EK\theta_2 + C_2$

TABLE B.—VALUES OF C_1, C_2, $C_1 + \frac{C_2}{2}$, AND $C_2 + \frac{C_1}{2}$ FOR VARIOUS TYPES OF LOADING

TYPE OF LOADING	C_1	C_2	$C_1 + \frac{C_2}{2}$ RIGHT END HINGED	$C_2 + \frac{C_1}{2}$ LEFT END HINGED
	$\frac{1}{8} Pl$	$\frac{1}{8} Pl$	$\frac{3}{16} Pl$	$\frac{3}{16} Pl$
	$\frac{1}{12} wl^2$	$\frac{1}{12} wl^2$	$\frac{1}{8} wl^2$	$\frac{1}{8} wl^2$
	$\frac{2}{9} Pl$	$\frac{2}{9} Pl$	$\frac{1}{3} Pl$	$\frac{1}{3} Pl$
	$\frac{5}{16} Pl$	$\frac{5}{16} Pl$	$\frac{15}{32} Pl$	$\frac{15}{32} Pl$
	$Pkl(1-k)$	$Pkl(1-k)$	$\frac{3}{2} Pkl(1-k)$	$\frac{3}{2} Pkl(1-k)$

TYPE OF LOADING	C_1	C_2	$C_1+\frac{C_2}{2}$ RIGHT END HINGED	$C_2+\frac{C_1}{2}$ LEFT END HINGED
	$\frac{Wl}{12}(1+2k-2k^2)$	$\frac{Wl}{12}(1+2k-2k^2)$	$\frac{Wl}{8}(1+2k-2k^2)$	$\frac{Wl}{8}(1+2k-2k^2)$
	$Pk(1-k)^2 l$	$Pk^2(1-k)l$	$\frac{1}{2}P(1-k)(2-k)kl$	$\frac{1}{2}P(1-k^2)kl$
	$wl^2k^2(3k^2-8k+6)$	$\frac{wl^2k^3}{12}(4-3k)$	$\frac{1}{8}wl^2k^2(2-k)^2$	$\frac{1}{8}wl^2k^2(2-k^2)$
	$\frac{l^2}{60}(3w_1+2w_2)$	$\frac{l^2}{60}(2w_1+3w_2)$	$\frac{l^2}{120}(8w_1+7w_2)$	$\frac{l^2}{120}(7w_1+8w_2)$
	$\frac{wk^2l^2}{60}(10-10k+3k^2)$	$\frac{wk^3l^2}{60}(5-3k)$	$\frac{wk^2l^2}{120}(20-15k+3k^2)$	$\frac{wk^2l^2}{120}(10-3k^2)$

Comparing equation (63) with equation (61) it is seen that for this condition $\theta_2 = -\frac{1}{2}\theta_1$.

In (*b*), with restraint equal at both ends, $\theta_2 = -\theta_1$, and

$$M_1 = 2EK\theta_1 \tag{64}$$

In (*c*), with the right end completely restrained, $\theta_2 = 0$, and

$$M_1 = 4EK\theta_1 \tag{65}$$

In (*d*), with restraint of such character that a point of inflection occurs at the midpoint of the member $\theta_2 = \theta_1$ and

$$M_1 = 6EK\theta_1 \tag{66}$$

The values of M_2 may be derived in terms of either θ_1 or θ_2 as desired, for conditions (*b*) and (*d*), and in terms of θ_1 for condition (*c*). A summary of the various equations is given in

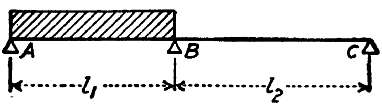


FIG. 67.

Table *A*. These equations apply to members with level supports only. If the supports are not on the same level, owing to the displacement of one or both of them, equation (61), (62), (61*a*), or (62*a*) must be employed.

The more common conditions of loading are given in Table *B*, together with the corresponding values of C_1, C_2, $C_1 + \frac{C_2}{2}$, and $C_2 + \frac{C_1}{2}$. For other loadings similar values may be obtained by determining the area of the simple-beam moment diagram due to such loads, and the distance of its centroid from the end of the member under consideration. Making the proper substitutions in the last terms of equations (59) and (60), the desired values of C_1 and C_2 are determined. It should be noted that for symmetrical loads the last term of these equations reduces to $\frac{F}{l}$.

131. Application of the Slope-deflection Methods to Simple Cases. (*a*) Applying the equations of the preceding article to a

beam of two spans l_1 and l_2, respectively, resting freely upon its supports and sustaining a uniform load w over the span l_1 (see Fig. 67):

From equation (62*a*),

$$M_{BA} = 3EK_1\theta_B + \frac{w_1 l_1^2}{8}$$

and

$$M_{BC} = 3EK_2\theta_B$$

Since there is equilibrium at the joint B,

$$M_{BA} + M_{BC} = 0$$

or

$$(3EK_1 + 3EK_2)\theta_B + \frac{w_1 l_1^2}{8} = 0$$

and

$$\theta_B = -\frac{1}{3EK_1 + 3EK_2} \cdot \frac{w_1 l_1^2}{8}$$

Substituting the value of θ_B in equation (62*a*),

$$M_{BC} = -M_{BA} = -\frac{K_2}{K_1 + K_2}\left(\frac{w_1 l_1^2}{8}\right)$$

If the load covers the span l_2 instead of the span l_1, a similar analysis gives

$$M_{BA} = -M_{BC} = \frac{K_1}{K_1 + K_2}\left(\frac{w_2 l_2^2}{8}\right)$$

For both spans sustaining loads w_1 and w_2, respectively,

$$M_{BC} = -M_{BA} = -\left[\frac{K_2(w_1 l_1^2) + K_1(w_2 l_2^2)}{8(K_1 + K_2)}\right]$$

which for equal loads, equal spans, and equal moments of inertia becomes $-\frac{1}{8}wl^2$, the negative moment over the center support of a beam of two equal spans resting freely on its supports.

(*b*) In Fig. 68, a beam of two spans fixed at both ends rests freely on the intermediate support.

From Tables A and B,

$$M_{BC} = 4EK_2\theta_B$$

and

$$M_{BA} = 4EK_1\theta_B + Pk^2(1 - k)l$$

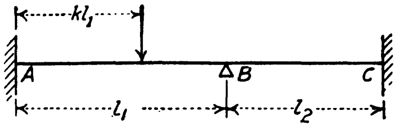


FIG. 68.

from which

$$\theta_B = -\frac{1}{4EK_1 + 4EK_2}Pk^2(1 - k)l$$

and

$$M_{BC} = -M_{BA} = -\frac{K_2}{K_1 + K_2}Pk^2(1 - k)l$$

132. Building Frames. Reinforced concrete building frames are composed of columns and slabs or of columns, beams, girders, and slabs. In the latter case the girders and columns or the

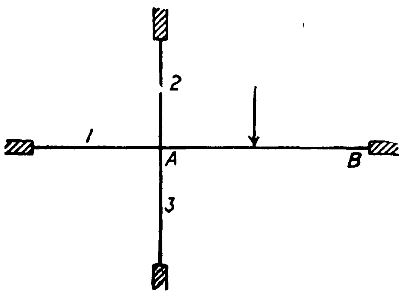


FIG. 69.

beams and the columns may be considered to form a more or less rigid frame. The columns may be of the same size throughout the structure or their cross-sections may vary with the load that they must sustain. Similarly, uniformity or variation may be found in the beams and girders of a structure.

Consider Fig. 69 to illustrate a portion of a building frame. The point A is the junction of the members shown. The degree

of rigidity of A depends upon the relative stiffnesses of the members intersecting in the joint and upon the degree of restraint imposed upon the farther ends of the several members. Also the moment in AB at A and the moment in members 1, 2, and 3 caused by the load on AB depend upon these same considerations. If any or all of the members 1 to 3 are infinitely rigid, the point A is fixed, while if none of the members are at all rigid, the point A may be considered hinged. In the latter case, the moment

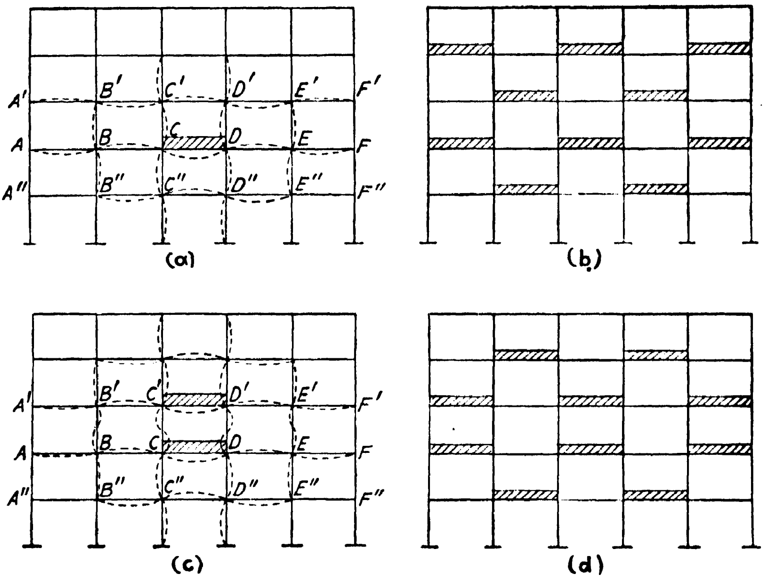


FIG. 70.

in AB at A is zero and no moment is transferred to the other members. In a building frame of reinforced concrete, each member of the frame is restrained to some extent at each end, due to the rigid connection existing between it and the other members of the frame. This restraint causes negative moments in the ends of the beams, tending to produce rotation at these points, and results in flexural stresses in the intersecting members.

Figure 70a shows one span of a building frame sustaining uniform load. The deformations caused in the various members of the frame by this loading are indicated by the broken lines. The deformations in the members immediately adjacent to C

and D are much greater than those in members farther removed from these points. If loads were added on BC and DE, there would be practically no deformations in the columns at C and D, and CD would be practically fixed. On the other hand, if loads were placed on AB and EF, the deformations of the columns at C and D would be increased. Still greater deformations of the columns at C and D could be obtained by the loading shown in Fig. 70*b*.

Another loading producing large stresses in the columns is that shown in Fig. 70*c*. This type of loading develops a point of contraflexure in the center of the columns between $C'D'$ and CD. A still further increase in stress occurs with the type of loading shown in Fig. 70*d*.

133. Moments in Beam and Girder Building Frames. In Fig. 71, which is a portion of the frame of Fig. 70, it is possible, by assuming various conditions of restraint at the terminals B, C', D', E, D'', and C'', to approximate any condition of loading. It will be assumed that the three girders have equal cross-sections and lengths, that the upper columns $C'C$ and DD' are equal in stiffness, and that the lower columns CC'' and DD'' are also equal in this respect. The stiffness of the girder is K_1, that of the column above C is K_2, and that below C is K_3.

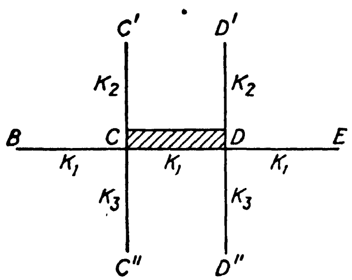


FIG. 71.

(*a*) *All Terminals Hinged.* From Table A

$$M_{CD} = 2EK_1\theta_C - C_{CD}$$
$$M_{CB} = 3EK_1\theta_C$$
$$M_{CC'} = 3EK_2\theta_C$$
$$M_{CC''} = 3EK_3\theta_C$$

For equilibrium, the sum of all the moments about the joint C must be zero, *i.e.*,

$$M_{CD} + M_{CB} + M_{CC'} + M_{CC''} = 0$$

Substituting the values for the several moments

$$\theta_C = \frac{C_{CD}}{E}\left(\frac{1}{5K_1 + 3K_2 + 3K_3}\right)$$

Substituting the value of θ_C in the moment equations

$$M_{CB} = C_{CD}\left(\frac{3K_1}{5K_1 + 3K_2 + 3K_3}\right)$$
$$M_{CC'} = C_{CD}\left(\frac{3K_2}{5K_1 + 3K_2 + 3K_3}\right)$$
$$M_{CC''} = C_{CD}\left(\frac{3K_3}{5K_1 + 3K_2 + 3K_3}\right)$$

and since

$$M_{CD} = -(M_{CB} + M_{CC'} + M_{CC''})$$
$$M_{CD} = -C_{CD}\left(\frac{3K_1 + 3K_2 + 3K_3}{5K_1 + 3K_2 + 3K_3}\right)$$

(*b*) *All Terminals Fixed.* From Table A

$$M_{CD} = 2EK_1\theta_C - C_{CD}$$
$$M_{CB} = 4EK_1\theta_C$$
$$M_{CC'} = 4EK_2\theta_C$$
$$M_{CC''} = 4EK_3\theta_C$$

As before,

$$\theta_C = \frac{C_{CD}}{E}\left(\frac{1}{6K_1 + 4K_2 + 4K_3}\right)$$
$$M_{CB} = C_{CD}\left(\frac{2K_1}{3K_1 + 2K_2 + 2K_3}\right)$$
$$M_{CC'} = C_{CD}\left(\frac{2K_2}{3K_1 + 2K_2 + 2K_3}\right)$$
$$M_{CC''} = C_{CD}\left(\frac{2K_3}{3K_1 + 2K_2 + 2K_3}\right)$$

and

$$M_{CD} = -C_{CD}\left(\frac{2K_1 + 2K_2 + 2K_3}{3K_1 + 2K_2 + 2K_3}\right)$$

In building frames the actual conditions of restraint at the ends of the various members are usually neither hinged nor fixed.

The conditions of restraint may approximate either the hinged or fixed state or be similar to one of those illustrated in Fig. 66*b* or 66*d*. The value of the moment in the girder itself varies but little for the different conditions of end restraint. The coefficient of the terms K_2 and K_3 is the same in both numerator and denominator, and that of K_1 in the denominator is never less than that in the numerator. Therefore, the actual moment is never greater than that in a continuous girder with restrained ends, while in all cases except those involving girders large in comparison with the columns, it will be nearly equal to the moment as determined for a continuous girder with restrained ends.

The conditions of loading and restraint producing the maximum probable moment in the columns will be discussed in the following articles.

134. Interior Columns. Unless the column spacings are very irregular, no moment will be developed in the interior columns by the dead load of the structure. In the usual case of bays equal or nearly so, the dead load is symmetrical with respect to the columns and the only moment that can be developed in the columns is that caused by the possible eccentricity of the live load.

An inspection of Fig. 70 shows that the maximum moment in an interior column such as $C'C$ occurs under a loading of the type shown in either (*b*) or (*d*). Either one of these loadings is extremely unlikely. Loadings of the type shown in either (*a*) or (*c*), however, produce the same effect and are much more probable.[1]

Considering the joint C of Fig. 70*a*, any one of the joints at B, C', D', E, D'', and C'' may have a condition of restraint varying between the hinged and fixed state. The maximum moment in the column CC' would be developed if C' and D' are fixed and the remaining joints hinged. The actual condition of restraint at the joints, however, is not hinged but more closely approaches

[1] F. E. Richart, in "A Study of Bending Moments in Columns," *Proc.*, A.C.I., vol. 20, p. 495, states that a loading such as (*c*) produces about 80 per cent of the moment in column $C'C$ as would be produced by a loading such as in (*d*).

the fixed state. With all terminals considered fixed, the moment

$$M_{CC'} = C_{CD}\left(\frac{4K_2}{6K_1 + 4K_2 + 4K_3}\right) \tag{67}$$

In the lower column the maximum moment is developed with C'' and D'' fixed and the other joints hinged, but for the same reasons as given above, all terminals will be considered fixed and

$$M_{CC''} = C_{CD}\left(\frac{4K_3}{6K_1 + 4K_2 + 4K_3}\right) \tag{68}$$

Similarly, considering the joint of Fig. 70*c*, the maximum moment in the column CC' occurs when all the joints except C' and D' are fixed, while the condition of restraint of these two joints is similar to that shown in Fig. 66*d*. In this case

$$M_{CC'} = C_{CD}\left(\frac{6K_2}{6K_1 + 6K_2 + 4K_3}\right) \tag{69}$$

Likewise, with the loading on $C''D''$ instead of on $C'D'$, C'' and D'' being in a condition of restraint similar to that shown in Fig. 66*d*, while the other joints are fixed,

$$M_{CC''} = C_{CD}\left(\frac{6K_3}{6K_1 + 4K_2 + 6K_3}\right) \tag{70}$$

The upper columns are never larger in cross-section than those below them, so that with equal story heights, K_3 is never less than K_2, but it is often greater. Therefore, the maximum probable moment in an interior column is never likely to be greater than would be obtained from equation (70). Conditions of restraint varying from the conditions assumed in the development of equation (70) would cause a slight variation in the moment in the column, but the conditions as assumed are usually actually realized, so the above equation is recommended for general use.

The basement columns of a building frame present a special case, in that the basement floor is usually not an integral part of the structure, and does not transfer its load to the frame. The maximum probable moment is developed when the basement columns are fixed, all other joints of the frame being likewise

assumed as fixed. The value of the moment is then given by equation (68).

Another special case occurs in the upper tier of columns. Any eccentric moment in the roof girder must be transferred to the adjacent girder and the supporting column, there being no upper column to aid in absorbing such moment. If the live load on the roof is nearly as great as that on the floor, so that with the type of loading shown in Fig. 70*c* a point of inflection may be assumed at the center of the columns, the moment at the top of the roof column is

$$M'_{CD}\left(\frac{6K_3}{6K_{1'} + 6K_3}\right) \tag{71}$$

in which M'_{CD} is the moment in the roof girder, and $K_{1'}$ the stiffness of the roof girder.

Usually the moment in the roof girder is so much smaller than the corresponding moment in the floor girder that the maximum moment in the roof column occurs at the bottom of the column. In such a case, the roof load being considerably less than the floor load, it is not reasonable to expect that a loading similar to that of Fig. 70*c* will cause a point of inflection at the center of the columns. If the top of such a column is considered fixed, however, it would seem that the moment computed according to this assumption is great enough to provide for the actual stresses developed. Therefore, the moment at the bottom of the column may be taken as

$$M_{CD}\left(\frac{4K_2}{6K_1 + 4K_2 + 4K_3}\right) \tag{72}$$

The critical cases are summarized in Fig. 72. When there is no live load on certain panels, while the remainder of the structure is sustaining its full live load, the effect on the columns adjacent to the unloaded panel or panels is similar to the effect caused by the loading of these panels only. The type of loading indicated in Fig. 72 produces larger total stresses in the columns than any of the loadings of Fig. 70, since each column must sustain all or nearly all of its design dead and live load. The M_1 of equations

shown in the figure is the value of the fixed end live-load moment in the girder as given in the table on page 202 or computed for some other type of load.[1]

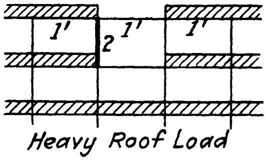
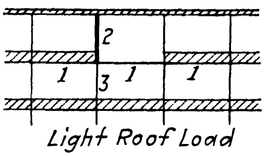
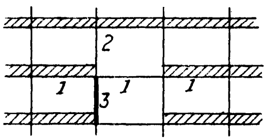
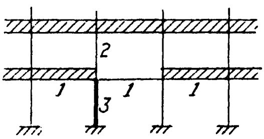
	Type of Loading	Moment and Direct Load
Roof Column		Moment at Top of Column $M_2 = M_{1'}\left(\frac{K_2}{K_1'+K_2}\right)$ Direct Load = Full Load $-\frac{1}{2}$ Live Roof Panel Load
		Moment at Bottom of Column $M_2 = M_1\left(\frac{3K_2}{3K_1+3K_2+2K_3}\right)$ Direct Load = Full Load
Intermediate Column		Moment at Top of Column $M_3 = M_1\left(\frac{3K_3}{3K_1+2K_2+3K_3}\right)$ Direct Load = Full Load $-\frac{1}{2}$ Live Floor Panel Load
Basement Column		Moment at Top of Column $M_3 = M_1\left(\frac{2K_3}{3K_1+2K_2+2K_3}\right)$ Direct Load = Full Load $-\frac{1}{2}$ Live Floor Panel Load

FIG. 72.—Moments in interior columns.

The values of the moments in the columns as determined by the application of the equations of Fig. 72 depend upon the relative stiffnesses of the columns themselves and the relation of the stiffnesses of girders to those of the columns. The sections of the columns seldom change rapidly and are often constant for several tiers.

[1] An approximate solution, which involves very little error may be obtained by omitting all of the coefficients of K in the equations of Fig. 72.

Figure 73 has been plotted for various ratios of the average stiffness of the columns to the stiffness of the girder. It is plotted for the type of loading shown in Fig. 70c, which produces the largest probable moment in an interior column.

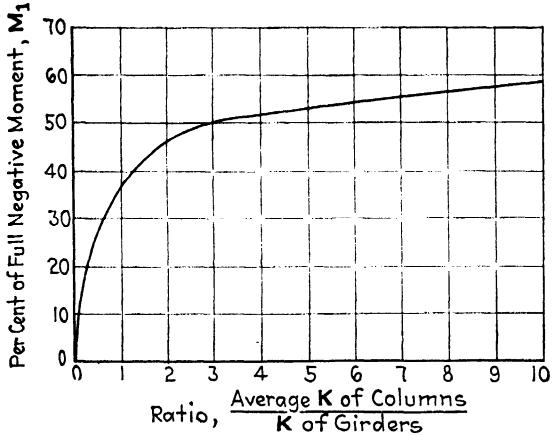


FIG. 73.

135. Exterior Columns. Both dead and live floor and roof loads are applied eccentrically to the exterior columns. A portion of the unbalanced moment thus produced may often be balanced by carrying the spandrel beams on the outside portion of the exterior columns. It is, however, very often impossible fully to compensate for the unbalanced moment due to the dead load alone by this type of construction. In any case, it is necessary to determine the amount of the moment in the column caused by the eccentricity of the roof and floor loads.

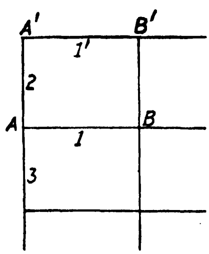


FIG. 74.

For the dead load, it is reasonable to assume that the farther ends of the columns are fixed, and that the inner end of the girder is in the same condition of restraint as the outer end. With these assumptions, the moment in the top of the lower column is (see Fig. 74)

$$M_{AB}\left(\frac{4K_3}{2K_1 + 4K_2 + 4K_3}\right) \tag{73}$$

At the roof level, the moment in the top of the upper column is

$$M_{A'B'}\left(\frac{4K_2}{2K_{1'} + 4K_2}\right) \tag{74}$$

If the full live load is assumed over all portions of the structure, the live-load moment in the column is determined in the same manner as the dead-load moment, that is, from equation (73). A greater moment in the column is, however, developed when the loading is similar to that of Fig. 70*c*. With this type of loading, a point of inflection occurs at the center of the columns, and the moment at the top of the column is

$$M_{AB}\left(\frac{6K_3}{2K_1 + 4K_2 + 6K_3}\right) \tag{75}$$

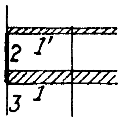
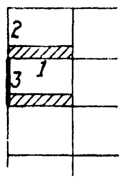
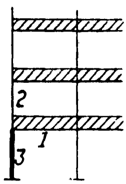
The stress caused by this moment, together with the corresponding direct load stress, is often the maximum stress occurring in a column near the roof. In a column of the lower stories, the decrease in the direct load with this type of loading so reduces the direct load stress that the maximum stress in the column usually occurs under full live load. In such cases equation (73) is applicable for both dead and live loads.

The critical cases are summarized in Fig. 75. The value of M_1 may be taken from the table on page 202 for the type of load sustained by the girder, the dead load being used in the determination of the moment in the column due to the dead load on the girder, and the live load for the corresponding live-load determination.[1] Figure 76 is a graphical representation of the two equations given for an intermediate exterior column in Fig. 75. The lower curve should in all cases be used to determine the dead-load moment, while either may be used for the determination of the live-load moment depending upon the type of loading assumed.

136. Moment of Inertia of Sections. The moment of inertia of a section subject to compressive stresses only has been used in Chaps. IV and V. It is obtained by making use of the so-called

[1] Coefficients of K in the equations of Fig. 75 may be omitted for an approximate solution as in the case of interior columns.

transformed section as explained in Chap. III, and the whole area of the section is included. In beams and slabs, however, where both tensile and compressive stresses occur on the section, and where in regions of high tensile stress cracks often appear in the

Type of Loading		Moment and Direct Load
Roof Column		Moment at Top of Column Live and Dead Loads $M_2 = M_{1'}\left(\frac{2K_2}{K_{1'} + 2K_2}\right)$
Column Below Roof Column		Moment at Top of Column Dead Load $M_3 = M_1\left(\frac{2K_3}{K_1 + 2K_2 + 2K_3}\right)$ Live Load $M_3 = M_1\left(\frac{3K_3}{K_1 + 2K_2 + 3K_3}\right)$ Direct Live Load = Full Live Load − ½ Live Roof Panel Load.[4]
Basement Column		Moment at Top of Column Live and Dead Loads $M_3 = M_1\left(\frac{2K_3}{K_1 + 2K_2 + 2K_3}\right)$

4. In some instances this type of loading may produce the maximum stress in a column further from the roof. In such a case the decrease in the direct load will of course be greater.

FIG. 75.—Moments in exterior columns.

concrete, it becomes a question whether the tension zone of the concrete should be included in the calculations.

In the usual beam of a building frame, the effective section at the support is that of a rectangular beam reinforced for compression, while the section at mid-span is a T-beam. It is certainly true that the flange of a T-beam, although not effective at the support, does increase the stiffness of the member, and

this should be given some consideration in the calculations. The stiffness of the member is the result desired, and in a beam of this type there is no actual section whose moment of inertia definitely determines the stiffness of the member for all conditions of stress and loading. It is recommended that the full concrete section (steel not included) at mid-span be used in the calculations for the moment of inertia, the calculations being made about the gravity axis of this section. The value so obtained is usually somewhat greater than that computed for the doubly reinforced

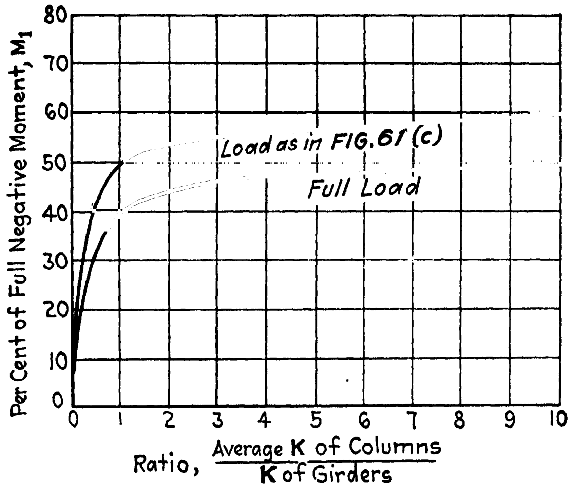


FIG. 76.

rectangular section at the support, and less than that for the section at the center with the steel included. For the columns, since all, or nearly all, of the steel and concrete sections are in compression, the moment of inertia of the entire steel and concrete sections should be used.

137. *Illustrative Problem.* Figure 77 shows a section of the concrete frame of a two-story (and basement) building, for which it is required to design the columns. The distance center to center of columns in the direction perpendicular to the section is 23 ft.-0 in. The roof has been designed for a live load of 40 lb. per sq. ft. and an additional dead load of 35 lb. per sq. ft. to allow for cinder concrete surfacing to provide for drainage.

The live load on the floors is 200 lb. per sq. ft. and allowance has been made for 1 in. of surface finish. The floors and roof have been designed for a 2000-lb. concrete. The columns are to be designed for a 2500-lb. concrete. f_s = 20,000. Interior columns

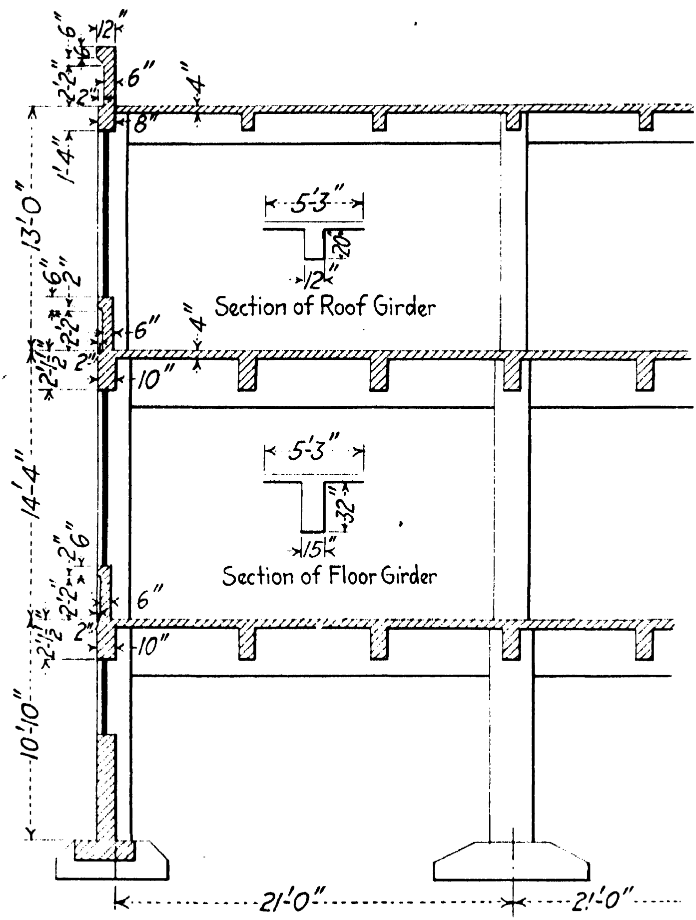


Fig. 77.

are to be round, with hot rolled spirals, and exterior columns are to be rectangular, with lateral ties.

The bending moments in the roof and floor girders are computed as follows:

Roof girder.

Dead load:

Slab = 85 lb. per sq. ft. = 595 lb. per ft. of beam.

Beam = 8 × 12 × 150 ÷ 144 = 100 lb. per ft. of beam.

Total (slab and beam) = 695 lb. per ft. of beam.

Concentrated load to girder = 695 × 23 = 15,985 lb.

Fixed end moment = $\frac{2}{9}$ × 15,985 × 21 = 74,600 ft.-lb.

Design moments (based on recommendations of Art. 125):

Center of span and first interior support = $\frac{12}{10}$ × 74,600 = 89,500 ft.-lb. ($\frac{8}{10}$ × $\frac{1}{3}$ × 15,985 × 21 = 89,500)

Exterior support = $\frac{12}{16}$ × 74,600 = 56,000 ft.-lb.

Girder = 20 × 12 × 150 ÷ 144 = 250 lb. per ft.

Fixed end moment = $\frac{1}{12}$ × 250 × 21^2 = 9200 ft.-lb.

Center of span and first interior support = $\frac{12}{10}$ × 9200 = 11,000 ft.-lb.

Exterior support = $\frac{12}{16}$ × 9200 = 6900 ft.-lb.

Live load:

40 lb. per sq. ft. = 280 lb. per ft. of beam.

Concentrated load to girder = 280 × 23 = 6440 lb.

Fixed end moment = $\frac{2}{9}$ × 6440 × 21 = 30,100 ft.-lb.

Center of span and first interior support = $\frac{12}{10}$ × 30,100 = 36,100 ft.-lb.

Exterior support = $\frac{12}{16}$ × 30,100 = 22,600 ft.-lb.

Summary of Moments

	Fixed end, in.-lb.	Center and 1st sup., in.-lb.	Ext. sup., in.-lb.
Dead load	1,005,000	1,306,000	755,000
Live load	361,000	433,000	271,000
Total	1,366,000	1,739,000	1,026,000

Floor girder.

Dead load:

Slab = 62 lb. per sq. ft. = 434 lb. per ft. of beam.

Beam = 10 × 21.5 × 150 ÷ 144 = 224 lb. per ft. of beam.

Total (slab and beam) = 658 lb. per ft. of beam.
Concentrated load to girder = 658 × 23 = 15,134 lb.
Fixed end moment = $\frac{2}{9}$ × 15,134 × 21 = 70,600 ft.-lb.
Center of span and first interior support = $\frac{12}{10}$ × 70,600 = 84,700 ft.-lb.
Exterior support = $\frac{12}{16}$ × 70,600 = 53,000 ft.-lb.
Girder = 15 × 32 × 150 ÷ 144 = 500 lb. per ft.
Fixed end moment = $\frac{1}{12}$ × 500 × 21^2 = 18,400 ft.-lb.
Center of span and first interior support = $\frac{12}{10}$ × 18,400 = 22,000 ft.-lb.
Exterior support = $\frac{12}{16}$ × 18,400 = 13,800 ft.-lb.

Live load:
200 lb. per sq. ft. = 1400 lb. per ft. of beam.
Concentrated load to girder = 1400 × 23 = 32,200 lb.
Fixed end moment = $\frac{2}{9}$ × 32,200 × 21 = 150,300 ft.-lb.
Center of span and first interior support = $\frac{12}{10}$ × 150,300 = 180,400 ft.-lb.
Exterior support = $\frac{12}{16}$ × 150,300 = 112,700 ft.-lb.

SUMMARY OF MOMENTS

	Fixed end, in.-lb.	Center and 1st sup., in.-lb.	Ext. sup., in.-lb.
Dead load	1,068,000	1,280,000	802,000
Live load	1,804,000	2,165,000	1,352,000
Total	2,872,000	3,445,000	2,154,000

The gravity axis of the roof girder section is [(63 × 4)22 + (12 × 20)10] ÷ [(63 × 4) + (12 × 20)] = 16 in. from the lower surface of the beam. The moment of inertia about this axis is

$$I = \frac{1}{12} \times 63 \times 4^3 + (63 \times 4)6^2 + \frac{(4^3 + 16^3)12}{3} = 26{,}000 \text{ in.}^4$$

and

$$K = 26{,}000 \div 252 = 103$$

Similarly for the floor girder the gravity axis is $[(63 \times 4)34 + (15 \times 32)16] \div [(63 \times 4) + (15 \times 32)] = 22$ in. from the lower surface,

$$I = \frac{1}{12} \times 63 \times 4^3 + (63 \times 4)12^2 + \frac{(10^3 + 22^3)15}{3} = 94{,}900 \text{ in.}^4$$

and

$$K = 94{,}900 \div 252 = 377$$

Interior Columns. The computations necessary for the investigation of the bending stresses in the interior columns are given on page 222. The dead load supported by the upper interior column consists of the weight of the cinder concrete, the roof slab, the stem of three roof beams, and the stem of one girder. The dead load brought to the columns at each floor is obtained in a similar manner. The weights of the columns are obtained from Table 10, the height being taken as the distance from the top of floor or footing to the bottom of the beam in the roof or floor above. The cross-section of the columns required to sustain the direct load may be selected with the aid of Table 8.

The roof column is the smallest column for which a spiral is obtainable; and if a 12-in. column had been used, the allowable stress when bending was considered would have been exceeded. The size of the second floor column necessary to carry the direct load proved satisfactory in the bending investigation, but the steel in the basement column had to be increased from ten 1-in. square to twelve 1-in. square bars to satisfy the latter requirements.

Exterior Columns. The computations necessary for the design of the exterior columns are given on page 225.

The dead load supported by the roof column consists of one-half of the dead load supported by the interior column plus one-half the weight of the stem of one roof beam and the weight of the parapet wall. The dead load brought to the columns at each floor is obtained in a similar manner.

The width of the column parallel to the wall is made the same for all columns. Their lengths are the same as those of the

INTERIOR COLUMNS

1	2	3	4	5	6	7	8	9	10	11
Column supporting	Loads	Diameter, in.	Reinforcement	p_g	I in.^{4}	K	Moment in column, in.-lb.	$f_c = \frac{N}{A} + \frac{Mr}{I}$	$\frac{e}{d}$	Allowable f_c (Diagram 7)
Roof	Dead 53,200 Live 19,300 72,500 Col. 1,900 74,400	14	6-$\frac{5}{8}$ ϕ $\frac{3}{8}$ ϕ spiral $1\frac{3}{4}$-in. pitch	$\frac{1.84}{154} = 0.0120$	1886 206 2090	$\frac{2090}{140} = 15$	Top: $361{,}000\left(\frac{15}{103 + 15}\right) = 46{,}000$ Bottom: $1{,}804{,}000\left(\frac{45}{1131 + 45 + 58}\right) = 66{,}000$	$\frac{74{,}400}{154 + 11 \times 1.84} + \frac{66{,}000 \times 7}{2090} = 659$	0.063	805
Second floor	74,400 Dead 55,900 Live 96,600 226,900 Col. 2,600 229,500	16	8-1 ϕ $\frac{3}{8}$ ϕ spiral 2-in. pitch	$\frac{628}{201} = 0.0312$	3217 1045 4260	$\frac{4260}{146} = 29$	Top: $1{,}804{,}000\left(\frac{87}{1131 + 30 + 87}\right) = 126{,}000$	$\frac{226{,}900 - 48{,}300}{201 + 11 \times 6.28} + \frac{126{,}000 \times 8}{4260} = 898$	0.044	925
First floor	229,500 Dead 55,900 Live 96,600 382,000 Col. 2,900 384,900	20	12-1 sq. $\frac{3}{8}$ ϕ spiral $2\frac{1}{4}$-in. pitch	$\frac{12.00}{314} = 0.0382$	7,854 3,499 11,350	$\frac{11{,}350}{106} = 107$	Top: $1{,}804{,}000\left(\frac{214}{1131 + 58 + 214}\right) = 275{,}000$	$\frac{382{,}000 - 48{,}300}{314 + 11 \times 12.00} + \frac{275{,}000 \times 10}{11{,}350} = 990$	0.044	975

corresponding interior columns. The moments of inertia in column (5) are computed with the aid of Tables 10 and 12.

The outside faces of the columns and beams are in the same vertical plane. This causes a compensating moment to be developed in each column. In the roof column this is due to the eccentricity of the beam only. The center line of the beam is $\frac{18 - 8}{2} = 5$ in. from the axis of the column. The load brought to the column by the two wall beams framing into the column is 19,200 lb., and the moment caused by the eccentricity is 96,000 in.-lb. This moment acts in the opposite direction from the fixed end moment in the girder, and the difference between

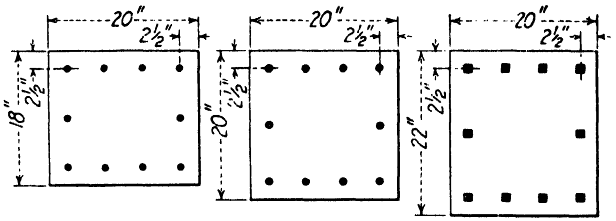


FIG. 78.

the two moments is distributed to the columns above and below. In the lower columns there is an additional compensating moment due to the fact that the columns are eccentric with respect to each other.

At the top of the column supporting the second floor, the eccentric beam brings in a dead load of 15,134 + 8600 = 23,700 lb., and the moment is $23,700\left(\frac{20 - 10}{2}\right) = 118,000$ in.-lb. The dead-load moment due to the eccentricity of the roof column is $39,100 \times 1 = 39,000$ in.-lb. and the total dead-load compensating moment 157,000 in.-lb. Similarly, for the live load, from the beam $16,100\left(\frac{20 - 10}{2}\right) = 81,000$ in.-lb. and from the column $9700 \times 1 = 10,000$ in.-lb. or a total compensating live load moment of 91,000 in.-lb. The similar moments at the top of the basement column are:

$$\text{Beam } [24{,}300 \text{ (dead)} + 16{,}100 \text{ (live)}]\left(\frac{22-10}{2}\right) = 243{,}000 \text{ in.-lb.}$$

$$\text{Column } [83{,}000 \text{ (dead)} + 58{,}000 \text{ (live)}] \times 1 = \underline{141{,}000} \text{ in.-lb.}$$

$$\text{Total} = 384{,}000 \text{ in.-lb.}$$

For large eccentricities, the values of K from Diagrams 16 to 20 vary considerably for the different values of $\frac{d'}{a}\cdot$ Therefore interpolation between Diagrams 18 and 19 is necessary for an exact solution in the case of the roof column.

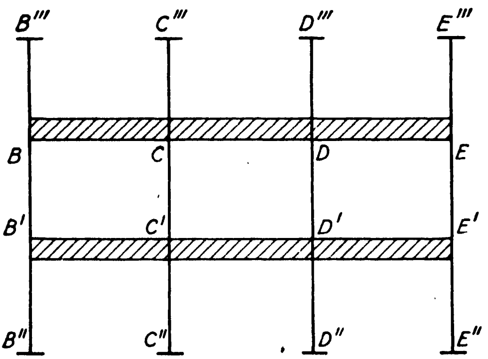


FIG. 79.

Furthermore, the diagrams do not extend far enough for this exceptional case. The upper portions of the np_g curves are practically straight lines. In Diagram 18,

$\frac{e}{a}$	K	Actual $\frac{e}{a} = 0.99$	
0.74	5.50	0.99	$\frac{0.25}{0.21} \times 1.50 = 2.06$
0.53	4.00	0.74	
Diff. 0.21	1.50	0.25	K for $0.99 = 5.50 + 1.79 = 7.29$

In Diagram 19,

0.65	5.50	0.99	$\frac{0.34}{0.29} \times 2.50 = 2.93$
0.36	3.00	0.65	
Diff. 0.29	2.50	0.34	K for $0.99 = 5.50 + 2.93 = 8.43$

Interpolating between diagrams, K for $\frac{d'}{a} = 0.18 = 7.97$.

The cross-sections of the columns are shown in Fig. 78.

EXTERIOR COLUMNS

1	2	3	4	5	6	7	8	9	10	11	12	13
Column supporting	Loads	Cross-section, in.	Reinforcement	$I = I_c + (n-1)I_s$ in.^{4}	K	Moment in column, in.-lb.	$\frac{e}{a}$	$\frac{d'}{a}$	np_g	K	f_c	Allowable f_c (Diagram 12)
Roof	Roof: Dead 27,800 Live 9,700 Wall: 6,900 44,400 Col. 4,400 48,800	18 × 20	10-3/4 φ $p_g = 0.0167$	$20 \times 446 + 8 \times 25 \times 11 = 11{,}920$	$\frac{11{,}920}{140} = 85$	From girder 1,366,000 From eccentricity, $19{,}200\left(\frac{18-8}{2}\right) = \frac{96{,}000}{1{,}270{,}000}$ $1{,}270{,}000\left(\frac{170}{103+170}\right) = 791{,}000$	$\frac{791{,}000}{44{,}400 \times 18} = 0.990$	0.18	0.20	7.97	983	990
Second floor	48,800 Floor: Dead 30,200 Live 48,300 Wall: 8,600 135,900 Col. 5,100 141,000	20 × 20	10-7/8 φ $p_g = 0.0150$	$20 \times 667 + 8 \times 34 \times 11 = 16{,}330$	$\frac{16{,}330}{146} = 112$	Dead: From girder 1,068,000 From eccentricity 157,000 911,000 $911{,}000\left(\frac{224}{377+170+224}\right) = 265{,}000$ Live: From girder 1,804,000 From eccentricity 91,000 1,713,000 $1{,}713{,}000\left(\frac{336}{377+170+336}\right) = 658{,}000$ Total = 917,000	$\frac{917{,}000}{126{,}200 \times 20} = 0.363$	0.16	0.18	2.82	890	890
First floor	141,000 Floor: Dead 30,200 Live 48,300 Wall: 9,200 228,700	22 × 20	10-1 1/8 sq. $p_g = 0.0227$	$20 \times 887 + 8 \times 92 \times 11 = 25{,}836$	$\frac{25{,}836}{106} = 244$	From girder, 2,872,000 From eccentricity, 384,000 2,488,000 $2{,}488{,}000\left(\frac{488}{377+224+488}\right) = 1{,}114{,}000$	$\frac{1{,}114{,}000}{228{,}700 \times 22} = 0.221$	0.15	0.35	1.62	842	880

A closer approximation to the actual moments in the beams than the arbitrary assumption of the coefficients of Art. 125 may be obtained as follows: In Fig. 79 assume the roof and footing terminals of the columns as fixed. While this condition is not absolute, any other assumption makes but a slight difference in the moments. Then,

$$
\begin{aligned}
M_{BC} &= 2EK_1(2\theta_B + \theta_C) - 239{,}300 \qquad (2{,}872{,}000 \text{ in.-lb.})\\
&= 1508E\theta_B + 754E\theta_C - 239{,}300\\
M_{BB'''} &= 4EK_2\theta_B = 324E\theta_B\\
M_{BB'} &= 4EK_3\theta_B = 448E\theta_B\\
&\overline{2280E\theta_B + 754E\theta_C - 239{,}300 = 0}
\end{aligned}
$$

and, since $\theta_C = -\theta_D$,

$$
\begin{aligned}
M_{CD} &= 2EK_1\theta_C - 239{,}300 = 754E\theta_C - 239{,}300\\
M_{CB} &= 2EK_1(2\theta_C + \theta_B) + 239{,}300\\
&= 1508E\theta_C + 754E\theta_B + 239{,}300\\
M_{CC'''} &= 4EK_2\theta_C = 60E\theta_C\\
M_{CC'} &= 4EK_3\theta_C = 116E\theta_C\\
&\overline{2438E\theta_C + 754E\theta_B = 0}
\end{aligned}
$$

From which

$$E\theta_B = +117.0 \qquad \text{and} \qquad E\theta_C = -36.2$$

and

$$
\begin{aligned}
M_{BC} &= 1508 \times 117.0 - 754 \times 36.2 - 239{,}300 = 90{,}100 \text{ ft.-lb.}\\
&= 1{,}081{,}000 \text{ in.-lb.}\\
M_{CD} &= -754 \times 36.2 - 239{,}300 = -266{,}600 \text{ ft.-lb.} = 3{,}199{,}000 \text{ in.-lb.}
\end{aligned}
$$

These moments are less than the design moments computed on page 220. A similar computation for the first floor shows a greater value for $B'C'$ than for BC and a smaller value for $C'D'$ than for CD. In some cases the differences are greater, which shows the desirability of making an analysis where all members are considered as a part of the rigid frame.

CHAPTER VII

FOUNDATIONS

138. Definitions and Essential Requirements. The foundation of a structure may be defined as that part of it which is usually placed below the surface of the ground, and which distributes the load upon the earth beneath it. Two essential requirements in the design of foundations are that the settlement of the structure shall be as small as possible, and that settlement, if any, shall be uniform throughout the structure. The first requirement may be provided for by distributing the load over an area large enough so that the safe bearing power of the soil will not be exceeded. Uniform settlement may be secured by designing the foundations so that the soil pressure over the entire base of the structure is uniform. Failure to provide for the equalizing of the unit foundation pressures is the principal cause of the cracks which disfigure so many buildings. A large number of buildings have been designed with massive bearing piers carrying nearly the whole weight from the floors. Between these piers, smaller piers or columns carrying very little load have been placed. Often the area of the base of all foundations has been the same. The result has been a settlement of the heavier piers, a shearing of window caps and lintels, and many unsightly cracks. Since it is essentially the dead load that causes the greatest amount of settlement, footings should be proportioned for equal unit pressures under dead load, or in some cases, dead load plus partial live load. The method of proportioning footing areas is given in Art. 169.

In a reinforced concrete building the required bearing area is furnished by widening the base of each column or wall. The widened portion is called the footing. Building footings may be divided into three main classes: (1) wall footings, (2) single

column footings, (3) multiple column footings. Since the stresses in footings are mainly compressive, concrete, either plain or reinforced, is particularly adapted to such use.

139. Bearing Capacity of Soils. The sustaining power of earth formations depends mainly upon the composition, the amount of moisture contained, and the degree of confinement in the mass. Sand, if securely confined, or artificially protected against the possibility of lateral displacement, can sustain heavy loads with negligible compression. The supporting power of clays is extremely variable. Certain deposits are known to be compact and hard, and have a high supporting power, while others are plastic and easily compressed. The chief characteristics which render clay more or less unstable as a foundation material are its property of retaining water which is once admitted, and its tendency to soften gradually as the amount of water increases. The depth of the foundation is an important factor in determining the allowable pressure on a clay bed: the greater the depth the less likelihood of lateral displacement of the clay, and a more nearly constant moisture content. When clay is mixed with other materials, such as coarse sand or gravel, its supporting power is materially increased, being greater in proportion as the other materials are in excess, up to the point of forming a cemented mass in which the clay is just sufficient in quantity to act as a cement in binding the other materials together. The allowable pressure on solid rock is usually governed by the strength of the masonry rather than by that of the rock itself.

No definite values can be given to the safe bearing capacity of different classes of soils because of the many variables which of necessity are considered. Unless the bearing capacity of the material at a given site is already known, it should be determined by direct tests, if this is at all feasible.[1] In the absence of any

[1] The factor of safety to be allowed in determining the safe bearing power of the soil will vary from about 1 to 5, depending upon the superstructure and the character of the load coming to the foundation. The blue clay that underlies the State Capitol Building at Albany, N. Y., was found by careful and elaborate tests to sustain a load of 6 tons per sq. ft. It was decided to

satisfactory test, the following limiting values taken from the Building Code of the National Board of Fire Underwriters may be used as a guide in selecting the bearing capacity of any given foundation bed.

RECOMMENDED BEARING CAPACITIES, TONS PER SQUARE FOOT

Soft clay	1	Coarse sand	4
Firm clay	2	Gravel	6
Wet sand	2	Soft rock	8
Sand and clay, mixed or in layers	2	Hardpan	10
Fine dry sand	3	Medium rock	15
		Hard rock	40

140. Factors Affecting the Design of Concrete Footings. In ordinary constructions the load on a wall or column is transmitted vertically through the wall or column to the footing, which in turn is supported by the upward pressure of the soil on which it rests. Uniformity of this upward pressure is assumed in the design of all symmetrically loaded footings, although the probability of obtaining uniformity is dependent to a large extent upon the material in the foundation bed. Recent investigations have shown that the total soil resistance is made up of two parts, the direct resistance to compression of the soil under the footing, and the shearing resistance of the soil around the perimeter of the footing—the so-called perimeter effect. The perimeter effect is almost negligible in most sand beds, owing to the lack of cohesion among the sand particles, so that uniform upward pressure is apt to be approached, if not actually realized, under symmetrically loaded footings resting on sand formations. In clays, however, the cohesive resistance around the perimeter of the loaded area contributes a variable but consequential part of the total resistance, and, as a result, the upward pressure near the perimeter is greater than that near the middle of the footing.

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adopt 2 tons as the safe load to be used in the design of the foundations. The foundations of the Congressional Library at Washington, D. C., were designed to limit the actual soil pressure to 2½ tons per sq. ft., although the yellow clay supporting them was found capable of carrying a total load of 13½ tons per sq. ft.

Single-column footings should be centered under the columns which they support, in order to avoid unsymmetrical resisting pressures, which would result in uneven settlement of the footing. Footings supporting more than one column should be so proportioned that any settlement which might take place will be uniform, in order to prevent the obvious damage to the superstructure which would otherwise occur.

Assuming uniform pressure distribution as an actuality, the manner in which this pressure is resisted is a more or less uncertain factor, especially in column footings, where bowl-shaped deformation occurs. In order to obtain information which would permit a rational treatment of the problem, a series of tests was made at the Engineering Experiment Station of the University of Illinois, the results of which were published in *Bulletin* 67, by Arthur N. Talbot. The conclusions and recommendations contained in this bulletin form the basis of the following discussion of footing design.

141. Plain Concrete Footings. The area of the base of a plain concrete footing is obtained by dividing the total load on the footing, including its own weight, by the allowable unit soil pressure. The top area must be large enough to provide for proper distribution of the load from the wall or column (see Art. 153). Where there is a great difference between the area of the top and base, the upper surface of the footing is usually sloped or stepped.

The depth of the footing must be sufficient to keep the tension in the concrete within the allowable limit, which is given in the Joint Code as $0.03f'_c$. The depth must also be sufficient so that the unit shearing stress, computed at sections as prescribed in Art. 149 for reinforced concrete footings, shall not exceed $0.02f'_c$. The bending moment is computed in the manner outlined in Art. 144 for wall footings and in Art. 147 for column footings. The critical shear is computed as in Art. 144 for wall footings and in Art. 149 for column footings.

142. Pedestals. Quite frequently a footing is required to be placed at some distance below the column that it supports. This is particularly true of footings for structural-steel columns, where the footing has to be located at some distance below the basement floor in order to rest on a suitable bearing stratum.

In such cases, a pedestal is placed on the footing and the column rests on top of the pedestal. The allowable compressive unit stress on the gross area of a plain concrete pedestal, as specified in the Joint Code, is $0.25f'_c$. Where this stress is exceeded, reinforcement must be provided, and the pedestal designed as a reinforced concrete column.

Short pedestals are also frequently required on top of these footings supporting reinforced concrete columns, in order to provide sufficient length of embedment for the dowels that are used to complete the transfer of stress from the columns to the footing (see Art. 153). Short pedestals of this nature are required to be poured monolithically with the footings of which they are a part.

143. Reinforced Concrete Footings. In the majority of cases reinforced concrete is preferable to plain concrete for footing construction due to the saving in excavation, in material, and in weight of the foundation itself. This is the result of the smaller depth required to provide for the existing bending and shearing stresses. The following articles will be devoted to analysis of the various types of reinforced concrete footings.

144. Analysis of Wall Footings.[1] The principles of beam action are, in general, applicable to wall footings. Figure 80 shows a wall footing and a typical set of external forces acting upon it. Although it is evident that the maximum bending moment occurs at a section which passes through the middle of the wall, an analysis of the resisting moments justifies the assumption that the critical section for moment will occur at the face of the wall.[2] The results of the tests bear out this assumption, and

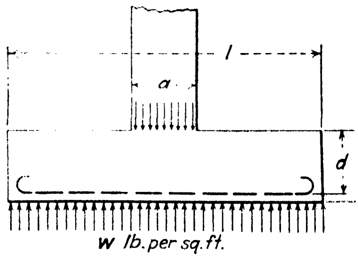


FIG. 80.

[1] The analysis of wall footings is substantially from *Bull.* 67, Engineering Experiment Station, University of Illinois.

[2] This applies to footings under concrete walls only. For footings supporting masonry walls the Joint Code specifies that the critical section for moment shall be taken halfway between the middle and edge of the wall.

measurements of deformation in the reinforcement indicate that the calculated tensile stress in the bars at this section is probably somewhat higher than the maximum tensile stress developed. The maximum bending moment is given by the equation

$$M = \tfrac{1}{8}w(l - a)^2$$

The calculation for bond stress in the tested footings, based on the total external vertical shear at the section at the face of the wall, and calculated as for ordinary beams, evidently gives stresses higher than actually occur. As bond resistance, however, is so important an element of strength in a short cantilever beam, this method of calculation and the use of the working value of bond stress ordinarily assumed in design seem only reasonably conservative, and may be recommended for general practice. Anchorage of bars by bending upward and back in a long curve, or by looping in a horizontal plane, was found to add materially to bond resistance.

The Joint Code requires that standard hooks (see Art. 82) be formed on each end of each bar in all footing slabs. The allowable unit bond stress is the ordinary stress for beams (Art. 82), increased by 50 per cent because of the extra anchorage. The resulting stresses are $0.075f'_c$ for deformed bars and $0.06f'_c$ for plain bars.

The tests indicate that the vertical shearing stresses developed at the face of the wall, calculated by the usual method, are higher than the vertical shearing stresses which are found to exist in simple beams with concentrated loading when diagonal tension failures are developed. It was found that these failures start at a point some distance away from the section at the face of the wall. This observation, and certain analytical considerations such as the probable greater proportion of shear taken in the compressive area at sections near the face of the wall, show that in calculating the vertical shearing stress which shall be used as a basis for judging the resistance to diagonal tension, a section some distance from the face of the wall should be used. The tests and the discussion indicate that a section d distant from the face of the wall (d being the distance from the center of the reinforcing

bars to the top of the footing at the edge of the wall) may properly be used as the critical section for calculating the vertical shearing stress for this purpose, and that at this section the working stresses generally specified for ordinary beams may be used for calculating resistance to diagonal tension failure. Web reinforcement, while adding to diagonal tension resistance, is not especially effective, and since it is very inconvenient to place, it is usually better to design the footing so that the vertical shearing stress is within the limit of the working stress permitted in beams without web reinforcement.

The allowable shearing unit stress recommended in the Joint Code, which includes an allowance for the added resistance furnished by the hooks on the bars, is $0.03f'_c$, with a maximum of 75 p.s.i.

145. Design of a Typical Wall Footing. A 16-in. wall supports a total load of 23,100 lb. per lin. ft. and rests on soil whose safe bearing power is 2 tons per sq. ft. Design a footing for this wall that will satisfy the requirements of the Joint Code. A 2000-lb. concrete is to be used; $f_s = 18{,}000$ p.s.i.

Assuming the weight of the footing as 900 lb. per lin. ft., the total width of footing required $= \dfrac{24{,}000}{4{,}000} = 6.0$ ft. The net upward pressure on the footing $= \dfrac{23{,}100}{6} = 3850$ lb. per sq. ft. The moment at the critical section is

$$M = 3850 \times \frac{(6 - 1.33)^2}{8} \times 12 = 126{,}000 \text{ in.-lb.}$$

With allowable unit stresses of 18,000 and 800, Table 6 gives $K = 139$ and $j = 0.867$

$$d = \sqrt{\frac{126{,}000}{139 \times 12}} = 8.7 \text{ in.}$$

An effective depth of 9 in. is used, which, with 3-in. insulation, gives a total thickness of 12 in. The weight per linear foot is then 900 lb. as assumed.

$$A_s = \frac{126,000}{18,000 \times 0.867 \times 9} = 0.90 \text{ sq. in. per ft.}$$

Assuming deformed bars anchored at both ends by means of hooks,

$$\Sigma_0 = \frac{2.33 \times 3850}{150 \times \frac{7}{8} \times 9} = 7.6 \text{ in. per ft.}$$

These requirements are satisfied by using ½-in. square bars, 3 in. center to center. Investigating for diagonal tension, the total external shear on a section 9 in. from the face of the wall is

$$\frac{23,100}{6} \times \frac{19}{12} = 6100 \text{ lb.}$$

and the unit shear, which is a measure of diagonal tension, is

$$v = \frac{6100}{12 \times \frac{7}{8} \times 9} = 64 \text{ p.s.i.}$$

This is but 4 lb. in excess of the allowable ($0.03 \times 2000 = 60$) and the design may be considered satisfactory.

Single-column Footings

146. Computation of Bearing Area. Ordinarily, in single-column footings the load may be considered as applied uniformly over the bearing area of the column, and if the footing is symmetrically placed under the column the upward pressure is usually considered to be uniformly distributed over the base of the footing. The footing is then analogous to a cantilever slab supported at the top over a central area and loaded with a uniform upward load. As the projecting portion of the footing deflects upward, its surface assumes the shape of a bowl. Steel is required in the bottom of the footing to resist the resulting tension stresses. This steel is placed in two directions, as shown in Fig. 84,[1] the bars in one direction resting directly on top of those in the other direction.

[1] Occasionally footing reinforcement is placed in four directions, as shown in Fig. 85. This is unusual in modern practice, however. The method of design is the same as for two-way footings, except that the steel area required

The required area of the footing is obtained by dividing the total load on the footing (including its own weight) by the allowable unit soil pressure. The weight of a single-column footing will generally vary from 6 to 10 per cent of the column load. For maximum economy, a footing supporting a square or a round column should be square, whereas the sides of a footing supporting a rectangular column should be somewhat longer on the side parallel to the long side of the column than in the other direction, the sides of the footing being approximately in the same ratio as the sides of the column.

147. Bending Moment. In Fig. 81, assume that the footing is cut along the four diagonal lines, AD, BC, etc., the cuts extending from the corners of the column to the nearest corners of the footing. The footing now consists of four cantilevers, $ABCD$, etc., each loaded with a uniform unit upward pressure equal to the load on the base of the column divided by the area of the footing. The maximum moment in each cantilever is equal to the total load on the cantilever (*e.g.*, the net unit soil pressure multiplied by the area $ABCD$) multiplied by the distance from the center of gravity of the trapezoid to the edge of the column. The weight of the footing is not considered in this computation (or in any others except those for determining the required bearing area), because the downward weight of the footing compensates for the additional upward soil pressure caused by this weight.

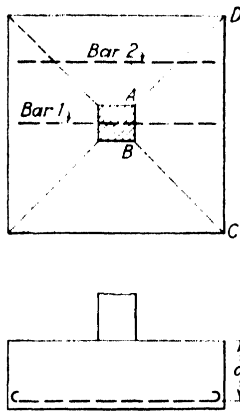


FIG. 81.

If the footing had actually been cut, as assumed above, the moment at AB would have to be resisted by a rectangular section with a width equal to AB and a depth equal to the distance from the top of the footing to the center of the steel near the bottom of

in each direction is divided into one band parallel to the axis of the footing and two half bands in the diagonal direction, with practically all of the bars passing under the column.

the footing, *i.e.*, the as yet unknown distance d in Fig. 81. However, since the cutting is purely imaginary, it is not unreasonable to assume that a greater width than AB is effective in resisting the bending moment on the trapezoid $ABCD$.

The tests indicate also that a bar such as bar 2, Fig. 81, which is placed some distance from the column, receives practically the same stress as bar 1, which is directly under the column. The required reinforcement can therefore be spread out over a fairly large width without losing its effectiveness.

The method of computing bending moment as specified in the Joint Code is slightly more severe than is indicated in the preceding analysis. According to this Code, the moment at any section is determined by passing through the section a vertical plane, which extends completely across the footing, and computing the moment of the forces acting over the entire area of the footing on one side of the plane. The critical section for moment is at the face of the column (or pedestal) for footings supporting a single concrete column (or pedestal), or halfway between the face of the column and the edge of the metallic base for footings under metallic bases. The width resisting compression at any section is taken as the full width of the top of the footing at the section under consideration.

The effective depth required to prevent overstressing the concrete in compression is computed from the rectangular-beam equation [equation (6), Art. 47], $M = Kbd^2$. The allowable extreme fiber stress in compression is the same as for ordinary beams. Because of the large effective width, except for sloped-top footings (see Art. 152), the effective depth obtained from this computation would not be sufficient to provide adequate resistance to diagonal tension, and usually the depth selected is obtained from the latter requirement, as explained in Art. 149.

After the effective depth has been selected, the steel area required in each direction is obtained from equation (5), Art. 47, $M = A_sf_sjd$, an approximate value of $j = 0.9$ being used. Further refinement in computing the value of j is not justified, because of the assumptions made in computing the moment and in selecting the effective width b. The moment to be used in the

computation of A_s in each direction is specified as 85 per cent of the moment obtained in the manner described in the preceding paragraph.

148. Placing the Reinforcement. In square footings the required reinforcement in each direction is distributed uniformly across the full width of the footing. In rectangular footings the reinforcement in the long direction is distributed across the full width of the footing; in the short direction a portion ΔA_s of the total required reinforcement A_s is uniformly distributed in a band having a width equal to the length of the short side of the footing and centered with respect to the center line of the column. The portion ΔA_s is computed from the following equation:

$$\Delta A_s = A_s\left(\frac{2}{S + 1}\right)$$

in which S is the ratio of the long to the short side of the footing. The remainder of the required reinforcement in the short direction is uniformly distributed in the outer portions of the footing.

The Joint Code requires that the reinforcement in footings or other principal structural members in which the concrete is deposited against the ground shall have not less than 3 in. of concrete between it and the ground contact surface. In ordinary computations, the effective depth is taken as 4 in. less than the total thickness of the footing, and this value is used in computations for both directions, even though actually the d in one direction is less than that in the other direction by an amount equal to the diameter of the bars.

149. Diagonal Tension. Tests indicate that diagonal tension develops in critical amounts at a distance from the face of the column equal to the effective depth d of the footing. Hence, the critical section for shear is assumed as a vertical section obtained by passing a series of vertical planes through the footing, each of which is parallel to a corresponding face of the column or pedestal and located a distance therefrom equal to the effective depth of the footing. According to the Joint Code, each face of the critical section for shear shall be considered as resisting an external shear equal to the net upward pressure on an area

bounded by that face, two diagonal lines drawn from the corners of the column or pedestal at angles of 45 degrees with the principal axes of the footing, and that portion of the corresponding edge or edges of the footing intercepted between the two diagonals. Thus, in Fig. 82, the critical section for shear is the vertical section $ABCD$, the shear resisted by the face AB is the net upward load on the shaded area ab, and the shear resisted by the face BC is the net upward load on the shaded area bc.

The unit shear, which is a measure of diagonal tension, is computed from equation (10), Art. 70, $v = \frac{V}{bjd}$, using for b the width of the face of the critical section, and for d the depth at the face of the critical section above the plane of the reinforcement. For single block footings the effective depth d is the same for all sections in the footing, but for footings with a sloping top, the effective depth d obviously varies with the location of the section. The allowable unit shear is specified in the Joint Code as $0.03f'_c$, with a maximum of 75 p.s.i.

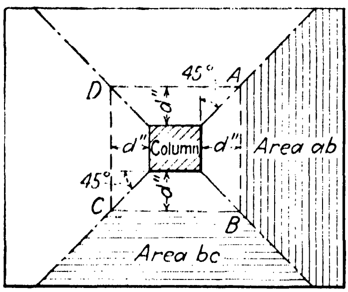


FIG. 82.

The depth of the footing is usually governed by the shearing strength required; hence, in design, the above equation is used to determine the unknown effective depth for the known values of v, b, and V. This is in reality a "cut-and-try" process, since the values of b and V depend upon the values of d, which must therefore be assumed before the computations can be completed.

Web reinforcement is ordinarily undesirable in a single-column footing because of the uncertainty of its effectiveness and the difficulty of placing it. The specified maximum value of v, as given above, is therefore the value specified for ordinary beams without web reinforcement but with the bars hooked, and with an added arbitrary limit of 75 p.s.i.

150. Bond Stresses. Bond resistance is one of the most important factors governing the strength of concrete footings,

and probably much more important than has been appreciated by the average designer. The calculations of bond stress in footings of ordinary dimensions where large reinforcing bars are used (*i.e.*, small ratio of perimeter to area) show that the bond stress may be the governing element of strength. The Joint Code recognizes the importance of bond resistance by specifying that standard hooks shall be formed on each end of each reinforcing bar.

The critical sections for bond are assumed at the face of the column or pedestal and at vertical planes where changes in section occur. The critical unit bond stress for each band of reinforcing bars is computed from equation (16), Art. 81, $u = \frac{V}{\Sigma_0 jd}$, considering the circumference of all of the bars in the band and assuming V as 0.85 of the total net upward load on the rectangle specified in the Joint Code (see Art. 147) for computing the bending moment, *i.e.*, the shaded rectangle at the right side of Fig. 83.

The allowable unit bond stress as specified for ordinary beams with hooked bars is reduced by 25 per cent because of the loss in bond area at the intersections of the bars. The resulting allowable unit stress is $0.045f'_c$ for plain bars and $0.056f'_c$ for deformed bars.

151. Footings Supporting Round Columns. The preceding discussions have been based upon the assumption that the footings are supporting square or rectangular columns or metallic bases. In computing the stresses (bending, shear, and bond) in footings which support a round or octagonal concrete column or pedestal, the "face" of the column or pedestal shall be taken as the side of a square having an area equal to the area enclosed within the perimeter of the column or pedestal. Computations are carried out in the same manner as for footings supporting square or rectangular columns, the equivalent square column being used in place of the actual round column (see Art. 226).

152. Stepped and Sloped Footings. If the depth required for moment is large, the footing may be sloped between the edge of the column (preferably 3 or 4 in. from the edge of the column) and

the edge of the footing, provided the actual depth to the steel at the critical section for diagonal tension is made sufficient to provide fully for this diagonal tension in accordance with the method outlined in Art. 149, and further provided that the top area of the footing is made sufficiently large to satisfy other requirements as given in Art. 153.

The Joint Code specifies that in sloped footings the thickness above the reinforcement at the edge of the footing shall not be less than 6 in. for footings on soil, or less than 12 in. for footings on piles. The top of the footing may be stepped instead of sloped, provided the steps are so placed that the footing will have at all sections a depth at least as great as that required for a sloping top. Stepped footings must be cast monolithically. The value of b (see Art. 147) to be used in analyzing sloped or stepped footings for bending shall not exceed the width of the flat top of the footing, since this is the maximum width over which the full effective depth d, for moment, is available.

The complete design of a sloped footing reinforced in two directions is given in Art. 155.

153. Transfer of Stress at Base of Column. The compressive stress in the longitudinal reinforcement at the base of a reinforced concrete column is transferred to the pedestal or footing by means of dowels. There should be at least one dowel for each column bar, and the total sectional area of the dowels should not be less than the sectional area of the longitudinal reinforcement in the column. If f'_c is 3000 p.s.i. or greater, the dowels should extend into the column and into the pedestal or footing not less than 30 diameters of the dowel bars for plain bars, or 24 diameters for deformed bars. If f'_c is less than 3000 p.s.i., both of these distances must be increased 25 per cent. If the footing thickness is not sufficient to furnish the required embedment, plus insulation, a pedestal must be used; the height of the pedestal must be sufficient to provide for the deficiency in the footing thickness, and the area must be sufficient to satisfy the requirements of the following paragraph.

The Joint Code specifies that the permissible compressive unit stress r_a on top of the pedestal or footing directly under the

column shall not be greater than that determined by the formula

$$r_a = 0.25f'_c\sqrt[3]{\frac{A}{A'}}$$

in which A = total area of top of pedestal or footing.
A' = loaded area of pedestal or footing at the column base.

In sloped or stepped footings, A may be taken as the area of the top horizontal surface of the footing or as the area of the lower base of the largest frustum of a pyramid or cone contained wholly within the footing and having for its upper base the loaded area A', and having side slopes of 1 vertical to 2 horizontal. The Code further specifies that the unit stress on the gross area of a concentrically loaded pedestal shall not be greater than $0.25f'_c$. Pedestals used for the purpose of furnishing depth for embedment of dowels must be poured monolithically with the footing.

154. Design of a Two-way Block Footing. A column 24 in. square supports a total load of 400,000 lb. Design a single-slab concrete footing, reinforced in two directions, to support this column on a soil which has a safe bearing capacity of 5000 lb. per sq. ft. A 2500-lb. concrete and intermediate grade steel are to be used.

Assuming the weight of the footing as 25,000 lb., the bearing area required is $\frac{425,000}{5000} = 85$ sq. ft. A base 9 ft.-3 in. square, furnishing 85.5 sq. ft., is selected. The unit pressure due to the load on the column only is $\frac{400,000}{85.5} = 4680$ lb. per sq. ft.

The depth of the footing is governed by the shearing stresses; but, in order to locate the critical section for shear, the depth must be known or assumed. An approximate value can be obtained by first computing the depth required for the bending moment.

The maximum moment (see Fig. 83) is

$$M = 4680 \times 9.25 \times 3.625 \times \frac{3.625}{2} \times 12 = 3,420,000 \text{ in.-lb}$$

From Table 6, $K = 164$; and from equation (6), Art. 47,

$$d = \sqrt{\frac{3,420,000}{9.25 \times 12 \times 164}} = 13.7 \text{ in.}$$

According to preceding discussions, the effective depth required for shear will be greater than 13.7 in. Assume that 18 in. will be adequate. The critical section for shear (Fig. 83) is then 18 in.

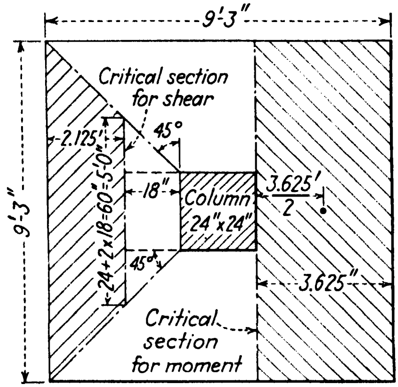


Fig. 83.

from the face of the column, the width of this section is $24 + 2 \times 18 = 60$ in., and the total external shear on the section is

$$V = \frac{5.0 + 9.25}{2} \times 2.125 \times 4680 = 71,000 \text{ lb.}$$

The allowable unit shear is 75 p.s.i. Hence,

$$v = \frac{71,000}{60 \times 0.9 \times d} = 75$$

from which, $d = 17.6$ in., or 18 in., as assumed.

Any required revision in the assumed value of d would change not only the total shear V but also the effective shearing width b in the above equations. The total height of the footing, allowing 4 in. of insulation below the center of the steel, as recommended in Art. 148, is 22 in., and the weight of the footing is 23,600 lb., approximately as assumed. No revision of the bearing area is necessary.

The moment to be used in computing the steel area required in each direction is 0.85 × 3,420,000 = 2,910,000 in.-lb.

$$A_s = \frac{2,910,000}{20,000 \times 0.9 \times 18} = 9.0 \text{ sq. in.}$$

The allowable unit bond stress is 0.056 × 2500 = 140 p.s.i. The maximum shear to be used in the equation for unit bond

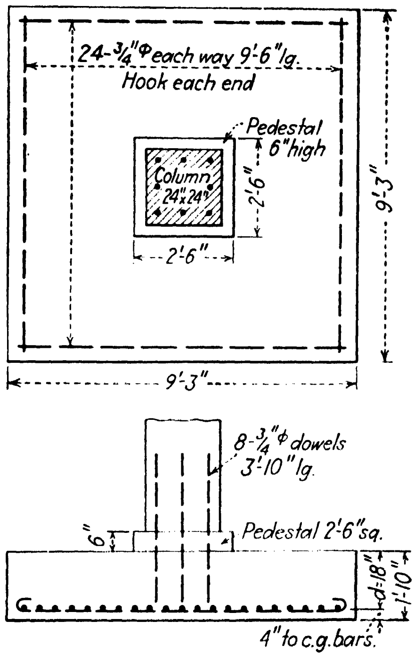


FIG. 84.—Two-way block footing.

stress is 0.85 of the net upward load on the shaded rectangular area in Fig. 83, or 0.85 × 4680 × 9.25 × 3.625 = 133,000 lb.

$$\Sigma_0 \text{ (required)} = \frac{133,000}{140 \times 0.9 \times 18} = 58.5 \text{ in.}$$

Twenty-four ¾-in. round bars furnish an area of 10.6 sq. in. ~~and a total perimeter~~ of 56.6 in. The slight discrepancy in perimeter is not serious. Allowing about 4 in. of insulation to

the edges of the footing, the spacing of the bars is approximately $4\frac{1}{2}$ in., which is satisfactory.

Dowels will be placed in the footing to transfer the stress from the column bars to the footing. According to Art. 153, these dowels should be of the same size and the same number as the bars in the column. They should extend into the footing and into the column a distance of 30 diameters. Assuming that the column is reinforced with eight $\frac{3}{4}$-in. round bars, eight $\frac{3}{4}$-in. round dowels 3 ft.-10 in. long will be required. In order to furnish sufficient depth for embedment in the footing, a pedestal 6 in. high is required. The required area of the pedestal (see Art. 142) is $\frac{400{,}000}{625} = 640$ sq. in. The pedestal will be made 30 in. square (area 900 sq. in.), which will give a 3-in. projection beyond the column. The unit stress on the loaded area is $\frac{400{,}000}{24 \times 24} = 692$ p.s.i., and the allowable stress on this area (see Art. 153) is

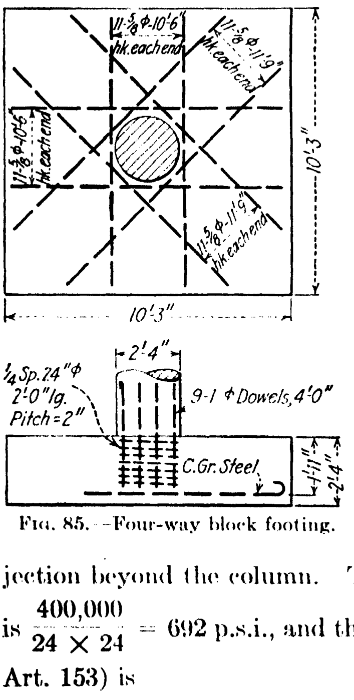


FIG. 85.—Four-way block footing.

$$r_a = 0.25 \times 2500\sqrt[3]{900/640} = 700 \text{ p.s.i.}$$

Complete details of the footing are shown in Fig. 84.

The design of a two-way square block footing supporting a round column is given in Art. 226, and the design of a two-way rectangular block footing supporting a rectangular column is given in Art. 227. A typical footing with reinforcement in four directions is shown in Fig. 85.

155. Design of a Two-way Sloped Footing. A square sloped footing with two-way reinforcement is to support a column which is 26 in. square and which is reinforced with twelve 1-in. round

bars and ¼-in. lateral ties at 12 in. on centers. The total load at the base of the column is 400,000 lb. The footing is to be made of 2500-lb. concrete and the allowable unit stress in the reinforcement is to be 18,000 p.s.i. The allowable soil pressure is 4000 lb. per sq. ft. Design the footing.

Assume weight of footing = 0.08 × 400,000 = 32,000 lb.

$$\text{Bearing area required} = \frac{432,000}{4000} = 108 \text{ sq. ft.}$$

Size of base = 10 ft.-6 in. square (area = 110.25 sq. ft.)

The flat top of the footing will be made 34 in. square, thus allowing for a projection of 4 in. beyond each face of the column. According to Art. 153, the allowable unit pressure on the area directly under the column is

$$r_a = 0.25 \times 2500\sqrt{\frac{34 \times 34}{26 \times 26}} = 748 \text{ p.s.i.}$$

The actual unit pressure is

$$\frac{400,000}{26 \times 26} = 592 \text{ p.s.i.}$$

and the size of the top is satisfactory.

The unit pressure at the base of the footing due to the load on the column only (the net pressure), is

$$\frac{400,000}{110.25} = 3630 \text{ lb. per sq. ft.}$$

The critical moment is

$$M = 3630 \times 10.5 \times 4.17 \times \frac{4.17}{2} \times 12 = 3,980,000 \text{ in.-lb.}$$

In a sloped-top footing the effective width b is taken as the width of the flat top, as recommended in Art. 152, and the depth required for moment is usually the governing factor. With $K = 173$ (Table 6) and $b = 34$ in., the effective depth required is

$$d = \sqrt{\frac{3,980,000}{173 \times 34}} = 25 \text{ in.}$$

The total thickness of the footing is 29 in. At the edge, the thickness above the reinforcement must be at least 6 in. (see Art. 152); the total edge thickness will be made 12 in. The weight of the footing is then 27,000 lb. The revised required bearing area is 106.7 sq. ft., but the assumed side dimensions will still be necessary if the usual practice of detailing footing widths in multiples of not less than 3 in. is followed.

The critical section for shear is 25 in. from the face of the column; the width of the section is $26 + 2 \times 25 = 76$ in. (= 6.33 ft.); the distance from the reinforcement to the top of the footing at the section is 17.2 in.; and the total external shear on the section is $\frac{6.33 + 10.5}{2} \times \frac{25}{12} \times 3630 = 63{,}500$ lb. The actual unit shear is then

$$v = \frac{63{,}500}{76 \times 0.9 \times 17.2} = 54 \text{ p.s.i.}$$

which is less than the allowable value of 75 p.s.i.

The maximum moment to be used in computing the required steel area in each direction is $0.85 \times 3{,}980{,}000 = 3{,}380{,}000$ in.-lb.

$$A_s = \frac{3{,}380{,}000}{18{,}000 \times 0.9 \times 25} = 8.33 \text{ sq. in.}$$

In investigating for bond stress,

$$V = 0.85 \times 3630 \times 10.5 \times 4.17 = 135{,}000 \text{ lb.}$$

$$\Sigma_0 \text{ (required)} = \frac{135{,}000}{140 \times 0.9 \times 25} = 43 \text{ in.}$$

Nineteen ¾-in. round bars furnish an area of 8.40 sq. in. and a total perimeter of 44.8 in. The spacing is approximately 6½ in., which is satisfactory.

Twelve 1-in. round dowels 5 ft.-0 in. long will be placed in the footing. These must extend $30 \times 1 = 30$ in. into the column and 30 in. into the footing. Since the footing is only 29 in. thick, a 6-in. pedestal is required to provide, with the available footing

thickness, for the embedment of the dowels and still furnish adequate insulation at the bottom of the dowels. The size of the pedestal is the same as that of the flat top of the footing. It must be poured monolithically with the footing. If, as is usually the case, the elevation of the bottom of the column is fixed, it will be necessary to place the bottom of the footing lower than it would

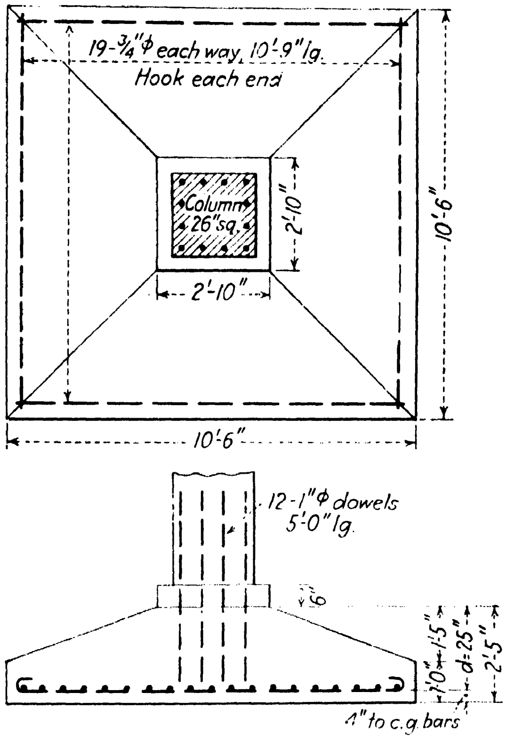


FIG. 86.—Two-way sloped footing.

be if no pedestal were required, by an amount equal to the height of the pedestal. Details of the above footing are shown in Fig. 86.

MULTIPLE-COLUMN FOOTINGS

156. Types of Multiple-column Footings. It is sometimes desirable, and quite frequently necessary to support more than one column on the same footing. A multiple-column footing is necessary when two columns with fairly large loads are so close

together that there is not sufficient space between the columns for two independent footings of economical proportions, or when the face of an exterior column coincides with, or is close to, the building line, thus making it impossible to center an independent footing under this column. In the latter case, an eccentric footing would result in unequal distribution of the pressure on the soil, with the possibility of uneven settlement, and bending in the columns; a single footing supporting the exterior column and the adjacent interior column may be so proportioned as to make the center of gravity of the footing area coincide with the center of gravity of the column loads and thus theoretically secure the uniform soil pressure which is essential to a satisfactory footing design. A footing of this type is called a combined footing.

A combined footing may be rectangular in plan, or trapezoidal. The former shape is suitable if the interior column has the greater load and if the footing may be extended beyond this column as far as necessary. The trapezoidal shape is required if the column loads are unequal and if for any reason the footing cannot extend any appreciable distance beyond *either* column. The critical sections for diagonal tension in combined footings are the same as for single-column footings (see Art. 149), and the transfer of the stress at the base of the columns is accomplished in the same manner. The methods of computing critical moments are explained in Arts. 157 and 158.

Another expedient that may be used, in case an independent footing cannot be centered under an exterior column, is to connect the eccentric exterior-column footing with the nearest interior-column footing by means of a concrete beam or strap, poured monolithically with the two footings, as shown in Fig. 91, the entire construction being so proportioned as to resist the tendency of the exterior column to overturn the footing under that column. A footing of this type is called a cantilever footing. A complete design is given in Art. 159.

157. Design of a Combined Footing Supporting Two Equal Column Loads. Two columns, each 24 in. square and each carrying a total load of 310,000 lb., are spaced 10 ft.-0 in. center to center, as shown in Fig. 87. Design a rectangular combined

footing to support these two columns. The allowable soil pressure is 4000 lb. per sq. ft.; a 2000-lb. concrete is to be used; $f_s = 18{,}000$ p.s.i.

The proposed method of design considers that the load from each column will be carried by a beam in the direction of the width of the footing (*i.e.*, at right angles to the line of columns) and that this load will then be distributed longitudinally throughout the length of the footing by a band of steel in the longitudinal direction. Assuming the weight of the footing as $0.06 \times 2 \times 310{,}000 = 37{,}000$ lb., the bearing area required is

$$\frac{2 \times 310{,}000 + 37{,}000}{4000} = 164.3 \text{ sq. ft.}$$

The proper selection of width and length is governed by several factors. If the footing is too long, the moment in the portions projecting beyond the columns in the longitudinal direction will be excessive and the unit shearing and bond stresses in the transverse beam will be unduly large. If the footing is too short, the negative moment between the columns in the longitudinal direction will be too great for an economical design. A short footing will necessitate an excessive width, which will cause an unduly large moment at the edges of the columns in the transverse direction. Two or three trial designs may be necessary to secure the proportions which will give the most economical footing, considering excavation, concrete, and steel. The size finally selected in the present case is 18 ft.-9 in. by 8 ft.-9 in. (area = 164.1 sq. ft.).

The effective width of the longitudinal band (for both tension and compression) is equal to the full width of the footing. The effective compression width of the transverse beam can be assumed equal to from one and one-half to two times the width of the column, whereas the steel in this direction can be spread out over a width somewhat greater than the column width plus twice the effective depth.

Design of Transverse Direction. In concrete combined-footing designs it is customary to consider, as in single-footing designs, that the critical sections for moment in the cantilever portions

of the footing are at the edges of the columns or pedestals. At the outer edge of each column,

$$M = \frac{310,000}{8.75} \times \frac{(3.375)^2}{2} \times 12 = 2,420,000 \text{ in.-lb.}$$

Assuming $b = 2 \times 24 = 48$ in., the required effective depth is

$$d = \sqrt{\frac{2,420,000}{139 \times 48}} = 19 \text{ in.}$$

Later investigation shows that an effective depth of 21 in. is required, to avoid the necessity of using web reinforcement in the transverse beam. This increased depth will not only avoid the necessity of stirrups but it will also reduce the steel area required.

$$A_s = \frac{2,420,000}{18,000 \times 0.9 \times 21} = 7.12 \text{ sq. in.}$$

At the edge of the column, $V = \dfrac{310,000}{8.75} \times 3.375 = 119,500$ lb.

Since the transverse bars will be in contact with the bottom longitudinal bars, the allowable unit bond stress, assuming proper anchorage at each end of each bar, as recommended in Art. 150, is 112 p.s.i.

$$\Sigma_0 = \frac{119,500}{112 \times 0.9 \times 21} = 56.4 \text{ in.}$$

Twenty-nine ½-in. square bars are selected, and placed in a width of 72 in., thus resulting in a spacing of about 2½ in.

The total shear at a distance of 21 in. from the edge of the column is

$$V = \frac{310,000}{8.75}\left(3.375 - \frac{21}{12}\right) = 57,500 \text{ lb.}$$

$$v = \frac{57,500}{48 \times 0.9 \times 21} = 63 \text{ p.s.i.}$$

If no web reinforcement is used, the allowable unit shear, with anchored bars, is $0.03 \times 2000 = 60$ p.s.i. The above value may be considered satisfactory.

Design of Longitudinal Direction. The critical moment in this case is at the outer edge of the transverse beam, and

$$M = \frac{620{,}000}{18.75} \times \frac{(2.375)^2}{2} \times 12 = 1{,}120{,}000 \text{ in.-lb.}$$

At the center of the footing

$$M = \left(310{,}000 \times \frac{9.375}{2} - 310{,}000 \times 5\right)12 = 1{,}200{,}000 \text{ in.-lb.}$$

The latter value governs, and the effective depth required is

$$d = \sqrt{\frac{1{,}200{,}000}{139 \times 8.75 \times 12}} = 9.1 \text{ in.}$$

While a total thickness of $9\frac{1}{2} + 3 = 12\frac{1}{2}$ in. would be satisfactory for this part of the footing, the required steel area would be reduced by a greater thickness, and the shearing stresses would also be lowered. An effective depth of 12 in. will be selected. The bottom longitudinal bars will be placed underneath the transverse bars. Allowing 3 in. of insulation below the center of the lower bars, the total thickness of the longitudinal slab is 15 in.

At the center of the footing,

$$A_s = \frac{1{,}200{,}000}{18{,}000 \times 0.9 \times 12} = 6.18 \text{ sq. in.}$$

An examination of the bending-moment diagram for the longitudinal direction shows that the maximum bond stress on these bars (the top bars between the columns) occurs at the point of inflection. Assuming this point to be at a distance of $\frac{1}{4} \times 10 = 2\frac{1}{2}$ ft. from the center line of the column, the shear at this point is

$$V = \frac{620{,}000}{18.75}(4.375 + 2.5) - 310{,}000 = -83{,}000 \text{ lb.}$$

The allowable unit bond stress, assuming hooked ends, is 150 p.s.i., since these bars are not in contact with bars at right angles to them.

$$\Sigma_0 = \frac{83{,}000}{150 \times 0.9 \times 12} = 51.1 \text{ in.}$$

Twenty-six ½-in. square bars are selected. These are placed 3 in. from the top of the footing and will extend between the column center lines, with standard hooks at each end.

At the outer edge of the transverse beam,

$$A_s = \frac{1,120,000}{18,000 \times 0.9 \times 12} = 5.78 \text{ sq. in.}$$

$$V = \frac{620,000}{18.75} \times 2.375 = 78,500 \text{ lb.}$$

The allowable unit bond stress here is 112 p.s.i., since there are two intersecting layers of bars.

$$\Sigma_0 = \frac{78,500}{112 \times 0.9 \times 12} = 64.8 \text{ in.}$$

At the outer edge of the column,

$$M = \frac{620,000}{18.75} \times \frac{(3.375)^2}{2} \times 12 = 2,250,000 \text{ in.-lb.}$$

$$d \text{ (required)} = \sqrt{\frac{2,250,000}{139 \times 8.75 \times 12}} = 12.4 \text{ in.}$$

The effective depth furnished at this section, assuming ½-in. bars, is ½ in. greater than that of the transverse beam, or 21½ in.

$$A_s = \frac{2,250,000}{18,000 \times 0.9 \times 21.5} = 6.45 \text{ sq. in.}$$

$$V = \frac{620,000}{18.75} \times 3.375 = 111,500 \text{ lb.}$$

$$\Sigma_0 = \frac{111,500}{112 \times 0.9 \times 21.5} = 51.4 \text{ in.}$$

The above computations show that, for the bottom longitudinal bars, the required area is governed by the section at the outer edge of the column (A_s = 6.45 sq. in.), and the required total perimeter is governed by the section at the outer edge of the transverse beam (Σ_0 = 64.8 in.). Thirty-two ½-in. square bars, hooked at the ends, are required. The approximate spacing is about 3¼ in. These bars should extend from the end of the footing to a section about 20 diameters beyond the point of

inflection. Since this would leave but a short length between the bars near the center of the footing, in order to simplify the placing, without adding materially to the weight of the steel, the bottom longitudinal bars will be detailed to extend the full length of the footing.

For the longitudinal direction, the critical sections for shear are at distances of $21\frac{1}{2}$ in. from the faces of the columns and 12 in. from the edges of the transverse beams. The total shears at the

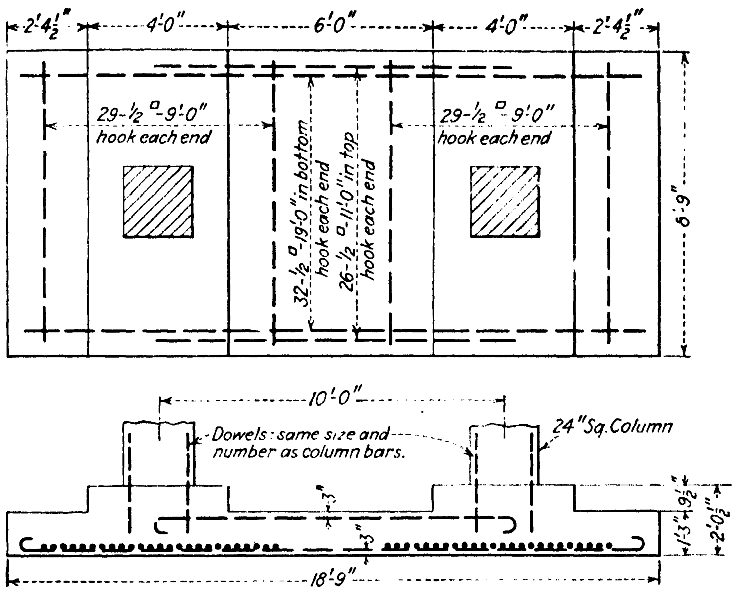


FIG. 87.—Combined footing with two equal column loads.

former sections are greater than those at the latter, and hence the diagonal tension investigation is made at a distance of $21\frac{1}{2}$ in. from the faces of the columns, which sections are $9\frac{1}{2}$ in. from the edges of the transverse beams. Between the columns,

$$V = \frac{620{,}000}{18.75}\left(2.375 + 4.0 + \frac{9.5}{12}\right) - 310{,}000 = -73{,}000 \text{ lb.}$$

$$v = \frac{73{,}000}{8.75 \times 12 \times 0.9 \times 12} = 64 \text{ p.s.i.}$$

This is but slightly higher than the allowable unit stress for beams without web reinforcement (60 p.s.i.). Stirrups are unnecessary.

Outside of the columns,

$$V = \frac{620,000}{18.75}\left(2.375 - \frac{9.5}{12}\right) = 53,800 \text{ lb.}$$

Stirrups are obviously unnecessary in this region.

Complete details are given in Fig. 87. The actual weight of the footing is 39,000 lb. The soil pressure is $\frac{620,000 + 39,000}{164.1} =$ 4020 lb. per sq. ft. which is but 0.5 per cent greater than the specified maximum and may be considered satisfactory.

158. Design of a Rectangular Combined Footing Supporting Two Unequal Column Loads. An exterior column, 24 by 18 in. in cross-section (see Fig. 89), supports a total load of 200,000 lb. The adjacent interior column is 24 by 24 in. in cross-section and carries a total load of 300,000 lb. The two columns are to be supported on one rectangular combined footing, one end of which cannot extend beyond the outer face of the exterior column. The distance center to center of columns is 18 ft.-0 in. The allowable soil pressure is 4000 lb. per sq. ft. A 2000-lb. concrete and structural grade reinforcing bars are to be used in the footing.

Assuming the weight of the footing[1] as 0.12(300,000 + 200,000) = 60,000 lb., the bearing area required is $\frac{560,000}{4000} = 140.0$ sq. ft.

In order to secure uniform soil pressure, the center of gravity of the footing must coincide with the center of gravity of the column loads. The latter is $\frac{300,000}{500,000} \times 18 = 10.8$ ft. from the center of the exterior column. The length of the footing must be 2(10.8 + 0.75) = 23.1 ft. A length of 23 ft.-3 in. is selected. The width required is then $\frac{140.0}{23.25} = 6.0$ ft. The proposed method of design assumes the loads from the columns to be carried by transverse beams under the columns, these beams in turn distributing the loads to the longitudinal beam.

[1] The weight of a combined footing of this type will usually vary from 8 to 15 per cent of the sum of the column loads, the distance between the columns being an important factor, as a careful analysis of the following design will indicate.

Design of Longitudinal Direction. The net upward pressure per linear foot is $\frac{500,000}{23.25} = 21,500$ lb. The maximum negative moment between the columns occurs at the section of zero shear. Let x be the distance from the outer edge of the exterior column to this section. Equating to zero the expression for the shear at x, the following equation is obtained:

$$21,500x - 200,000 = 0$$

from which $x = 9.32$ ft.
The moment at this section is

$$M = \left[-200,000(9.32 - 0.75) + 21,500 \times \frac{(9.32)^2}{2} \right] \times 12$$
$$= -9,360,000 \text{ in.-lb.}$$

The moment at the right edge of the interior column (see Fig. 88) is

$$M = 21,500 \times \frac{(3.5)^2}{2} \times 12 = 1,575,000 \text{ in.-lb.}$$

For a 2000-lb. concrete, $f_c = 800$ p.s.i., and $n = 15$. From Table 6, $K = 139$ and $j = 0.867$. The effective width of the longitudinal beam, for both compression and tension, is equal to the full width of the footing, or $6.0 \times 12 = 72$ in. From equation (6),

$$d = \sqrt{\frac{9,360,000}{139 \times 72}} = 30.6 \text{ in.}$$

An effective depth of 31 in. is selected; with 3 in. insulation, the total thickness of the footing is 34 in. and the weight is 59,500 lb., which agrees closely with the assumed value.

Between the columns[1] the area of steel required is

$$A_s = \frac{9,360,000}{18,000 \times 0.867 \times 31} = 19.3 \text{ sq. in.}$$

[1] In computing the area of longitudinal steel at this section, the use of the theoretical value of j is justified, since the design is analogous to that of an ordinary rectangular beam of definite width and ideal steel ratio. At other sections in the longitudinal beam, and in the transverse beam, an approximate value of $j = 0.9$ will be used.

The critical sections for bond, for these bars, are at the inner edge of the exterior column and at the point of inflection (see Fig. 88), which is approximately 0.1 ft. from the edge of the interior column. The latter section governs in this design, and the total shear is

$$V = 181{,}750 - \frac{21{,}500}{10} = 179{,}600 \text{ lb.}$$

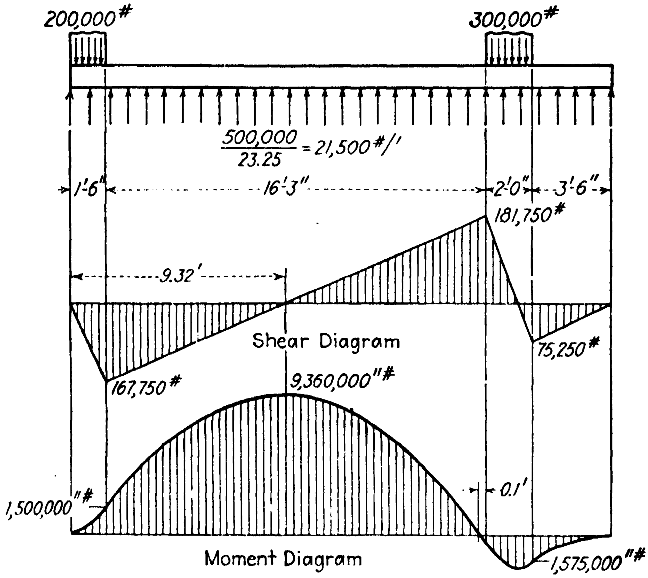


FIG. 88.

With anchored bars, the allowable unit bond stress (steel in one direction only) is $1.5 \times 0.05 \times 2000 = 150$ p.s.i.

$$\Sigma_0 = \frac{179{,}600}{150 \times 0.9 \times 31} = 43 \text{ in.}$$

The area requirement governs, and twenty 1-in. square bars are selected. The spacing is approximately $3\frac{1}{2}$ in.

For the portion of the longitudinal beam which projects beyond the interior column,

$$A_s = \frac{1{,}575{,}000}{18{,}000 \times 0.9 \times 31} = 3.13 \text{ sq. in.}$$

The critical section for bond is at the edge of the column, at which section $V = 21{,}500 \times 3.5 = 75{,}300$ lb. Since these bars are in the bottom of the footing and hence are in contact with the transverse bars, the allowable unit bond stress, with anchored bars, is $0.75 \times 150 = 112$ p.s.i.

$$\Sigma_0 = \frac{75{,}300}{112 \times 0.9 \times 31} = 24.1 \text{ in.}$$

Thirteen $\frac{5}{8}$-in. round bars are selected.

The critical section for diagonal tension is at a distance of d in. from the columns. Near the inner edge of the exterior column, $V = 167{,}750 - 21{,}500 \times {}^{31}\!/_{12} = 112{,}300$ lb. and the unit shear is

$$v = \frac{112{,}300}{72 \times 0.9 \times 31} = 56 \text{ p.s.i.}$$

With anchored bars the allowable unit shearing stress for beams without web reinforcement is $0.03f'_c = 60$ p.s.i., and no web reinforcement is required in this portion of the footing.

At a distance of 31 in. from the left edge of the interior column, the total shear is $181{,}750 - 21{,}500({}^{31}\!/_{12}) = 126{,}300$ lb., and the unit shear is 63 p.s.i. This is sufficiently close to the allowable value of 60 p.s.i. so that no web reinforcement need be placed in this region.

At a distance of 31 in. from the right edge of the interior column the total shear is 16,800 lb., and the unit shear is 9 p.s.i. No stirrups are required in this region.

Design of Transverse Beam under Interior Column. The moment at the edge of the interior column, in the transverse direction, is

$$M = \frac{300{,}000}{6} \times \frac{2^2}{2} \times 12 = 1{,}200{,}000 \text{ in.-lb.}$$

Assuming that the effective width for compression is equal to $1\frac{1}{2}$ times the column width, or $1\frac{1}{2} \times 24 = 36$ in., the required effective depth is

$$d = \sqrt{\frac{1{,}200{,}000}{36 \times 139}} = 15.5 \text{ in.}$$

The actual value of d which is furnished (see Fig. 89) is $31.0 - \frac{5}{8} = 30\frac{3}{8}$ in.

$$A_s = \frac{1,200,000}{18,000 \times 0.9 \times 30\frac{3}{8}} = 2.45 \text{ sq. in.}$$

At the edge of the column, $V = \dfrac{300,000}{6} \times 2 = 100,000$ lb.

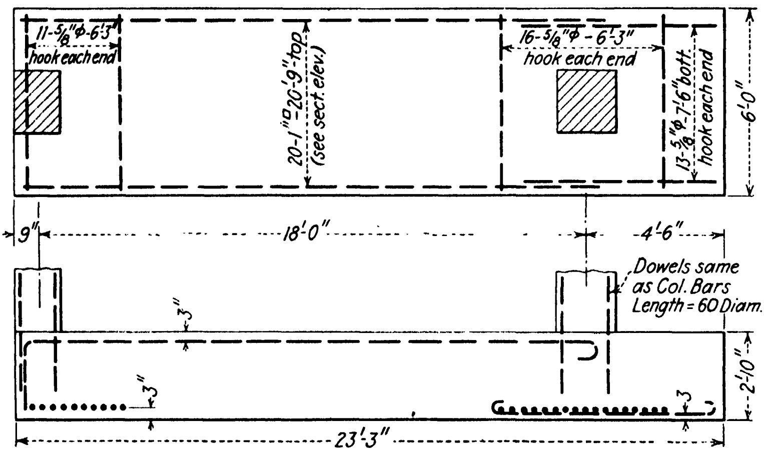


FIG. 89.—Combined footing with two unequal column loads.

With anchored bars, $u = 112$ p.s.i., and

$$\Sigma_0 = \frac{100,000}{112 \times 0.9 \times 30\frac{3}{8}} = 32.8 \text{ in.}$$

Sixteen $\frac{5}{8}$-in. round bars are selected and placed arbitrarily in a width of 60 in. The spacing is about 4 in., and all bars fall well within a 45-degree line from the edge of the column.

No investigation for diagonal tension is necessary, since a section d inches from the edge of the column falls outside of the footing.

Design of Transverse Beam under Exterior Column. The moment at the edge of the exterior column is

$$M = \frac{200,000}{6} \times \frac{2^2}{2} \times 12 = 800,000 \text{ in.-lb.}$$

Assuming that the effective width for compression (concrete on one side of the column only) is $1\frac{1}{4} \times 18 = 22$ in., the effective depth required is

$$d = \sqrt{\frac{800{,}000}{139 \times 22}} = 16.2 \text{ in.}$$

The actual d furnished is $30\frac{3}{8}$ in.

$$A_s = \frac{800{,}000}{18{,}000 \times 0.9 \times 30\frac{3}{8}} = 1.68 \text{ sq. in.}$$

At the edge of the column, $V = \dfrac{200{,}000}{6} \times 2 = 66{,}700$ lb.

$$\Sigma_0 = \frac{66{,}700}{112 \times 0.9 \times 30\frac{3}{8}} = 21.8 \text{ in.}$$

Eleven $\frac{5}{8}$-in. round bars are selected and placed arbitrarily in a width of 44 in. For the same reason as given in the design of the transverse beam under the interior column, no investigation for diagonal tension is necessary.

Complete details are shown in Fig. 89.

159. Design of a Cantilever Footing. In the cantilever type of construction the wall-column footing is connected to the nearest interior-column footing by means of a beam or strap. The eccentric load from the exterior column is resisted by a downward pressure from the interior column, the effect of which is transmitted through the strap. The wall-column load is supported directly by the strap, and is distributed to the soil by means of bars, at right angles to the strap, in the exterior footing.

In order to illustrate the principles involved in the design of a cantilever footing, the columns of the problem in Art. 158 will be so supported. As in Art. 158, the allowable soil pressure is 4000 lb. per sq. ft., a 2000-lb. concrete will be used, and f_s will be 18,000 p.s.i. The footing for the interior column is designed in the manner indicated in Art. 150, and it will be assumed that this design has been made. The resulting details are shown in Fig. 80.

The area required for the exterior-column load, allowing about 25 per cent for the weight of the footing and strap, is $\frac{250,000}{4000} =$ 62.5 sq. ft. A base 6 ft.-0 in. by 10 ft.-6 in. is selected.

Considering the strap as a free body, the external forces acting on it are as shown in Fig. 90. The eccentricity of the exterior-column load is resisted by a downward pressure P from the interior column, thus counterbalancing the tendency of the exterior footing to overturn. The load on the exterior column is assumed to be uniformly distributed over the column base, and the upward reaction R is assumed to be uniformly distributed over the 6-ft. width of the exterior footing.

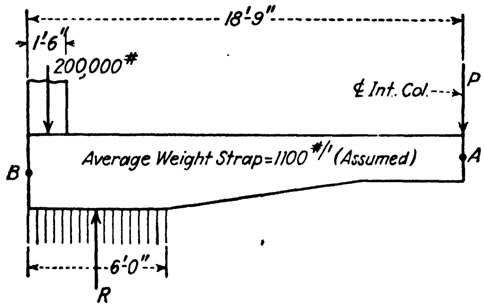


FIG. 90.

Assuming the weight of the strap as 1100 lb. per lin. ft., the reaction R is determined by taking moments about A (Fig. 90), as follows:

$$-200,000 \times 18.0 - 1100 \times 18.75 \times \frac{18.75}{2} + R \times 15.75 = 0$$

from which

$$R = 240,800 \text{ lb.}$$

The downward pressure P is determined by taking moments about B, as follows:

$$200,000 \times 0.75 + 1100 \times 18.75 \times \frac{18.75}{2} + P \times 18.75 - 240,800 \times 3.0 = 0$$

from which

$$P = 20{,}200 \text{ lb.}$$

The maximum moment in the strap occurs at the point of zero shear, which is x feet from B. The value of x is obtained as follows:

$$-200{,}000 + \frac{240{,}800}{6}x - 1100x = 0$$

from which

$$x = 5.1 \text{ ft.}$$

The moment at this section (the maximum moment) is then,

$$M = \left[-200{,}000(5.1 - 0.75) - 1100 \times \frac{(5.1)^2}{2} + \frac{240{,}800}{6} \times \frac{(5.1)^2}{2}\right]12$$

$$= -4{,}360{,}000 \text{ in.-lb.}$$

$$bd^2 \text{ (required)} = \frac{4{,}360{,}000}{139} = 31{,}400 \text{ in.}^3$$

If $b = 30$ in., $d = 33$ in.

The maximum shear occurs at the inner face of the exterior column.

$$V = -200{,}000 - 1100 \times 1.5 + \frac{240{,}800}{6} \times 1.5 = 141{,}500 \text{ lb.}$$

The allowable unit shearing stress is 120 p.s.i. The critical width for shear may be taken in the plane of the longitudinal bars, and this width includes not only the width of the strap, but also the extra width of the exterior footing at this plane (see Fig. 91). With the proposed arrangement of the exterior footing, the shearing width in the plane of the longitudinal reinforcement is approximately $30 + 2 \times \frac{1}{8} \times 48 = 42$ in., and the depth required for shear is

$$d = \frac{141{,}500}{120 \times 0.9 \times 42} = 31 \text{ in.}$$

The total depth of the strap at the point where it joins the exterior footing, and from this point to the end of the strap, is made $33 + 3 = 36$ in., the governing factor being the moment requirement. The bottom surface will be sloped from the bottom of the inner edge of the exterior footing to the bottom of the left edge of the interior footing as shown in Fig. 80. The depth required for moment and shear at any section along the sloped portion is considerably less than that furnished.

The exterior footing will be sloped away from the strap to a total depth of 12 in. at the edges of the footing, as shown in Fig. 91. The actual weight of the exterior footing, exclusive of the strap, is then 14,000 lb., and the average soil pressure under that footing is

$$\frac{240{,}800 + 14{,}000}{10.5 \times 6} = 4030 \text{ lb. per sq. ft.}$$

This is so little greater than the specified maximum of 4000 lb. per sq. ft. that it may be considered satisfactory.

The area of steel (A_s) and the total perimeter of bars (Σ_0) required in the strap are as follows:

$$A_s = \frac{4{,}360{,}000}{18{,}000 \times 0.9 \times 33} = 8.15 \text{ sq. in.}$$

$$\Sigma_0 = \frac{141{,}500}{150 \times 0.9 \times 33} = 31.8 \text{ in.}$$

Eight 1-in. square bars, hooked at both ends, are selected; these bars will be placed in one row, the center of which will be 3 in. below the top of the strap.

Half of these bars could be discontinued at some section between the columns. The exact point of cut-off would be determined by computing the area of steel required at two or three sections, and plotting a curve for these values; or the moment could be expressed in terms of a distance x from the center of the interior column and equated to the resisting moment of four bars. Both of these methods have been explained in Chap. III.

Stirrups are required from the inner edge of the exterior column, the section of maximum shear, to the section where the unit shear

is 60 p.s.i. Let x be the distance from this section to the outer face of the exterior column (point B, Fig. 90). Writing the value of the total shear at this section, which will obviously fall within the width of the exterior footing, substituting in the equation for unit shear with the effective width equal to 42 in., and equating the resulting expression to 60, the following results are obtained:

$$v = 60 = \frac{200{,}000 - \dfrac{240{,}800}{6}x + 1100x}{42 \times 0.9 \times 33}$$

from which

$$x = 2.1 \text{ ft.}$$

The required spacing of $\frac{5}{8}$-in. round double-looped stirrups at the inner edge of the exterior column is

$$s = \frac{4 \times 0.3068 \times 18{,}000 \times 0.9 \times 33}{141{,}500 - (60 \times 42 \times 0.9 \times 33)} = 9.8 \text{ in.}$$

Only one stirrup is necessary, but two will be used, the first being placed about 4 in. from the inner edge of the exterior column and the second 8 in. from the first.[1] The unit shearing stress at the inner edge of the exterior footing is considerably less than 60 p.s.i., and no web reinforcement other than that mentioned above is required.

The unit net upward pressure on each of the cantilever portions of the exterior footing is

$$\frac{240{,}800}{10.5 \times 6} = 3820 \text{ lb. per sq. ft.}$$

The maximum moment in the cantilever, at the edge of the strap, is

$$M = 3820 \times 4 \times 6 \times 4\tfrac{1}{2} \times 12 = 2{,}202{,}000 \text{ in.-lb.}$$

The total area of steel required in the exterior footing, per-

[1] The above design is conservative; it probably would be safe to omit the stirrups entirely.

pendicular to the strap, is then

$$A_s = \frac{2,202,000}{18,000 \times 0.9 \times 33} = 4.13 \text{ sq. in.}$$

Fourteen ⅝-in. round bars, hooked at each end, are selected. The unit bond stress on these bars is

$$u = \frac{3820 \times 4 \times 6}{14 \times 1.964 \times 0.9 \times 33} = 112 \text{ p.s.i.}$$

This is well below the allowable value of 150 p.s.i.

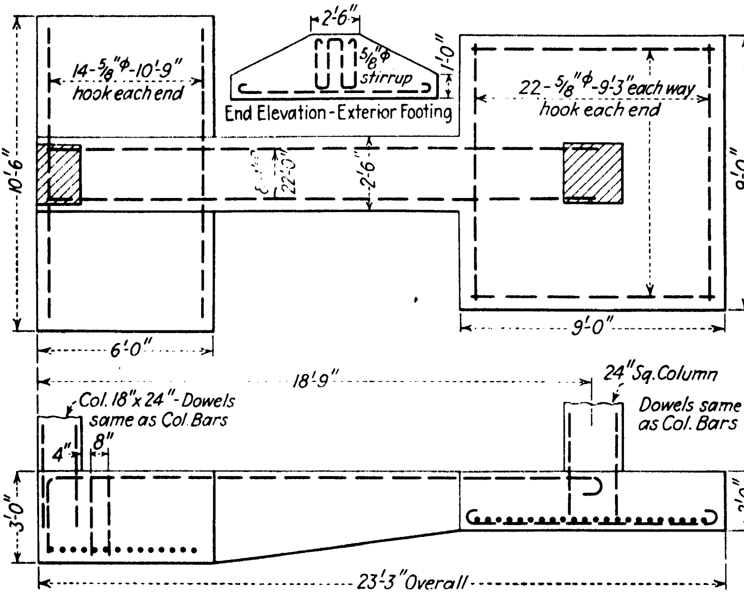


FIG. 91.—Cantilever footing.

Complete details of the footing are shown in Fig. 91.

160. Miscellaneous Foundations. In designing foundations to rest on soil, the safe bearing power of which is very small, it sometimes becomes necessary to extend the footings to cover practically all of the area of the building, one connected to the other. Such foundations may consist of a solid flat slab of concrete, a series of beams with slabs at the top or bottom, or a

series of connected multiple-column footings. In some cases it may be necessary to support the individual footings on piles.

Pile Foundations

161. Concrete Foundations on Piles. Where a soil of a highly compressible nature is encountered, and where the amount of excavation which would be required to reach a firm stratum would be excessive, economy might dictate the use of a foundation supported on piles, the latter being long enough either to reach the firm substratum or to develop sufficient skin friction to overcome the loads to which they are subjected.

This type of foundation consists essentially of a concrete slab, usually reinforced, supported directly on the piles. The heads of the piles are allowed to project a short distance above the ground so that the concrete may encase these portions of the piles and form with them a solid unit. A minimum embedment of 6 in. is considered satisfactory in most cases. If desirable, the material around the piles may be excavated, the depth of excavation depending upon soil conditions, and the space thus made filled in with gravel or other solid material, on which the concrete is laid as stated above. Such procedure utilizes the increased bearing power of the earth surrounding the piles.

The essential difference in the design of a concrete foundation supported on piles, and the design of a footing resting directly on the soil is in the manner in which the load on the footing is resisted by the foundation bed. In the former case, a series of concentrated upward loads must be considered, while in the latter case, uniform distribution under the entire concrete area is assumed.

162. Bearing Power of Piles. When piles are supported entirely by the friction between their sides and the earth, the load is transmitted to a deep ground level in a conoid of pressure through the earth above it. Such piles should be driven so far apart, or to such a depth, that the increased area of bearing developed by the conoid of pressure, which has the required altitude to contain the frictional resistance, reaches a level at which the material will afford the required support before it

intersects the corresponding conoid of an adjacent pile. In good practice, bearing piles are never spaced closer than 2½ ft. center to center, and preferably not closer than 3 ft.

The bearing value of a pile which is supported by the friction of the earth into which it is driven will depend, among other things, upon the soil conditions, the size and spacing of the piles, and the depth to which they are driven.[1] The absolute maximum loads usually permitted on timber and concrete piles are 20 tons and 40 tons, respectively. The actual bearing power can be determined by a loading test, in which a pile or group of piles is loaded with a static load placed in increments until appreciable settlement is noted, or by a driving test, in which the measured penetration caused by several blows of the hammer is used to compute the probable bearing power.

The most widely used formulas for computing the bearing power from penetrations observed in driving are the *Engineering News* formulas. A suitable factor of safety is included in these formulas, so that the resulting values are the safe bearing capacities rather than the ultimate. When a drop hammer is used, with a free fall not exceeding 20 ft., the equation for the safe bearing power is

$$P = \frac{2WH}{s + 1}$$

When a single-acting steam hammer is used, the safe load per pile is

$$P = \frac{2WH}{s + 0.1}$$

[1] Goodrich recommends that "the best practice is to assume a given load per pile, to design all footings accordingly, and to require the superintendent of construction to provide and drive piles which will sustain this assumed load. In that case the designer's care will be to provide just the proper number under each footing, and to space them so that each pile will develop its full proportion of the given load. To this end, groups should be made as nearly circular as possible, especially when they consist of any considerable number of piles. The corner piles of square groups of 16 piles might just as well be omitted."

When a double-acting steam hammer is used, the safe load is

$$P = \frac{2H(W + Am - b)}{s + 0.1}$$

in which P = safe bearing power per pile, in pounds.

W = weight of a drop hammer or the weight of the striking part of a steam hammer, in pounds.

H = height of fall of a drop hammer, or the length of the stroke of a steam hammer, in feet.

s = average penetration in inches per blow for at least three consecutive blows of the hammer.

A = effective area of piston in square inches.

m = mean effective pressure of the steam on the downward stroke, in pounds per square inch.

b = total back pressure in pounds.

A constant of 0.3 instead of 0.1 in the above equations is sometimes used.

When a pile is driven through soft material to a hard substratum, the safe bearing value is computed by treating the pile as a column, with an unsupported length equal to two-thirds of the penetration in distinctly soft material.

163. Bending Moment. As in the case of footings resting on soil, the external moment on any section is determined by passing through the section a vertical plane which extends completely across the footing and computing the moment of the forces acting over the entire area of the footing on one side of this plane. The forces are the net reactions from the piles, each reaction being equal to the total load on the column divided by the number of piles. The critical sections for moment are the same as for footings resting on soil, as given in Art. 147. The required tensile reinforcement is determined in the same manner as for footings on soil, and the placing of this reinforcement is governed by the same rules.

164. Shear. The critical section for shear to be used as a measure of diagonal tension is assumed as a vertical section obtained by passing a series of vertical planes through the footing, each of which is parallel to a corresponding face of the column or

pedestal, and located at a distance therefrom equal to one-half of the effective depth d of the footing. Each face of the critical section shall be considered as resisting an external shear equal to the net reactions from the piles which lie wholly or partly within the area bounded by this face, two diagonal lines drawn from the column or pedestal corners and making 45-degree angles with the principal axes of the footing, and that portion of the corresponding edge or edges of the footing intercepted between the two diagonals. Allowable shearing stresses are the same as for footings on soil, as given in Art. 149.

If, as is usually the case, a footing of uniform thickness is used, the thickness required will be governed by the shearing stresses. A minimum depth above the reinforcement of 12 in. is specified in the Joint Code or, for footings without reinforcement, a minimum of 14 in. above the tops of the piles.

165. Bond. Critical sections for bond are the same as for footings on soil (Art. 150); allowable unit stresses are the same; and the method of computing bond stresses is also the same, except that the concentrated net reactions from the piles are used in determining the total shear instead of the uniformly distributed net reaction from the soil.

166. Design of Footing Supported on Piles. A concrete footing resting on a pile foundation is used to support a column 18 in. square, the total load on which is 225,000 lb. The safe bearing power of each pile is 11 tons. Design the footing in accordance with the specifications of the Joint Code, using an ultimate compressive strength of concrete of 2000 p.s.i. and structural grade steel.

Assuming the weight of footing as 26,000 lb., the number of piles required $= \dfrac{251{,}000}{22{,}000} = 12$.

In order to keep a minimum spacing of piles of 2 ft.-6 in. and to distribute the load equally between the 12 piles as nearly as practicable, the arrangement shown in Fig. 92 is adopted. The net load per pile is $\dfrac{225{,}000}{12} = 18{,}750$ lb.

The maximum moment in the vertical section through one face of the column is

$$M = 18{,}750(4 \times 6 + 2 \times 36) = 1{,}800{,}000 \text{ in.-lb.}$$

The effective width is 120 in., and the effective depth required for moment is

$$d = \sqrt{\frac{1{,}800{,}000}{120 \times 139}} = 10.4 \text{ in.}$$

As stated in Art. 149, the depth required for shear will be the governing depth; but as long as the depth is greater than about

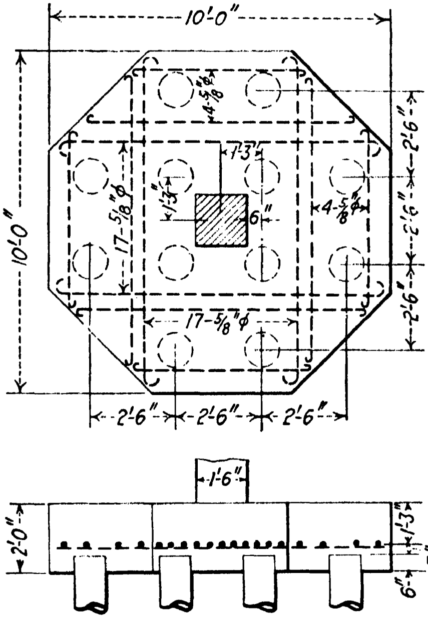


FIG. 92.

12 in. and less than about 42 in. (assuming piles with heads about 12 in. in diameter), the net reactions from only two piles need be considered in computing the effective external shear on one face of the critical section. Assuming a value of d equal to 15 in., the width of the critical section for shear is $18 + 2 \times 15 = 48$ in., and the unit shear is

$$v = \frac{2 \times 18{,}750}{48 \times 0.9 \times 15} = 58 \text{ p.s.i.}$$

This is so close to the allowable unit shearing stress (without web reinforcement) of 60 p.s.i. that obviously no reduction in the assumed effective depth can be made.

The total thickness of the footing, allowing a 6-in. embedment of the piles and 3 in. from the tops of the piles to the center of the reinforcement, is 24 in. The actual weight of the footing is then 26,000 lb., which checks the assumed value.

The area of steel required in each band must be sufficient to furnish a resisting moment of $0.85 \times 1{,}800{,}000 = 1{,}530{,}000$ in.-lb.

$$A_s = \frac{1{,}530{,}000}{18{,}000 \times 0.9 \times 15} = 6.3 \text{ sq. in.}$$

According to the specifications in Arts. 150 and 165, the total shear to be used in computing the bond stress for each band of steel is the net reaction from 4 piles, or $4 \times 18{,}750 = 75{,}000$ lb., and the allowable unit stress is 112 p.s.i.

$$\Sigma_0 = \frac{75{,}000}{112 \times 0.9 \times 15} = 49.5 \text{ in.}$$

Twenty-five $\frac{5}{8}$-in. round bars are selected ($A_s = 6.45$ sq. in., and $\Sigma_0 = 49.0$ in.). The bars are spaced uniformly across the entire width of the footing. The distance center to center of bars is approximately $4\frac{1}{2}$ in., which is satisfactory. Standard hooks are formed on each end of each bar, as required by the Code.

Details of the footing are shown in Fig. 92.

Proportioning Footing Areas for Uniform Pressure

167. Methods of Securing Uniform Settlement. In all of the preceding discussions and problems, the bearing areas of footings have been selected merely with the idea of keeping the unit soil pressures within specified limits, thus ignoring to a great extent the possibility of slight settlements and the effect of such settlements on the integrity of the structure above the footings.

A safe soil pressure is usually considered to be some value smaller than the unit load at which appreciable settlement takes place. All soils are compressible to some extent and, with unit

pressures such as are commonly used in practice, a moderate amount of settlement under each footing must be expected. If the amount of settlement of each footing in a large building is the same as that of the other footings, and if all of the settlements are simultaneous, no particular harm is done even if, in time, the total settlement is noticeable; the entire building merely lowers a fraction of an inch, an inch, 2 in., or more, without apparent damage. If however, uneven settlement takes place, floors will get out of level, doors will stick, cracks are apt to form, and even more serious damage may result. The problem confronting the designer is therefore not so much to prevent settlement as to make sure that whatever settlement takes place will be uniform throughout the entire structure.

If the footings rest on a fairly compressible soil, it is reasonable to expect that a great part of the settlement will be realized before the structure is made to serve its useful purpose, *i.e.*, before the live load is placed on it. Hence, in some cases, the areas of the footings are proportioned to give equal unit pressures under dead load alone. With a properly selected unit pressure, it is considered better practice, however, to assume that a part of the live load is necessary to produce noticeable settlement, and to proportion the footings for uniform pressures under dead plus partial live loads. It is seldom desirable to consider the full live load in the proportioning of footing areas, first, because of the reason given above, second, because the full design live load is rarely realized except occasionally in such structures as warehouses, etc., and third, because the live load is generally a spasmodic load and hence the settlement that may be caused by it is not in direct proportion to the amount of the load. The most generally adopted practice is to proportion footing areas so that the unit pressures under all footings will be the same when the footings are loaded with the dead load plus one-third or one-half of the probable live load.

168. Probable Live Load on Footings. The probable live load is the live load which is used in the design of the basement column. This is not necessarily the sum of the full live panel loads on each floor above the basement floor. Each floor panel

must, of course, be designed for the full live load that may at some time be placed on it, but in most structures, especially office buildings, hotels, apartment houses, etc., the full live load will never exist on all floors simultaneously. To allow for this condition, in computing the design loads for columns and footings, some reduction in the specified live load is permitted, the amount of reduction depending primarily on the number of stories in the building. The New York City building code is a typical example of conservative practice. This code contains the following:

(*a*) In structures intended for storage purposes all columns, piers or walls, and foundations may be designed for 85 per cent of the full assumed live load.

(*b*) In structures intended for other uses the assumed live load to be used in the design of all columns, piers or walls, and foundations may be as follows:

100 per cent of the live load on the roof.
85 per cent of the live load on the top floor.
80 per cent of the live load on the next floor.
75 per cent of the live load on the next floor below.

On each successive lower floor, there may be a corresponding decrease in percentage, provided that in all cases at least 50 per cent of the live load shall be assumed.

169. Method of Proportioning Footing Areas. A method for determining the bearing area A which is required under each individual footing to give uniform pressures under dead plus partial live load is as follows: Select one footing as the critical footing. If a given bearing pressure is not to be exceeded, the critical footing will be that footing with the greatest percentage of live load. The area of the base of the critical footing is A_c, the total load on it is T_c, the live load L_c, and the dead load D_c. The live load on any other footing is L, the dead load D, and the area A. Then, in order not to exceed the given value for the safe bearing capacity of the soil, B, the required area of the critical footing is

$$A_c = \frac{T_c}{B}$$

The bearing pressure B_c at the base of the critical footing, when that footing is loaded with its full dead load, D_c, plus a definite part of its live load, ΔL_c, is

$$B_c = \frac{D_c + \Delta L_c}{A_c}$$

In order to secure equal unit pressures at the bases of all the footings in the given structure when these footings are loaded with their full dead loads D plus the stated part of their respective live loads ΔL, the area A of each footing must be equal to

$$A = \frac{D + \Delta L}{B_c}$$

If the footings are to be proportioned for equal unit pressures under dead load plus one-half live load, the value of Δ in the above discussion is $\frac{1}{2}$; for dead load plus one-third live load, $\Delta = \frac{1}{3}$. With areas computed from this equation the maximum unit pressure under the critical footing will be equal to B, the maximum unit pressure under any other footing will be less than B, and the unit pressure under dead load plus one-half (or one-third or any other specified fraction) of the live load will be the same under all footings. Once the area of each individual footing is determined as above, the remainder of the design of each footing must be based on the total load.

170. *Illustrative Problem.* Four typical footings in a given building support dead and probable live loads as tabulated below. If the maximum safe bearing power of the soil on which these footings are to rest is 5000 lb. per sq. ft., what must be the area of each of these footings in order that uniform settlement may be expected, when the footings are loaded with the full dead load plus one-third of the live load?

Footing	Dead load, lb.	Live load, lb.	Live load, per cent	$DL + \frac{1}{3}LL$, lb.
A	500,000	300,000	37	600,000
B	400,000	210,000	34	470,000
C	400,000	300,000	42	500,000
D	500,000	180,000	26	560,000

The critical footing is C, and the required area is

$$\text{Footing } C = \frac{400{,}000 + 300{,}000}{5000} = 140 \text{ sq. ft.}$$

A footing 12 ft square is selected, the area of which is 144 sq. ft.

The unit pressure B_c under this footing when it is loaded with the full dead load plus one-third of the probable live load is

$$\frac{500{,}000}{144} = 3470 \text{ lb. per sq. ft.}$$

The required areas of the other footings are, therefore,

$$\text{Footing } A = \frac{600{,}000}{3470} = 173 \text{ sq. ft. (13 ft.-3 in. square)}$$

$$\text{Footing } B = \frac{470{,}000}{3470} = 135 \text{ sq. ft. (11 ft.-9 in. square)}$$

$$\text{Footing } D = \frac{560{,}000}{3470} = 161 \text{ sq. ft. (12 ft.-9 in. square)}$$

CHAPTER VIII

REINFORCED CONCRETE BUILDINGS

171. Adaptability of Reinforced Concrete. Reinforced concrete has gradually become one of the leading building materials of the present day, chiefly because of its durability, its fire-resisting qualities, its adaptability to various types of design, and its pleasing architectural appearance. When used with any other type of construction, as for example, the floors in a steel frame structure, or by itself in a building all of whose constituent structural parts are of reinforced concrete, its suitability is well recognized.

In determining the type of structure to be used for any particular building, usually the two most important considerations are the time required before the building may be occupied, and the relative economy of the selected type as compared with the other available structures. While the actual erection of a steel frame building may be completed in considerably less time than a reinforced concrete building, in most cases the length of time necessary for the fabrication of the steel will result in the lapse of a longer period of time from the letting of the contract to the completion of the structure than in the case of all-reinforced-concrete construction.

Steel frame structures in which no attempt is made to encase the steel may be lower in first cost than those of reinforced concrete. If, however, an attempt is made to have the steel structure as fireproof as the reinforced concrete structure, the ratio of relative first costs may be reversed. This is especially true of certain types of buildings in which long spans and heavy loads exist. A real comparison between buildings of different materials should be made only after a consideration of the first cost and subsequent annual expenditures.

172. Floor and Roof Loads. The minimum live loads for which the floors and the roof of any building must be designed are always specified in the building code that governs the site of the construction.

The range of minimum live-load values in pounds per square foot of floor or roof area, as given in several typical building codes, is as follows:

Apartments	40
Auditoriums and theaters:	
With fixed seats	50–80
Without fixed seats	100
Dwellings	40
Hospitals	40
Hotels:	
Rooms	40
Corridors, lobbies, dining rooms	100
Manufacturing buildings:	
Light manufacturing	75–125
Heavy manufacturing	125–200
Office buildings:	
Office space	50–60
Corridors, public spaces	100–125
Roofs, flat	30–40
School buildings:	
Classrooms	50–60
Corridors, public spaces	100
Garages:	
All types of vehicle	100–175
Passenger cars only	75–125
Store buildings:	
Retail	75–125
Wholesale	100–125
Warehouses:	
Light storage	75–150
Heavy storage	200–250

The specified minimum live loads cannot always be used. The type of occupancy should be considered, and the probable loads should be computed as accurately as possible. Warehouses for heavy storage may be designed for loads as high as 500 lb. or more per sq. ft.; unusually heavy operations in manufacturing buildings may require a large increase in the 200-lb. maximum specified above; and special provision must be made for all definitely located heavy concentrated loads.

Some building codes provide for occasional probable concentrated loads, such as the weight of a heavy safe, by requiring that in addition to being adequate to support the specified minimum uniform live load, the floor system shall also be capable of supporting a single concentrated load of 2000 lb. (or more) on an area $2\frac{1}{2}$ ft. square when this load is placed in any position on the floor. The concentrated load and the uniform load are, however, not assumed to act simultaneously.

In all cases the dead weight of the floor must be included in the total design load. When plastered ceilings are specified, as is frequently the case where ribbed floor construction is used, an additional allowance of from 10 to 15 lb. per sq. ft. is made for the lath and plaster. In certain types of buildings, particularly office buildings, where all partitions are not definitely located on the plans, an extra allowance of from 10 to 20 lb. per sq. ft. of floor area is frequently made for the weight of these partitions.

Reinforced Concrete Floors and Roofs

173. Types of Floor Systems. The different floor systems that are commonly used in reinforced concrete buildings may be divided into five general classes, as follows:

1. Beam-and-girder floors.
2. Beam-and-slab floors.
3. Flat-slab floors.
4. Ribbed floors, with clay-tile, gypsum-tile, or steel-tile fillers.
5. Steel-joist floors.

Each of these types is described in the following articles. Type 5 is more commonly used with structural-steel frames than with concrete frames, but it can be adapted to the latter type of construction.

174. Beam-and-girder Floors. A beam-and-girder floor consists of a series of parallel beams supported at their extremities by girders which in turn frame into concrete columns placed at more or less regular intervals over the entire floor area. This framework is covered by a reinforced concrete slab, the load from which is transmitted first to the beams and thence to the girders and columns. The beams are usually spaced so that they come at the midpoints, at the third points, or at the quarter points of the girders, as shown in Fig. 93. The arrangement of beams and spacing of columns should be determined by economical and practical considerations. These will be affected by the use to which the building is to be put, the size and shape of the ground area, and the load which must be carried. A comparison of a number of trial designs and estimates should be made if the size of the building warrants, and the most satisfactory arrangement

selected. As the slabs, beams, and girders are built monolithically, the beams and girders are designed as T-beams and advantage is taken of continuity.

Beam-and-girder floors are adapted to any loads and to any spans that might be encountered in ordinary building constructions. The normal maximum spread in live-load values is from 40 to 400 lb. per sq. ft., and the normal range in column spacings

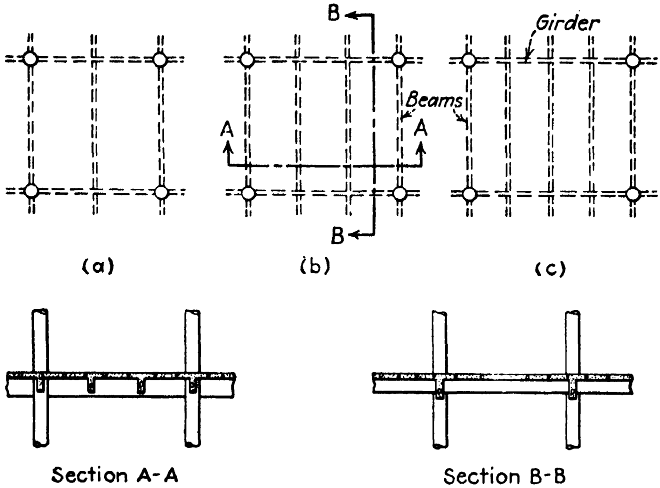


FIG. 93.—Framing of beam-and-girder floors.

is from 16 to 32 ft. A complete design of a typical beam-and-girder floor panel is given in Arts. 195 to 198.

175. Beam-and-slab Floors. Beam-and-slab floors are those in which a solid reinforced concrete slab is supported on concrete beams which transmit the loads directly to the columns. The beams are framed into the columns, usually in both rectangular directions; the slab is poured monolithically with the beams, with reinforcement parallel to each side of the floor panel. Occasionally the beams in one direction are omitted and the slab is supported on the beams between the columns in the other direction; the main slab reinforcement is then all placed at right angles to the beams, and only shrinkage reinforcement is placed in the other direction.

Beam-and-slab floors with beams in one direction only are heavy and expensive for ordinary spans, and are seldom used.

Those in which beams are placed in two directions are somewhat lighter, provided the panels are approximately square, and they furnish greater lateral support to the entire building frame.

176. Flat-slab Floors. A flat-slab floor consists of a reinforced concrete slab supported directly on concrete columns without the aid of beams or girders. This type is discussed in detail in Arts. 210 to 217. In general, flat-slab construction is economical for live loads of 100 lb. or more per square foot and for spans up to about 30 ft. For lighter loads, such as are used in apartment houses, hotels, and office buildings, some form of ribbed-floor construction will usually be cheaper than a flat-slab floor. For spans longer than about 30 ft. beams and girders are desirable because of the greater stiffness which can be secured with them. Flat-slab floors can be used only for constructions in which the column spacings are fairly constant, and in which the panel length in one direction is approximately the same as, and never more than 1.33 times, the panel length in the other direction.

177. Ribbed Floors. A ribbed floor consists of a series of small, closely spaced, reinforced concrete T-beams framing into beams or girders which in turn frame into the supporting columns. The T-beams (called joists or ribs) are formed by placing rows of fillers in what would otherwise be a solid slab. The fillers may be hollow clay-tile or gypsum-tile blocks, special steel cores, or ordinary wood forms. The girders which support the joists are built as regular T-beams with a maximum flange thickness equal to the total thickness of the floor, as shown in Fig. 99*a*.

Ribbed floors are economical for buildings such as apartment houses, hotels, and hospitals, where the live loads are fairly small and the spans comparatively long. They are not suitable for heavy construction such as is necessary in warehouses, printing plants, and heavy manufacturing buildings.

178. Ribbed Floors with Clay-tile Fillers. Figure 94 shows a ribbed floor in which structural clay-tile blocks are used as fillers. The blocks are usually 12 in. square and they can be obtained in thickness varying from 4 to 12 in. Specially formed blocks are also manufactured for use as floor fillers, but these

have little, if any, advantage over the standard square form. The blocks are placed flatwise and end to end, in rows between and at right angles to the girder forms. They are supported either by a solid decking similar to that used for regular concrete slabs, or on 2 by 8 planks called soffit boards, which are centered under the space between the rows of tile. The decking or soffit boards are supported on light shoring from the floor below. The usual clear distance between rows is 4 in., thus making the distance center to center of rows 16 in. Concrete is poured on the decking or soffit boards to fill the space between the rows of tiles and also to cover the tiles to a depth[1] of 2 or $2\frac{1}{2}$ in. The resulting construction consists of a series of concrete T-beams, or joists, with tile fillers under the slab in the spaces between the stems of the joists. The joists frame into, and are supported by, the girders, and these in turn are supported by the columns. When the concrete has hardened, the forms are removed, but the tile fillers remain in place, securely anchored to the concrete by the projections on the surfaces of the tiles. The entire ceiling is then plastered. The joists are reinforced, usually with two longitudinal bars, one bent and one straight. The slab is reinforced, primarily for shrinkage stresses, with wire mesh, expanded metal, or small bars placed at right angles to the joists; the area of the reinforcement is usually about 0.25 per cent of the cross-sectional area of the slab.[2]

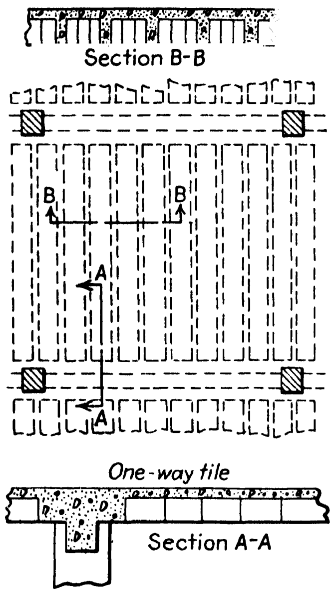


FIG. 94.—Details of one-way tile floor.

[1] The Joint Code permits a minimum slab thickness of $1\frac{1}{2}$ in., but not less than one-twelfth of the clear distance between joists. Normally a 2-in. minimum slab thickness is used.

[2] The Joint Code requires that the temperature reinforcement, placed

The method to be used in the design of a typical ribbed floor panel with clay-tile fillers is outlined in Arts. 204 to 206.

When a ceiling with a tile surface over the entire area is required, narrow flat-tile slabs are furnished, to be laid in the bottom of the joists before the concrete is poured. These tile facers are scored on both sides, the upper scoring serving to hold the facers to the concrete of the joists and the lower scoring serving as a bond for the ceiling plaster. When these tile facers are used, the effective depth of the joists is decreased by the thickness of the facers, about $\frac{5}{8}$ in., and to provide the necessary strength the total floor thickness must be increased accordingly. Tile facers are used primarily to prevent staining of the plaster by the concrete, but normally if the concrete is allowed to dry thoroughly before the plaster is applied, no plaster stains will be noticed under joists without facers.

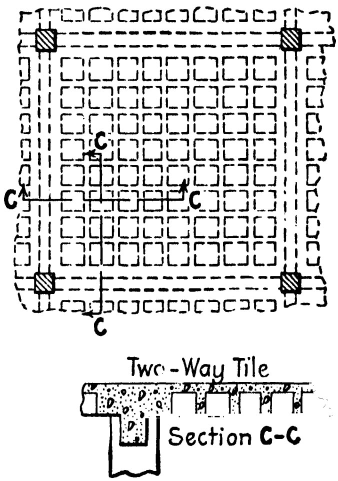


FIG. 95.—Details of two-way tile floor.

Special tiles with closed ends are also available for use in forming a two-way ribbed floor. These blocks are laid in rows, with a space of about 4 in. between adjoining blocks in the same row, and with a space of about 4 in. between adjacent rows; reinforcement is placed in the joists which are thus formed in the two directions. When such two-way tile construction is used, beams must be placed in both directions between the columns to furnish support for the joists. The essential details of a two-way ribbed floor are shown in Fig. 95.

179. Ribbed Floors with Gypsum-tile Fillers. Gypsum-tile ribbed-floor construction is similar to that in which clay-tile

at right angles to the joists, shall be of the amounts specified in Art. 59 for slabs reinforced in one direction.

fillers are used, except that the size and shape of the blocks are different. A cross-section of a typical gypsum filler tile is shown in Fig. 96. These tiles are made of dense specially prepared gypsum, in two sizes, one 12 in. wide and 30 in. long, with thicknesses of 3, 4, or 5 in., and the other 19 in. wide and 18 in. long, with thicknesses of 6, 8, 10, or 12 in. The 19- by 18-in. fillers are cast with one integral end, which provides a seal for each row at the girders. The 12-in. tiles are usually placed 16 in. on

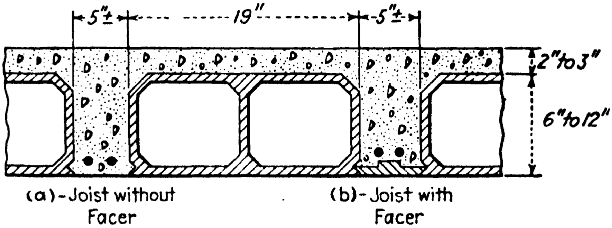


FIG. 96. Details of ribbed floor with gypsum tiles.

centers, thus providing a 4-in. joist, while the 19-in. tiles are usually placed 24 in. on centers, with a 5-in. joist width. Joist widths can, of course, be varied to suit any conditions.

Joist facers 5 in. wide, 1 in. thick, and 12 in. long, are available; these are laid on the forms between the rows of tiles and serve to give a uniform surface to the bottom of the floor. These joist facers are held in position by notches in the sides of the facers, which fit into corresponding notches in the sides of the fillers, as shown in Fig. 96*b*.

Open centering, consisting only of planks or soffit boards for the bottom forms of the joists, is usually used. These planks are wide enough to permit the edges of the tile blocks to rest on them. The soffit boards are supported on transverse 2 by 8 timber joists, from 3 to 4 ft. on centers; the 2 by 8 joists are supported on 4 by 4 posts about 4 ft. on centers.

Ribbed floors with gypsum-tile fillers are lighter in weight than those in which clay-tile fillers are used, and they are adapted to slightly longer spans. The tile pieces can readily be cut to fit conditions in the field, but they require careful handling to prevent excessive breakage.

180. Ribbed Floors with Steel-tile Fillers. When special steel cores are used, the formwork again consists of open centering, in which planks or soffit boards are placed only under the concrete ribs or joists. The steel cores are available usually in lengths of 30, 35, or 36 in., and in depths of 4, 6, 8, 10, 12, or 14 in. They

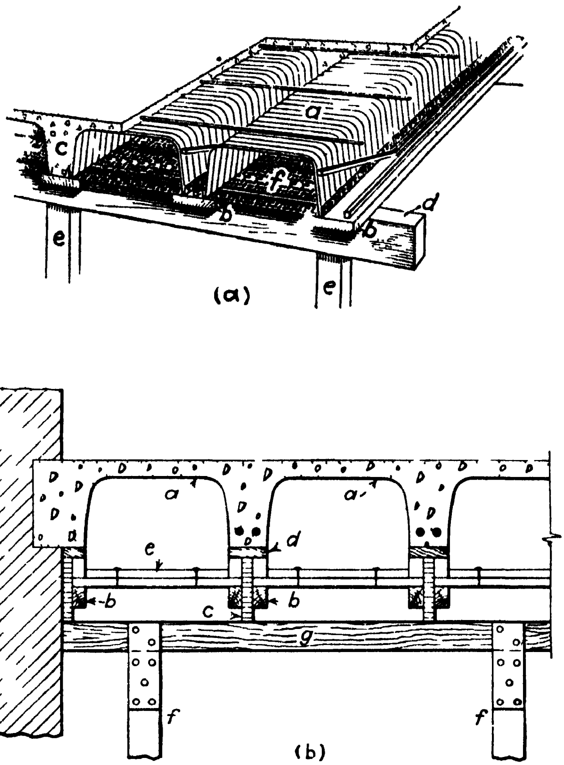


Fig. 97.—Details of ribbed floors with steel tiles.

are tapered in cross-section as shown in Fig. 97*a*, the most common width at the bottom is 20 in., although other widths between 10 and 31 in. are also obtainable. In some systems the individual core lengths *a*, Fig. 97*a*, rest on the edges of the soffit boards *b*, which serve as the bottom forms for the ribs or joists *c*, and which in turn are supported by transverse timber joists *d*, and vertical posts *e*. In other systems the steel tiles *a*, Fig. 97*b*, are supported by ledger strips *b*, which are fastened to the sides of timber joists *c*, laid on edge under the soffit boards *d*. With the latter system,

one depth of tile can be used for various depths of joists. In Fig. 97*b*, the bottoms of the tiles are held in the correct position by the separators *e*, which are made of two pieces of 1 by 2 strips, nailed together; the timber joists *c* rest on transverse joists *g*, and these are supported by the vertical posts *f*.

Concrete is placed on the soffit boards in the space between the rows of steel cores, and above the cores to form a slab from 2 to 3 in. in thickness.[1] After the concrete has hardened, the steel

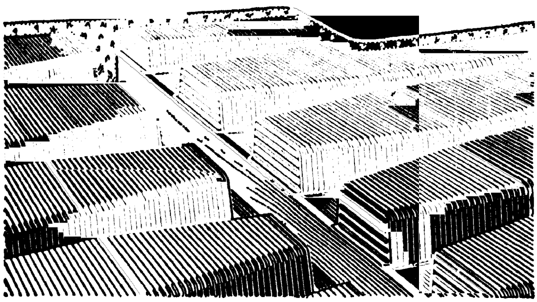


FIG. 98.—Steel tiles with tapered ends.

cores are generally removed, although some types are designed to be left in place permanently.

With permanent tiles, metal lath *f*, Fig. 97*a*, is placed under the tiles before they are put in position, and anchored to the concrete joists by suitable metal clips which project up into the joists. The entire ceiling is plastered after the formwork is removed. With removable tiles, the ceiling may be plastered by fastening metal lath to the concrete after the tiles are removed, or the joists may be left exposed. The joists are usually made 5 in. wide at the bottom, so that the distance center to center of joists is ordinarily 25 in. The reinforcement is similar to that which is used in ribbed floors in which clay-tile or gypsum-tile fillers are used.

Steel cores with the sides tapered in the direction of the length are available for use at the ends of the rows in designs where the

[1] The Joint Code specifies a minimum slab thickness of 2 in., but not less than one-twelfth of the clear distance between joists, with temperature reinforcement at right angles to the joists of an area not less than 0.049 sq. in. per ft. of width.

shearing stresses are relatively large. The width of tapered cores at the small end is 4 or 5 in. less than at the large end, so that the width of the joist near the support is increased accordingly. The increased width of the joist reduces the shearing stresses in the concrete. A typical construction showing tapered end lengths in position is illustrated in Fig. 98.

The girders which support the joists are rectangular beams or T-beams with a maximum flange thickness equal to the total floor thickness, as shown in Fig. 99*a*. If the maximum

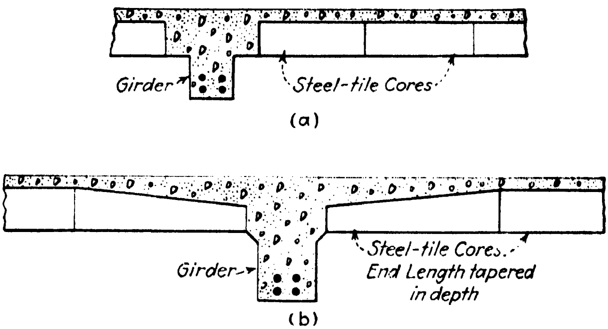


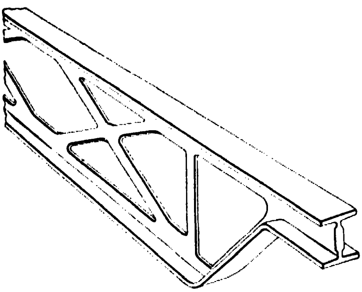
Fig. 99.—Details of girders for steel-tile floors.

flange thickness is to be used, the rows of tiles are stopped at the proper distance from the stem of the girder. If a flange thickness less than the maximum is desired, the end pieces of tile in each row are short sections which are dropped down on the forms, so that the tops of the pieces are at the proper elevation to furnish the required slab thickness above them. End caps are used at each end of each row of tile, as shown in Fig. 98, to hold the girder concrete in position.

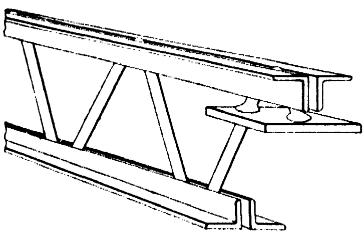
Tile pieces which are tapered in the depth are also available. These tiles are usually about 3 in. less in depth at one end than at the other, so that if one length of such tile is used at each end of each row, a slab from 5 to 6 in. thick at the edge of the girder is obtained, as shown in Fig. 99*b*. This thickness is often sufficient to furnish an adequate flange for the girder.

A complete design of a typical ribbed floor panel with steel-tile fillers is given in Arts. 199 to 203.

181. Steel-joist Floors. A steel-joist floor consists of a series of closely spaced, parallel, shallow joists or trusses of the Pratt, Warren, or double-Warren type, supported at the ends on steel or concrete beams, or on masonry walls, and covered with a thin concrete slab.



(a)



(b)

FIG. 100.—Typical steel joists.

Two common types of steel joists are shown in Fig. 100. The truss in view *a* is manufactured from one solid piece of specially rolled I-beam by a patented hot-slitting and rolling process. The webs of the I-beams are slit and the sections rolled out to the required depth at a temperature of 1800 deg. Fahrenheit. The resulting truss has T-shaped chords and a flat latticed web structure of uniform strength throughout. The bottom chords are bent up at the ends and welded to the top chords, in order to form a flat bearing surface and a rigid support. The truss of Fig. 100*b* is composed of angle chords and a continuous-bar web, assembled by the high-pressure electric welding method. The welds are designed to develop the full strength of the web members.

Steel joists are manufactured in depths of 8, 10, 12, 14, and 16 in., and are carried in stock in a range of depths and lengths to meet all usual conditions of load and span. The span limits vary from 4 to 32 ft.[1] The joists are spaced from 12 to 30 in.

[1] Long-span joists are also available, primarily for roofs, with depths up to 32 in., and for spans up to 64 ft. Roofs supported by these long-span joists have wood plank decks with built-up roofing, or concrete decks. The spacing of the joists is limited by the safe span of the deck. Long-span joists are bridged with cross bracing of not less than $1\frac{1}{4} \times 1\frac{1}{4} \times \frac{1}{8}$ in. angles, spaced not more than 10 ft. apart.

on centers, depending on the load and span, and are covered with a 2- or 3-in. concrete slab poured on metal rib lath which rests on and is fastened to the upper surfaces of the joists.

The trusses are braced transversely by some form of rigid diagonal bridging placed between adjacent trusses, at intervals of from 5 to 7 ft. One type of bridging is shown in position in

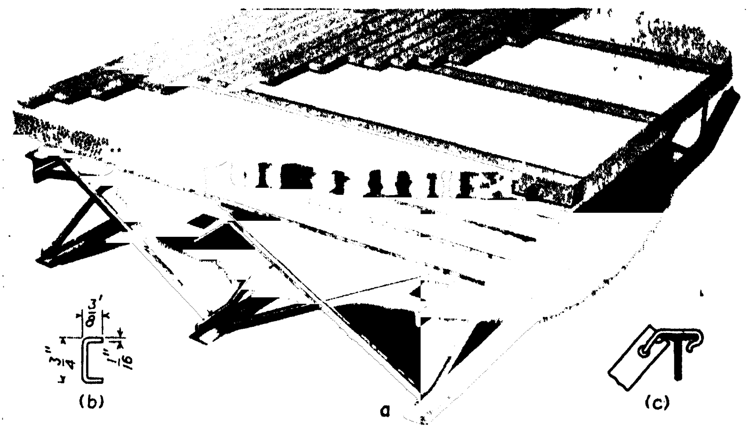


FIG. 101.—(a) Steel-joist floor. (b) Bridging members. (c) Wire clips.

Fig. 101*a*. The bridging members are light $\frac{3}{4}$-in. channels, as shown in Fig. 101*b*, which are notched at the ends to rest against the chords of the trusses. The channels are fastened to the chords of the trusses by means of No. 7 gauge wire clips, as shown in Fig. 101*c*.

Steel-joist floors are economical for light occupancies, but they are unsuitable for heavy or vibrating loads. They are easily installed and the cost of construction is low, because no forms are required for the concrete slab. Ceiling lath for plaster is attached directly to the lower chords of the trusses. Since the webs of the joists are open, all plumbing and service pipes may be concealed in the construction without interfering with the structural members. The method of design and the essential details are given in Arts. 207 to 209.

182. Concrete Slabs on Steel Beams. Reinforced concrete floors are the usual rule in steel-frame buildings. The slab

may rest on top of the beams or be supported by the lower flanges. If the concrete encases the steel completely, the fire-resisting qualities of the floor are greatly increased. The usual maximum spacing of beams in such construction is about 6 ft. The use

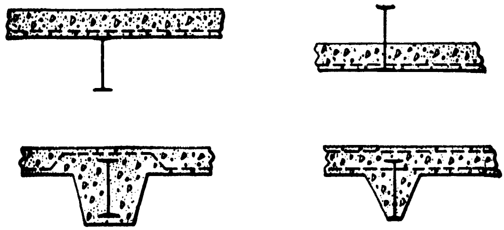


FIG. 102.—Methods of supporting concrete slabs on steel beams.

of one-way floors resting on steel beams is very common for longer spans up to about 20 ft. Illustrations of modifications of this type of construction are given in Fig. 102.

183. Floor Surfaces. A mortar or granolithic finish is probably the most common type of wearing surface for concrete floors. The usual proportions for such a surface are one part Portland cement, one part sand, and one part crushed stone which passes a ¼-in. sieve. A thickness of 1 to 2 in. should be used, depending on the time of pouring, 1 in. being sufficient if the main slab has not been allowed to set thoroughly before the placing of the mortar.

A wooden floor may be provided for if desired, by embedding beveled nailing strips or "sleepers," usually 2 by 4's laid flat,

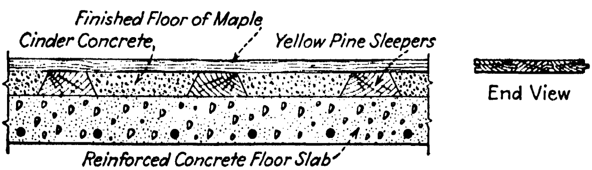


FIG. 103.—Details of a wooden floor on a concrete slab.

in a layer of cinder concrete on top of the main slab, as shown in Fig. 103. A spacing of 16 in. for the sleepers has been found satisfactory for the ordinary floor.

Small vitrified-clay flat tiles embedded in a 2-in. layer of Portland-cement mortar also provide a satisfactory floor surface.

Linoleum placed over a smooth cement base is also frequently used.

The durability of any wearing surface depends in a large measure upon the method of placing the surface. If not properly constructed, the granolithic finish might spall, the wood floor "dry-rot," the tiles curl, and the linoleum crack. In order to insure the maximum degree of serviceability from a given type of floor surface, a special study of methods which have proved successful for laying that particular type of floor should be made.

184. Concrete Roofs. The design of roofs is similar to the design of floors. In addition to the structural requirements, however, the roofs must be impervious to the passage of water, provide for adequate drainage, and furnish protection against condensation.

In order to provide adequate drainage, the roof slab may be pitched slightly, or a filling of some light material such as cinder concrete, covered with a suitable roofing material, may be used, the thickness of the filling being varied so as to give the required slope to the roof surface. The amount of slope required for drainage will depend upon the smoothness of the exposed surface. A value of 1/8 in. per ft. might be used with a surface of hard tile. Felt and gravel roofs should have a pitch of at least 1/4 in. per ft. Some form of flashing is required along the parapets to prevent the drainage from seeping into the building at the edges of the roof slab.

Condensation may best be guarded against by proper ventilation and insulation. The form of insulation to be used will depend upon the particular class of building under consideration. The main types of insulation[1] are:

Roofing felts and quilts.

Cinder fill (with cement finish upon which the roofing is laid).

Cinder-concrete fill covered with roofing.

Hollow tile (with mortar top coat upon which roofing is laid).

Combination hollow tile and cinder fill.

[1] See article on "Prevention of Condensation on Concrete Roof Surfaces," by Albert M. Wolf, *Concrete-Cement Age*, May, 1914.

Double concrete roof (light concrete slab above the main roof slab).

Suspended ceilings.

Imperviousness may best be provided for by the application of some form of separate roof covering, such as a combination of felt and gravel in alternate layers cemented together and to the slab by means of coal-tar pitch or asphalt, vitrified tile embedded in asphalt, or any of the commercial types of built-up roofings. Tin, corrugated iron, or copper roofings are sometimes placed on reinforced concrete buildings but are usually more expensive and less permanent than other types of coverings. If it is not desirable to use any separate roof covering, the main slab may be made reasonably waterproof by the methods mentioned in Art. 17. Such procedure is not recommended except for structures where absolute imperviousness is not essential, because of the difficulty of preventing entirely the formation of shrinkage cracks and the attending seepage.

Walls and Partitions

185. Panel or Curtain Walls. As a general rule, the exterior walls of a reinforced concrete building are supported at each floor by the skeleton framework of the building, their only function being to enclose the building completely. Such walls are called curtain walls or panel walls. They may be made of concrete, brick, concrete blocks, terra-cotta tile blocks, or hollow masonry blocks faced with brick or stone. The thickness will vary according to the material, the type of construction, and the building requirements governing the particular locality where the construction takes place.

The New York City Code (revised 1938) requires that panel walls in skeleton frame construction shall be at least 8 in. thick if such walls are constructed of solid masonry (brick or stone), at least 12 in. thick if of hollow masonry (*i.e.*, hollow concrete blocks or terra-cotta tiles), or at least 10 in. thick if the wall is a combination of hollow and solid masonry, constructed with at least 4 in. of solid masonry on the exterior face. The maximum height between supports of panel walls constructed of the mini-

mum thicknesses specified above shall be 13 ft. When such walls exceed 13 ft. in height, they shall be increased 2 in. in thickness for each 6½ ft., or fraction thereof, of height in excess of 13 ft.

The pressure of the wind is practically the only load that need be considered in determining the theoretical thickness of a reinforced concrete curtain wall. Designed as a slab supported on four sides for a wind pressure of 30 lb. per sq. ft., the walls in buildings of ordinary proportions need only be from 3 to 4 in. in thickness. This is too thin to permit of practical and economical construction and to assure complete protection against seepage and condensation. Most building codes require a minimum thickness of 8 in. for curtain walls of reinforced concrete.[1] The amount of steel necessary is usually governed by the necessity of guarding against the formation of cracks caused by expansion and contraction due to temperature changes. Small bars running horizontally and vertically are placed near each face of the wall and spaced not more than 12 or 18 in. center to center in both directions. On account of the probability of greater temperature variation on the exposed face, more steel should be placed near that face than on the inside unless the lateral pressure requirements govern these amounts.[2] These bars should extend into the columns and wall beams if the walls are poured at the same time as these members. If, as is usually the case, the walls are poured after the columns and beams, anchorage should be

[1] The Joint Code specifies a minimum thickness of 5 in., but not less than one-thirtieth of the distance between supporting or enclosing members.

[2] The Joint Code requires that the reinforcement in each direction (vertical and horizontal) shall have an area at least equal to 0.0025 times the cross-sectional area of the wall if bars are used, or 0.0018 times the area if electrically welded wire fabric is used, with wires not less than No. 10 W. and M. gauge. Walls more than 8 in. in thickness shall have the reinforcement for each direction placed in two layers parallel with the faces of the wall. One layer, consisting of not less than one-half and not more than two-thirds of the total required area, shall be placed not less than 2 in. and not more than one-third the thickness of the wall from the exterior surface. The other layer, comprising the remainder of the required reinforcement, shall be placed not less than ¾ in. and not more than one-third the thickness of the wall from the interior surface. Bars, if used, shall not be less than ⅜ in. in size, nor shall they be spaced more than 18 in. on centers.

provided for by means of dowels projecting from the latter units.[1] It is good practice in such cases to mortise the wall into the columns and wall beams by leaving grooves in the latter members when they are poured. The grooves may be formed by nailing wooden strips on the inside of the forms.

Where a small amount of window area is inserted in a curtain wall, the reinforcement may remain as for a solid wall, but additional bars should be placed near all edges of the openings. Where light is essential, as in a factory, practically the entire wall panel may be enclosed by windows or filled with glass blocks as in Fig. 121, construction then consisting of a wall beam or lintel at each floor and a spandrel between the wall beam and the window sill. Sometimes this spandrel is made of reinforced concrete and constructed as a part of the wall beam. In other instances the concrete spandrel is considered independent of the wall beam, and is reinforced only for temperature stresses. The advantage in using independent spandrels lies in the fact that they may be placed after the structural framework is completed and greater care can be taken in the finishing than would be possible if they were part of the load-bearing skeleton of the structure. The spandrel may also be made of brick or other suitable material.

Where a panel is made up primarily of window space, the New York City Code permits a minimum thickness of 8 in. for the portion of the wall below the window sill and above the window head, whether these portions are made of brick, hollow masonry blocks, or reinforced concrete. The portion of a panel wall between the window sill and the support of the panel wall is called an apron wall in that Code, and the portion above the window head is called a spandrel wall.

186. Bearing Walls. A bearing wall may be defined as one which carries any vertical load in addition to its own weight. Such walls may be constructed of stone masonry, brick, hollow building blocks, or reinforced concrete. Occasional projections

[1] The Joint Code requires that concrete walls shall be anchored to the columns, floors, pilasters, or buttresses with dowels at least ⅜ in. in size, spaced not more than 18 in. on centers, for each layer of wall reinforcement.

or pilasters add to the general appearance and strength of the wall. In small reinforced concrete commercial buildings and residences the bearing-wall type of construction may be used with economy and expediency. In larger commercial and manufacturing buildings where the element of time is an important factor, the delay necessary for the erection of the bearing wall and the attending increased cost of construction often dictate the use of some other arrangement.

Bearing walls may be of either single or double thickness, the advantage of the latter type being that the air space between the walls renders the interior of the building less liable to temperature variations, and makes the wall itself more nearly moisture-proof. On account of the greater gross thickness of the double wall, such construction reduces the available floor space. This feature is often sufficient in itself to warrant the selection of the solid wall unless the factors of condensation and temperature are of great importance. Hollow wall construction is usually limited to a total height of about 40 ft.

The thickness of bearing walls varies with the height. The New York City Building Code requires that, for bearing walls made of solid masonry units (brick, stone, sand-lime or concrete brick, etc.) and not more than 75 ft. in height, the thickness of the uppermost 55 ft. of height shall be at least 12 in. and the thickness below this shall be at least 16 in., except that for walls in buildings four stories in height a 12-in. thickness may be used throughout, and for walls in buildings of one, two, and three stories the top story thickness in each case may be 8 in. In computing wall thicknesses, a maximum story height of 13 ft. shall be assumed. For bearing walls of reinforced concrete, the thickness shall be at least one twenty-fifth of the unsupported height, but not less than 8 in. Reinforcement in concrete bearing walls shall be at least equal to the reinforcement specified in Art. 185 for curtain or panel walls.

According to the Joint Code, the working stress f_c in reinforced concrete bearing walls shall be $0.2f'_c$ for walls having a relation of height h to thickness t of 10 or less and shall be reduced to

$0.11f'_c$ for walls where $\frac{h}{t} = 25$ in accordance with the formula

$$f_c = 0.2f'_c\left(1.3 - 0.03\frac{h}{t}\right)$$

If the wall supports concentrated loads, the length of wall to be considered effective for each load shall not exceed the distance center to center of loads, nor shall it exceed the width of the bearing plus four times the wall thickness. If the wall reinforcement is designed, placed, and anchored in position as for tied columns, such reinforcement may be considered effective in resisting vertical loads in the same manner as for tied columns.

187. Basement Walls. In determining the thickness of basement walls, the lateral pressure of the earth, if any, must be considered in addition to the other structural features. If part of a bearing wall, the lower portion may be designed either as a slab supported by the basement and first floors, or as a retaining wall, depending upon the material used. If columns and wall beams are available for support, each basement wall panel of reinforced concrete may be designed to resist the earth pressure as a simple slab reinforced in either one or two directions.

A minimum thickness of 8 in. is generally specified for reinforced concrete basement walls, with minimum reinforcement as specified in Art. 185 for panel or curtain walls. In wet ground a minimum thickness of 12 in. should be used. In any case, the thickness cannot be less than that of the wall above.

Care should be taken to brace a basement wall thoroughly from the inside if the earth is backfilled before the wall has obtained sufficient strength to resist the lateral pressure without such assistance.

188. Parapet Walls. In the case of buildings with flat roof slabs on which drainage slopes are built, parapet walls are necessary architecturally to give a more finished appearance to the top of the structure, and practically to provide a backing for the drainage slopes. They are usually of brick or concrete or a combination of both. Concrete is, in most cases, preferable from

an economical standpoint. In order to give a better appearance it may have a veneer of brick or terra cotta.

The chief point to be considered in the construction of parapet walls is the necessity of providing adequate reinforcement to prevent cracks caused by excessive temperature changes (on account of exposed position) and expansion and contraction at corners.

The thickness is usually required to be the same as the wall below, but not more than 12 in. in any case. Parapet walls should extend at least 2 ft. above the roof surface.

189. Veneer for Exterior Walls. In order to give an attractive appearance to the building, it sometimes becomes necessary to cover up the entire exterior wall surface with a veneer of brick, stone, terra-cotta tile, marble, or other finishing material. A method of securing a covering of face brick to concrete consists in placing corrugated copper or galvanized iron ties, usually about $\frac{3}{4}$ in. wide and 6 in. long, at frequent intervals in the wall or column forms so that about 4 in. of the tie strip will project into the concrete when poured and the remaining 2 in. wall lie flat against the form and tacked lightly to it. When the form is removed, this latter portion is bent outward and is bonded into the brick veneer by means of the joint mortar. Terra cotta and stone facings are generally supported by ledges in the concrete or by angle irons, and are provided with anchors commensurate with the size of the veneer units.[1]

[1] The New York City Building Code includes the following:

(*a*) For anchorage of brick veneering on masonry, one substantial non-corroding metal wall tie shall be used for each 300 sq. in. of wall surface.

(*b*) For anchorage of architectural terra cotta and other moulded units on masonry, one non-corroding metal anchor, at least $\frac{5}{16}$ in. round or $\frac{1}{8} \times \frac{3}{4}$ in. flat shall be used for each piece, and two or more such anchors shall be used for all pieces over 18 in. in length or more than 300 sq. in. in superficial area.

(*c*) For anchorage of stone veneering on masonry, one non-corroding anchor at least $\frac{3}{16} \times 1$ in. flat shall be used for each piece over $\frac{1}{2}$ sq. ft. in face area, and at least two such anchors shall be used for all pieces over 24 in. in length or more than 400 sq. in. in superficial area.

(*d*) Veneering shall not be included in calculating bearing-wall

190. Partition Walls. Interior walls used for the purpose of subdividing the floor area may be made of concrete, metal lath and plaster, terra-cotta tile, plaster block, or brick. Reasonably adequate fire protection is afforded by a solid concrete wall 3 or 4 in. thick. If properly reinforced and anchored at the top and bottom, such a wall becomes desirable in nearly every respect. The reinforcement should be similar to that in curtain walls but need not be so great in quantity. Suitable anchorage may be obtained by permitting the vertical rods to project into the floor and ceiling. If it is convenient, as is usually the case, to pour the wall after the structural framework of the building is completed, a groove should be left in the floor and one in the ceiling to receive the partition. Two objectionable features of the solid concrete partition wall are its weight and cost of installation. In buildings where many lives would be endangered by a rapid spread of a fire once started, these objectionable features become insignificant.

The most common form of metal lath and plaster partition consists of some form of vertical steel studding suitably anchored to the floor and ceiling, with metal lath fastened to both sides. Each side is plastered with a mixture of lime and cement mortar, thus forming a hollow wall from 3 to 6 in. thick, which has, if proper bond is secured between the plaster and lath, a fair amount of fire resistance. This form may be modified by filling the space between the plaster sides with cinder concrete, or by omitting the metal lath on one side of the vertical studding and plastering both sides of the remaining sheet of lath. This results in a solid wall, usually about 2 in. thick, the reliability of which is rather uncertain. All openings should be framed with steel sections to which the wood frames or other trim may be fastened.

Terra-cotta tile partitions are usually made of blocks from 4 to 6 in. thick, although the blocks may be obtained in thicknesses varying from 2 to 12 in. This type of partition is light in weight, and satisfactory under ordinary conditions. The blocks may

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thicknesses.

(e) The maximum height of veneering on walls other than panel or curtain walls in buildings shall be 40 ft. above the foundations.

be plastered on one or both sides, the thickness of wall being increased by about 3/4 in. for each plastered side.

Partitions made of plaster blocks usually vary from 4 to 8 in. in thickness. The blocks are made of gypsum or plaster of paris, with an admixture of cinders, asbestos fiber, wood chips or vegetable fiber, and laid in gypsum plaster or cement mortar tempered with lime. They are light and easy to handle and place, but offer decidedly poor resistance to the action of fire and water.

The main use of brick inner walls is for the enclosing of stairs and elevator shafts, and in fire walls, the express purpose of which is to divide the floor area into sections to prevent the spreading of fire from one part of the structure to another. Reinforced concrete partitions are also used for the same purpose. These, as well as all other permanent partitions, should be independently supported at each floor on the fire-proof construction of the floor. The use of partitions of pressed metal and glass or of wood and glass should be restricted to the subdivision of rooms or spaces enclosed by fire-proof partitions.

STAIRWAYS

191. Types of Concrete Stairs. The simplest form of reinforced concrete stairway consists of an inclined slab supported at the ends upon beams, with steps formed upon its upper surface. Such a stair slab is usually designed as a simple slab with a span equal to the horizontal distance between supports. This method of design requires steel to be placed only in the direction of the length of the slab. Transverse steel, usually one bar to each tread, is used only to assist in distribution of the load and to provide temperature reinforcement. It sometimes becomes necessary to include a platform slab at one or both ends of the inclined slab. Many successful designs made as outlined above for the simple inclined slab indicate that the effect of the angle that occurs in a slab of this type can safely be disregarded.

It is advisable to keep the unsupported span of a stair slab reasonably short. If no break occurs in the flight between floors, intermediate beams, supported either by the structural framework of the building, or by additional short posts from the floor

below, may be employed. If the stair between floors is divided into two or more flights, beams as described above may be used to support the intermediate landing, and these in turn supported as above for the long straight flight, or the intermediate slab may be suspended from a beam at the upper floor level by means of rod hangers. Where conditions permit, the intermediate slab may be supported directly by the exterior walls of the building.

When it becomes necessary to use a stair slab of comparatively great length with no possibility of intermediate support, inclined side beams may be used. The stair slab is then reinforced with transverse bars to carry the load to the side beams, which are designed to transmit this load to the floor beams. This is normally not an economical form of construction.

192. Beams around Stair Wells. In buildings with flat-slab floor construction, it will be necessary to insert special beams at each floor level around the stairway opening as shown in Figs. 130 and 132, properly to support the stair slabs. In buildings with beam-and-girder floors, the regular floor beams, with dimensions modified as necessary, may be used to support the stair slabs, or special beams may be inserted for that purpose. In any case, the arrangement of beams must be such as to insure proper transmission of all loads to the columns or walls.

193. Building Code Requirements. The required number of stairways, and many of the details are governed to a large extent by the provisions of the governing building code. Among other things, these provisions stipulate the maximum distance from the most remote point in the floor area to the stairway, the minimum width of stairway, the maximum height of any straight flight, the maximum height (or rise) of a single step, the minimum distance (the run) between the vertical faces of two consecutive steps, and the required relation between the rise and the run to give safety and convenience in climbing.

In most codes the minimum width of any stair slab and the minimum dimension of any landing are about 44 in. The maximum rise of a stair step is usually specified as about $7\frac{3}{4}$ in., and the minimum run or tread width, exclusive of nosing, is $9\frac{1}{2}$ in. A rise of less than $6\frac{1}{2}$ in. is not considered generally satisfactory.

In order to give a satisfactory and comfortable ratio of rise to run, various rules have been adopted. One requires that for steps without nosings, the sum of the rise and run shall be 17½ in., but the rise shall not be less than 6½ in. or more than 7¾ in. The New York City Building Code requires that run and rise shall be so proportioned that the product of the run, exclusive of nosing, and the rise in inches, shall be not less than 70 or more than 75, but risers shall not exceed 7¾ in. in height, and treads, exclusive of nosing, shall be not less than 9½ in. wide.

The maximum height of a straight flight between landings is generally given as 12 ft., except for stairways serving as exits from places of assembly, where a maximum of 8 ft. is normally specified.

The number of stairways is governed by the width of the stair slab, the number of probable occupants on each floor, and the dimensions of the floor area. One typical code specification is that the distance from any point in an open floor area to the nearest stairway or exit shall not exceed 100 ft.; that the corresponding distance along corridors in a particular area shall not exceed 125 ft.; and that the combined width of all stairways in any story shall be such that the stairs may accommodate at one time the total number of persons occupying the largest floor area served by such stairs above the floor area under consideration on the basis of one person for each 22 in. of stair width and one and one-half treads on the stairs, and one person for each 3½ sq. ft. of floor area on the landings and halls within the stairway enclosure.

If the exact number of probable occupants is not known, the codes usually assume that there will be two occupants for every 10 sq. ft. of floor area in dance halls and places of assembly; one person for every 25 sq. ft. of floor area in stores, lodging houses, and reading rooms; one person for every 50 sq. ft. of floor area in offices and show rooms; one person for every 100 sq. ft. in hospitals, hotels, and residence buildings; and one for every 150 sq. ft. in warehouses and garages.

In all buildings over 40 ft. in height, and in all mercantile buildings regardless of height, the required stairways must be

completely enclosed by fire-proof partitions, and at least one stairway must continue to the roof. An open ornamental stairway can be used from the main entrance floor to the floor next above, provided it is not the only stairway. The live load usually specified for use in the design of stairways is 100 lb. per sq. ft. of horizontal area.

194. Construction Details. The usual practice is to construct the stairways after the main structural framework of the building has been completed, in which event recesses should be left in the beams to support the stair slab, and dowels should be provided to furnish the necessary anchorage. Occasionally, however, the stairs are poured at the same time as the floors, in which event negative-moment reinforcement should be furnished over the supports of the stair slab, as in any continuous beam construction. The steps are usually poured monolithically with the slab, but they may be molded after the main slab is in place. In the latter instance, provision must be made for securing the step to the slab. The nosing, where used, may be constructed by offsetting the upper portion of the vertical form of the step. A satisfactory wearing surface for the upper face of the step may be obtained by finishing with a 1-in. layer of cement mortar. Metal or slate treads embedded in the concrete are often used for a wearing surface. A complete design of a concrete stairway, with details of construction, is given in Art. 230.

Design of a Beam-and-girder Floor

195. Data and Specifications. In order to illustrate the application of the principles of reinforced concrete design to the design of a reinforced concrete floor of the beam-and-girder type, let it be required to design a typical interior floor bay to sustain a live load of 200 lb. per sq. ft. The columns supporting the floor are to be spaced 21 ft.-0 in. center to center in one direction and 23 ft.-0 in. center to center in the other direction. The beams span the 23-ft.-0-in. direction, and are placed one at each column and one at each third point of the supporting girders, thus making the distance center to center of beams 7 ft.-0 in. A 1-in. granolithic finish is to be included in the dead load on the slab,

but this finish is not to be considered as part of the effective depth of the slab. The allowable unit stresses are to be as specified in the Joint Code for a 3000-lb. concrete and intermediate grade steel. The span length of the slab, the beams, and the girders will be taken as the distance center to center of supports. The resulting design is conservative, since good practice ordinarily permits the use of clear spans in continuous beam constructions. The general arrangement of beams is shown in Fig. 107.

196. Design of Slab. Assuming the weight of slab and finish as 60 lb. per sq. ft., the total load is 260 lb. per sq. ft. and, for a 12-in. width of slab,

$$M = \tfrac{1}{12} \times 260 \times 7^2 \times 12 = 12{,}800 \text{ in.-lb.}$$

From Table 6, $k = 0.375$, $j = 0.875$, and $K = 197$.

$$d = \sqrt{\frac{12{,}800}{197 \times 12}} = 2.3 \text{ in.}$$

Selecting $d = 3$ in., and allowing 1 in. of insulation below the center of the steel, the total weight of the slab and finish is 62 lb. per sq. ft., which agrees closely with the assumed value. A thickness of slab less than 4 in. is not advisable in ordinary building construction.

The area of steel required for a 12-in. width of slab is

$$A_s = \frac{12{,}800}{20{,}000 \times 0.875 \times 3.0} = 0.25 \text{ sq. in.}$$

This is furnished by $\tfrac{3}{8}$-in. round bars 5 in. center to center.

One method of providing for continuity in the slab is to bend alternate bars up over the supports and to continue the remaining bars straight through the supports, as shown in Fig. 104, terminating the bars beyond the supports *B*2. This method furnishes only one-half as much steel area for negative moment at the supports *B*1 as for positive moment and is, therefore, not to be regarded as theoretically perfect. By adding $\tfrac{3}{8}$-in. round bars, 21 ft.-0 in. long, at 10 in. on centers in the top of the slab between the supports *B*2, the area of tension steel at the supports

*B*1 would be sufficient. The bars should be bent up approximately at the quarter points of the slab span.

In order to prevent shrinkage and temperature cracks, ⅜-in. round bars will be placed parallel to the beams about 18 in. apart, four in each slab panel. These bars also assist in distributing

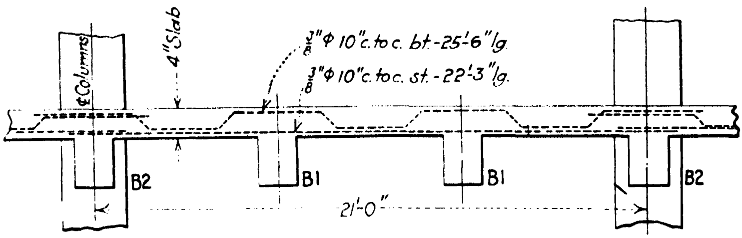


FIG. 104.

the load on the slab over a comparatively large width, and assist in binding the entire structure together.

197. Design of Beams. *Selection of Stem.* Since the slab and beams are poured at the same time and thoroughly bonded together, the latter may be designed as T-beams. The span of the beams is 23 ft.-0 in. The total load from the slab per linear foot = 7 × 260 = 1820 lb. Assuming the weight of the stem of the beam as 150 lb. per ft., the total load on the beam per linear foot is 1820 + 150 = 1970 lb.

$$M = \tfrac{1}{12} \times 1970 \times 23^2 \times 12 = 1{,}042{,}000 \text{ in.-lb.}$$
$$V = 1970 \times \tfrac{23}{2} = 22{,}600 \text{ lb.}$$
$$b'd = \frac{22{,}600}{\tfrac{7}{8} \times 180} = 144 \text{ sq. in.}$$
$$\text{If } b' = 8 \text{ in.,} \quad d \text{ (required)} = 18.0 \text{ in.}$$
$$\text{If } b' = 10 \text{ in.,} \quad d \text{ (required)} = 14.4 \text{ in.}$$

When the bars in beam *B*2 (see Fig. 107) are bent up over the support to provide for the negative moment there, they must be at a different distance from the top of the slab than the bent bars in girders *G*1, so as to allow one set to pass over or under the other set. In the present design the beam bars will pass below the girder bars; an insulation of 2½ in. to the center of the uppermost row of beam bars at the support will be used, with a distance of

2 in. vertically between the upper and lower rows. The center of the uppermost row of the girder bars can then be placed $1\frac{1}{2}$ in. from the top of the slab, which is sufficient insulation at this surface; with this arrangement the beam bars and girder bars will pass each other without interference.

The effective cross-section required for shear, as computed above, is the cross-section at the support, the value of d being measured from the bottom of the beam (the compression face) upward to the center of the top steel (the tension bars). In order to keep the ratio of $\frac{b'}{d}$ within the limits of $\frac{1}{2}$ to $\frac{1}{3}$, an effective section of 8 by 18 in. is selected. Allowing for two rows of bars, arranged at the support as indicated above, the total depth of the beam is $18 + 3.5 = 21.5$ in., and the depth below the slab is $21.5 - 4 = 17.5$ in. The weight of the stem is then 146 lb., which agrees closely with the assumed weight.

Design at Center. At the center of the beam an insulation of $2\frac{1}{2}$ in. to the center of the lowermost row of bars will be used, with 2 in. center to center vertically between the two rows. The effective depth at the center is then $21.5 - 3\frac{1}{2} = 18$ in. Assuming $jd = d - \frac{1}{2}t$, the area of steel required at the center is

$$A_s = \frac{1{,}042{,}000}{20{,}000(18.0 - 2.0)} = 3.26 \text{ sq. in.}$$

This is furnished by six $\frac{7}{8}$-in. round bars, the area of which is 3.61 sq. in. As explained in Art. 86, some excess of positive-moment steel is advisable in order to provide for the negative moment at the support, the required tension steel at the latter point being greater on account of the difference in values of j between the center and the support.

Design at Support. Three bars from each beam are bent up and carried over the support to the third point of the adjoining span. The three remaining bars of each beam are carried straight through the support into the adjoining span far enough to develop their strength in bond. This arrangement furnishes a total of six bars in both the top and the bottom over the support. The effective depth of the tension steel at the support is 18.0 in., as

indicated above, and the distance from the compression face to the center of the compression steel (*i.e.*, the value of d') is $3\frac{1}{2}$ in.

$$\frac{d'}{d} = \frac{3.5}{18} = 0.20, \qquad \text{and} \qquad np = np' = 10 \times \frac{3.61}{8 \times 18} = 0.250$$

From Diagram 6, $k = 0.422$ and $j = 0.838$.

$$f_s = \frac{1{,}042{,}000}{3.61 \times 0.838 \times 18} = 19{,}100 \text{ p.s.i.}$$

$$f_c = \frac{19{,}100 \times 0.422}{10(1 - 0.422)} = 1390 \text{ p.s.i.}$$

$$f'_s = 19{,}100 \times \frac{0.422 - 0.20}{1.0 - 0.422} = 7340 \text{ p.s.i.}$$

The allowable unit stress in the concrete at the support is $0.45f'_c = 1350$ p.s.i. The actual stress is about 3 per cent greater than this, but the design may be considered satisfactory because of the conservative assumption that was made in selecting the effective span.[1]

Arrangement of Reinforcement. The maximum unit bond stress on the tension bars is

$$u = \frac{22{,}600}{6 \times 2.749 \times \frac{7}{8} \times 18} = 87 \text{ p.s.i.}$$

The unit shear at the support is

$$v = \frac{22{,}600}{8 \times \frac{7}{8} \times 18} = 180 \text{ p.s.i.}$$

One bar from the middle of the upper row will be bent up first, and then the two outer bars in that row will be bent, as shown in Fig. 105. From Diagram 1, the first bar ($16\frac{2}{3}$ per cent of the total steel area) can be bent up at point s, Fig. 105, which is at a distance of $0.34 \times 23 \times 12 = 94$ in. from the center of the support, and the next pair of bars can be bent at point r, which is $0.21 \times 23 \times 12 = 58$ in. from the support.

Assuming that the negative moment is zero at the one-third point of the span and that the negative-moment curve is a

[1] See also Art. 101.

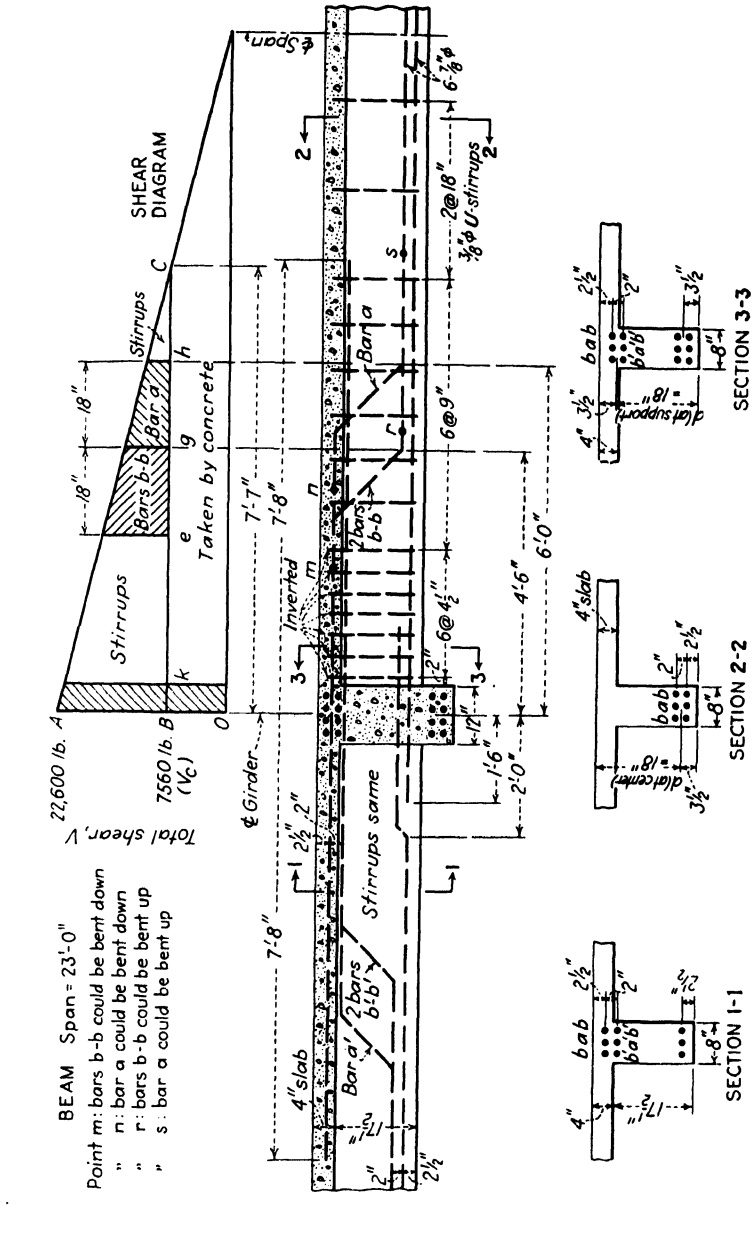


Fig. 105.—Details of beam.

straight line from the zero point to the maximum ordinate at the support, in order to provide fully for this moment at all points the single bar must reach the top of the beam not closer to the center of the support than

$$\frac{1}{2} \times \frac{23 \times 12}{3} = 46 \text{ in. (point } n\text{, Fig. 105)}$$

and the pair of bars must reach the top not closer to the support than

$$\frac{1}{3} \times \frac{23 \times 12}{3} = 31 \text{ in. (point } m\text{, Fig. 105)}$$

Diagonal Tension. The amount of the external shear that can be resisted by the concrete is $V_c = 60 \times 8 \times 7/8 \times 18 = 7560$ lb. From equation (13), Art. 76, the distance from the support over which web reinforcement is required is

$$x_1 = 23/2 - 7560/1970 = 7.6 \text{ ft. or } 91 \text{ in.}$$

This distance is also determined graphically in the shear diagram, Fig. 105, the distance BC being the required distance.

The bars will be bent up at 45 degrees and as close to the support as the bending stress requirements will permit, because of the greater shearing stresses there. The maximum distance over which the single bar can resist diagonal tension stresses without overstressing the bar in tension, from equation (15*a*), Art. 79, is

$$s = \frac{0.6013 \times 20{,}000 \times 7/8 \times 18}{0.7(V - 7560)}$$

in which V is the total external shear at the place where the bar is bent. Subsequent computations, made after the points of bending are selected, show that the distance s from the above equation is greater than the arbitrary maximum distance over which this bar can be assumed to resist diagonal tension stresses.[1] The latter distance, according to the specification in Art. 79 and Fig. 36, is 18 in.

[1] At point h, $V = 22{,}600 - 6 \times 1970 = 10{,}780$ lb., and $s = 84$ in.

The points at which the bars are bent are shown in Fig. 105, the distance between points of bending being 18 in. Bar *a* will then provide for diagonal tension over the distance *hg* in Fig. 105, and the two bars *b* will provide for it over the distance *ge*. Vertical stirrups are required in the portions *ke* and *hc*.

The maximum allowable spacing of $\frac{3}{8}$-in. round U-stirrups at the support is

$$s = \frac{2 \times 0.1104 \times 20{,}000 \times \frac{7}{8} \times 18}{22{,}600 - 7560} = 4.6 \text{ in.}$$

and at point *h*,

$$s = \frac{2 \times 0.1104 \times 20{,}000 \times \frac{7}{8} \times 18}{22{,}600 - 6 \times 1970 - 7560} = 21.5 \text{ in.}$$

The arbitrary maximum allowable spacing (see Art. 78) is $0.5 \times 18 = 9$ in.

The first stirrup is placed about 2 in. from the edge of the supporting girder. Throughout the distance *ke* a $4\frac{1}{2}$-in. spacing will be used, and in the distance *hC* a 9-in. spacing will be used. Stirrups will also arbitrarily be placed at 9 in. on centers in the distance *eh*. The latter stirrups are not required, nor would they necessarily be adequate if the bent bars were not available; but they do add an element of resistance which more than compensates for the comparatively small cost of the few extra stirrups involved. Conservative designers would also place stirrups throughout the central portion of the beam, at a spacing approximately equal to the effective depth of the beam, to assist in binding the web and flange together. These stirrups are shown in Figs. 105 and 107.

The bent bars are continued to the point of zero negative moment, assumed $\frac{1}{3} \times 23 \times 12 = 92$ in. from the girder center. The straight bars must be continued beyond the support for a distance of $\dfrac{7340}{4 \times 150} \times \dfrac{7}{8} = 11$ in. (see Art. 42), but they will be continued 1 ft.-6 in. beyond the center of the girder. The steel details are shown in Figs. 105 and 107.

198. Design of Girder. The girder has a span of 21 ft.-0 in., with concentrated loads of $2 \times 22{,}600 = 45{,}200$ lb. at each of the one-third points. The maximum moment due to the concentrated loads is

$$M_1 = \tfrac{2}{9} \times 45{,}200 \times 21 \times 12 = 2{,}530{,}000 \text{ in.-lb.}$$

Assuming the weight of the stem of the girder as 300 lb. per lin. ft., the maximum moment due to the uniform load is

$$M_2 = \tfrac{1}{12} \times 300 \times 21^2 \times 12 = 132{,}000 \text{ in.-lb.}$$

and the total maximum moment M is 2,662,000 in.-lb.

The total maximum shear is $45{,}200 + 300 \times 2\tfrac{1}{2} = 48{,}300$ lb.

$$b'd = \frac{48{,}300}{\tfrac{7}{8} \times 180} = 306 \text{ sq. in.}$$

Taking into consideration space for bars, economical depth, headroom, etc., the width of the stem is made 12 in., and the effective depth required is 25.5 in.

With the arrangement of steel proposed in the design of the beams, the center of the upper row of girder steel at the support is $1\tfrac{1}{2}$ in. from the top of the slab, and the vertical distance center to center of rows is 2 in. At the center of the girder an insulation of $2\tfrac{1}{2}$ in. below the center of the lower row of steel is provided, and the vertical distance center to center of rows is 2 in. The effective depth at the support (which is governed by the shear requirement) is therefore 1 in. greater than that at the center. With an effective depth at the center equal to 24.5 in. the value of d at the support is 25.5 in. which provides for the shear as computed above, and the total height of the girder, assuming two rows of steel, is $24.5 + 3.5 = 28.0$ in. The depth below the slab is 24 in., and the weight of the stem is 300 lb. per ft. as assumed.

The approximate required steel area at the center of the span is

$$A_s = \frac{2{,}662{,}000}{20{,}000 \times (24.5 - \tfrac{4}{2})} = 5.92 \text{ sq. in.}$$

Eight 1-in. round bars, furnishing 6.28 sq. in., are selected.

In order to provide for the negative moment, four bars from each girder are bent up and carried over the support to the point of inflection in the adjoining span. In addition, two more bars from each side are bent up and hooked into the column merely to assist in resisting diagonal tension stresses. The remaining bars are carried straight through the support a sufficient distance to develop their strength in bond. This arrangement furnishes eight 1-in. round bars in tension and four 1-in. round bars in compression at the support. The effective depth at the support is 25.5 in., and the value of d' is 2.5 in. since only one row of steel remains at the bottom at the support (see Fig. 106).

Investigating the unit stresses over the support,

$$\frac{d'}{d} = \frac{2.5}{25.5} = 0.10 \qquad np = 10 \times \frac{6.28}{12 \times 25.5} = 0.205,$$

and

$$np' = \tfrac{1}{2}np = 0.103$$

From Diagram 4, $k = 0.417$ and $j = 0.870$.

$$f_s = \frac{2,662,000}{6.28 \times 0.870 \times 25.5} = 19,200 \text{ p.s.i.}$$

$$f_c = 19,200 \times \frac{0.417}{10(1 - 0.417)} = 1370 \text{ p.s.i.}$$

$$f'_s = 19,200 \times \frac{0.417 - 0.10}{1 - 0.417} = 10,400 \text{ p.s.i.}$$

The maximum unit bond stress on the tension bars at the point of maximum shear is

$$u = \frac{48,300}{25.1 \times \frac{7}{8} \times 25.5} = 86 \text{ p.s.i.}$$

The inclined bars are so placed as to take as much of the diagonal tension as possible and thus keep the number of stirrups required a minimum.

The amount of external shear that can be resisted by the concrete is $V_c = 60 \times 12 \times \frac{7}{8} \times 25.5 = 16,000$ lb. at the support and $V_c = 60 \times 12 \times \frac{7}{8} \times 24.5 = 15,400$ lb. away from the

support. The shear diagram (Fig. 106) shows that web reinforcement is required only from the support to the first concentrated load.

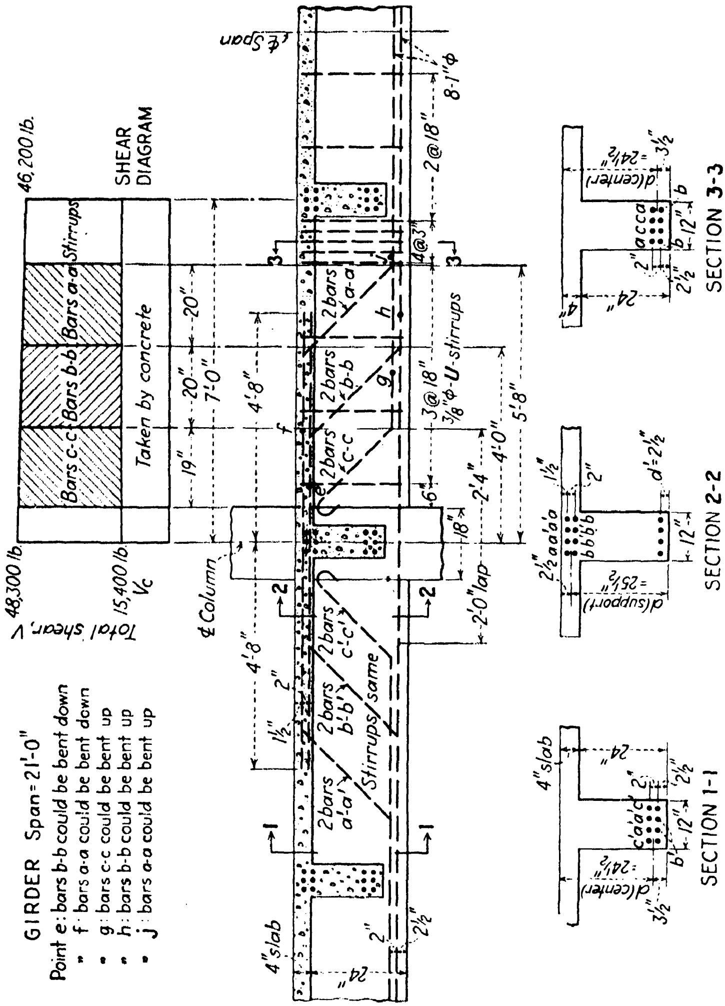


Fig. 106.—Details of girder.

The points where the bars may be bent up, in pairs, are determined by assuming that the positive moment is zero at a distance of $\frac{2}{3} \times l/3$ from the concentrated load nearest the support, measured toward the support, and that the moment diagram is a straight line between the maximum and zero values. The latter

assumption is in error only because of the uniformly distributed weight of the stem of the girder, and this is very small in comparison with the large concentrated loads. The first pair of bars may be bent up at a distance of $\frac{1}{4} \times \frac{2}{3} \times \frac{21 \times 12}{3} = 14$ in. to the left of the first concentrated load, or at point j in Fig. 106. The next pair may be bent at a distance of 28 in. from the first load, or at point h, and the third pair may be bent at point g, which is 42 in. from the first load.

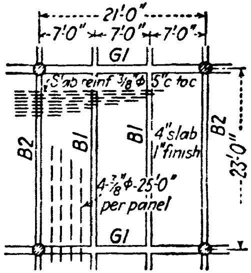
A similar assumption for the negative-moment variation can be made as for positive moment, the point of zero negative moment being, therefore, $\frac{2}{3} \times l/3 = 56$ in. from the support. The overlapping of the positive- and negative-moment diagrams which results from these assumptions provides for any variation in the moments that might result from any unequal placing of the live load on adjoining spans. Assuming that the eight bars over the support are stressed equally, one pair may be bent down at a distance of $\frac{1}{4} \times 56 = 14$ in. from the support, or at point e in Fig. 106, and another pair may be bent down 28 in. from the support, or at point f.

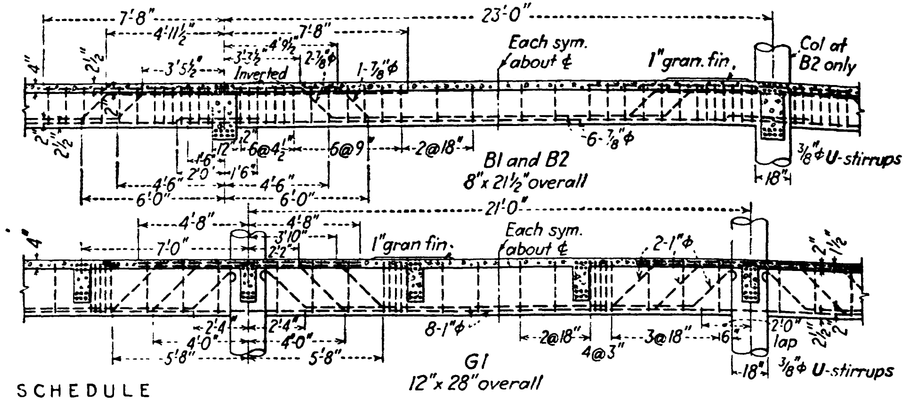
In order to stress the bent-up bars equally, they are bent up at about equal spacings, since the shear diagram, which is a measure of diagonal tension, is approximately a rectangle. Taking into account the allowable points of bending and assuming the width of column to be 18 in., inside of which no web reinforcement is required, the bars are bent, at 45 degrees, as shown in Fig. 106. It should be noted that the arbitrary maximum spacing of $d = 24.5$ in. has not been exceeded and that the pairs of bars are not overstressed by the inclined tension (see Art. 79).

Stirrups are required between the concentrated load and the point of bending of the first pair of bars, the required spacing being governed by the external shear at the point m. With $\frac{3}{8}$-in round U-stirrups.

$$s = \frac{2 \times 0.1104 \times 20{,}000 \times \frac{7}{8} \times 24.5}{(48{,}300 - 5.67 \times 300) - 15{,}400} = 3 \text{ in.}$$

It would be advisable to use $\frac{3}{8}$-in. round U-stirrups, spaced at





STEEL BENDING SCHEDULE

SIZE	LENGTH	MARK	NO PER BEAM	NO PER PANEL	DETAILS
7/8"φ	26'-0"	B1 B2	3 3	9	22'-6" 2" 3'-4" 2⅞ o to o
7/8"φ	39'-3"	B1 B2	1 1	3	38'4" overall 12'5½" 11'0" 12'7½" 1'2½" 1'0½"
7/8"φ	39'-3"	B1 B2	2 2	6	38'4" overall 10'11½" 14'0" 11'1½" 1'2½" 1'0½"
1"φ	26'-6"	G1	4	4	1'5" 2'0" 13'0" 1'10" 8'6" 1'1" o to o 1'5"
1"φ	28'-6"	G1	4	4	27'-8" overall 19'0" 6'10" 1'10" 1'11" o to o

STIRRUP SCHEDULE

SIZE	LENGTH	MARK	NO. PER BEAM	NO PER PANEL	DETAILS
3/8"φ	4'-0"	B1 B2	16 16	48	18" o to o 4½" INSIDE 4"
3/8"φ	4'-0"	B1 B2	14 14	42	18" o to o 4½" INSIDE 4"
3/8"φ	5'-6"	G1	20	20	25" o to o 8½" INSIDE 4"

FIG. 107. Details of beam-and-girder floor panel.

about 18 in., over the remaining portions of the girder to assist in securing unity of action of the two parts of the T-beam, as shown in Figs. 105 and 107.

The bent bars are continued 56 in. beyond the center of the column; the straight bars should be continued for a distance beyond the support at least equal to

$$\frac{10{,}400}{4 \times 150} \times 1 = 18 \text{ in.}$$

Complete details for the typical bay designed above are shown in Fig. 107.

Design of a Ribbed Floor with Steel-tile Fillers

199. Dead Loads. The weight of a ribbed floor can be computed from the known concrete dimensions. The general form of the cross-section of the steel tiles must be known, since the

Weight of Concrete in Ribbed Floors with Steel-tile Fillers

Depth of joist below slab, in.	Thickness of slab, in.	Average weight of floor, lb. per sq. ft.
4	2 2½	37 44
6	2 2½	44 50
8	2 2½	50 56
10	2 2½	56 62
12	2 2½ 3	62 68 73
14	2 2½ 3	73 79 85

taper of the sides and the chamfering of the upper corners affect the volume of concrete in the ribs or joists. The cross-section can be obtained from the manufacturers' catalogues. The average values in the table on page 313 can be used for joists (or ribs) 5 in. wide at the bottom and 24 in. on centers. These values assume that the steel cores will be removed. If permanent cores are used, from 1 to 2 lb. per sq. ft. should be added.

The values given in this table do not include the weight of an extra floor finish, or the weight of a plastered ceiling below the floor. The former can be computed from the specified thickness of finish or type of floor surface. In computing design loads, an allowance of 10 lb. per sq. ft. is usually made for a plastered ceiling. An additional allowance of from 10 to 20 lb. per sq. ft. is made for the weight of partitions, when these partitions are not definitely located on the architect's plans, or where there is a possibility of future rearrangement of partitions. The latter condition is very likely to occur in buildings of the types to which ribbed-floor construction is adapted. When definitely located partitions are parallel with the joists, a thicker joist than the normal 5-in. joist is usually placed under these partitions, or, if the partition is located between two joists, both of these joists are made thicker than the others and the slab thickness between them is also increased. The increased slab thickness is obtained by using shallower tiles in the one row, or by lowering the tiles in that row.

200. Data for Design. A typical interior panel of a hotel floor is to be built as a ribbed floor, using removable steel-tile cores. The joists are to be supported at the ends on concrete girders, as shown in Fig. 108. The span of the girders is 23 ft.-0 in., and the distance center to center of girders is 18 ft.-0 in. The live load is 60 lb. per sq. ft., and allowances for dead loads other than the weight of the concrete are to be made as follows: partitions, 10 lb. per sq. ft.; plastered ceiling, 10 lb. per sq. ft.; wood floor, with sleepers in cinder-concrete fill, 15 lb. per sq. ft. A 2000-lb. concrete is to be used in the floor, and structural grade deformed bars, for which f_s = 18,000 p.s.i., are to be used for reinforcement. It is required to design the panel.

201. Design of Joists. *Computation of Moments and Shears.* The joists are designed as regular continuous T-beams, with a flange width equal to the distance center to center of joists. In order to compute the weight of the floor, the depth of the joists below the slab and the thickness of the slab must be assumed. A combination of 8-in. joists and 2-in. slab will be tried. The weight of the concrete per square foot of floor is 50 lb., and the total load is $60 + 10 + 10 + 15 + 50 = 145$ lb. per sq. ft. With 5-in. joists, spaced 25 in. on centers, each joist supports $25/12 = 2.08$ sq. ft. of floor per foot of joist, and the total load on each joist is $145 \times 2.08 = 302$ lb. per lin. ft. The panel is fully continuous, and the effective span for moment is equal to the clear distance between the stems of the girders. If the width of the girder stems is assumed as 12 in., the joist span is $18 - 12/12 = 17$ ft. The maximum moment is, therefore,

$$M = 1/12 \times 302 \times 17^2 \times 12 = 87{,}300 \text{ in.-lb.}$$

In computing the maximum shear in the joists, the effective span can be taken as the clear distance between the flanges of the girders which support the joists. If, as in Fig. 108, the girder-flange width is assumed as 18 in., the effective span of the joists is $18 - 18/12 = 16.5$ ft., and the maximum shear is

$$V = 302 \times \frac{16.5}{2} = 2492 \text{ lb.}$$

Design for Shear. It is rather difficult to place stirrups in narrow joists, such as are used in this form of construction, hence the shearing unit stress should be kept below the maximum allowable value of $0.02 \times 2000 = 40$ p.s.i. The effective shearing width of a joist should be taken as the width at the bottom of the joist. If necessary, tapered tiles can be used at the ends of each row to increase the shearing area, or deeper tiles can be used to serve the same purpose. With 8-in. joists under a 2-in. slab, the effective depth is $10 - 1\frac{1}{4} = 8.75$ in., which will allow for a minimum clear insulation under the bars of about $\frac{3}{4}$ in.

If straight end tiles are used, the approximate maximum unit shearing stress is,

$$v = \frac{V}{b'jd} = \frac{2492}{5 \times 7/8 \times 8.75} = 65 \text{ p.s.i.}$$

This is greater than the allowable stress, and tapered end tiles, or deeper tiles, must be used. It should be remembered that the steel area required decreases with an increase in the depth of the joist, and, in determining whether to use tapered tiles or deeper tiles, both the cost of the steel and the cost of the concrete should

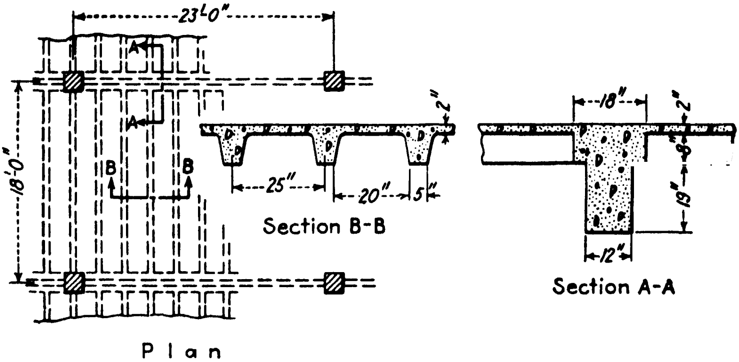


FIG. 108.—Details of ribbed floor with removable steel tiles.

be considered. The cost of the forms can be omitted in the comparison, because this cost does not vary materially.

Tapered end pieces with a taper of 4 in. in a length of 36 in. will be used in this design, mainly for the purpose of illustrating the effect on the design. The effective shearing width at the end of the joist is 5 + 4 = 9 in., and the approximate maximum unit shearing stress is

$$v = \frac{2492}{9 \times 7/8 \times 8.75} = 36 \text{ p.s.i.}$$

which is less than the allowable stress. At the beginning of the taper, which is another critical section for shear, the unit shearing stress is

$$v = \frac{2492 - 36/12 \times 302}{5 \times 7/8 \times 8.75} = 41 \text{ p.s.i.}$$

This is also satisfactory.

Computation of Steel Area. The approximate required area of steel reinforcement in each joist is

$$A_s = \frac{M}{f_s(d - \frac{1}{2}t)} = \frac{87{,}300}{18{,}000(8.75 - \frac{2}{2})} = 0.625 \text{ sq. in.}$$

Two ⅝-in. round bars furnish an area of 0.614 sq. in., and will be selected temporarily.

Investigation at Mid-span. It is necessary to review the joist at the center, in order to make sure that the compression stress in the concrete is less than the allowable value of $0.04 \times 2000 = 800$ p.s.i. With $n = 15$, $d = 8.75$ in., $A_s = 0.614$ sq. in., $b = 25$ in., and $t = 2$ in., the values of k and j as obtained from Diagram 2 are 0.251 and 0.918, respectively. Values of f_s and f_c are obtained from equations (23) and (18), as follows:

$$f_s = \frac{87{,}300}{0.614 \times 0.918 \times 8.75} = 17{,}700 \text{ p.s.i.}$$

$$f_c = \frac{17{,}700 \times 0.251}{15(1 - 0.251)} = 399 \text{ p.s.i.}$$

Both of these stresses are satisfactory.

Investigation at Support. One bar in each joist will be bent up at about the quarter-point of the span, and continued across the support to the point of inflection in the adjoining joist. This point may be taken at a distance from the edge of the stem of the girder equal to one-quarter of the clear span of the joist. The remaining bar is continued straight through the support to a point about 20 bar diameters past the opposite face of the stem of the girder. The joist at the support is thus a doubly reinforced beam, with $A_s = 0.614$ sq. in., $A'_s = 0.614$ sq. in., $b = 9$ in., and $d = 8.75$ in. When tapered tiles are used at the ends of the rows and a T-beam shape is given to the supporting girder, the investigation of the stresses in the concrete and in the steel at the end of the joist can be omitted. If it were desired to carry out this investigation, the method would be the same as in Art. 197.

Investigation of Bond Resistance. The critical section for bond in the negative reinforcement is at the edge of the flange of the

supporting girder. The unit bond stress at this section is

$$u = \frac{2492}{2 \times 1.964 \times \frac{7}{8} \times 8.75} = 83 \text{ p.s.i.}$$

The allowable unit bond stress is $0.05 \times 2000 = 100$ p.s.i.

202. Design of Girder. *Computation of Loads.* The loads which are brought to the girder from the joists are theoretically concentrated at the points where the joists frame into the girder. They are spaced so closely, however, that they may be considered as uniformly distributed throughout the length of the girder, without affecting materially the maximum moment in the girder. Each pair of adjoining joists, one on either side of the girder, transmits a load to the girder equal to twice the end shear of one joist, or $2 \times 2492 \times 4984$ lb. Since the joists are 25 in. or 2.08 ft. on centers, the uniform load per foot of girder is $\frac{4984}{2.08} = 2390$ lb. To this must be added the weight of the girder and the floor and ceiling loads directly over and under the flange of the girder. With an assumed cross-section as shown in Fig. 108, the weight of the flange is $\frac{18 \times 10}{144} \times 150 = 188$ lb. per ft., and the weight of the stem is $\frac{12 \times 19}{144} \times 150 = 238$ lb. per ft. The live load, the ceiling load, and the weight of the flooring and partitions on the 18-in. flange width are $(60 + 10 + 15 + 10)^{18}\!/_{12} = 142$ lb. per lin. ft. The total load on the girder is, therefore, $2390 + 188 + 238 + 142 = 2958$ lb. per ft. A value of 3000 lb. per ft. will be used, which will allow for the extra concrete in the tapered ends of the joists.

Computation of Moment and Shear. The girder is continuous at both ends, so that with 18-in. columns, the effective span is $23 - {}^{18}\!/_{12} = 21.5$ ft., and the maximum moment is

$$M = \tfrac{1}{12} \times 3000 \times (21.5)^2 \times 12 = 1{,}387{,}000 \text{ in.-lb}$$

The maximum shear is

$$V = 3000 \times \frac{21.5}{2} = 32{,}250 \text{ lb.}$$

Design at Center. The girder is a T-shaped beam, but, because of the comparatively thick flange, the neutral axis will probably be in the flange and the girder must be designed for moment as a rectangular beam with a width equal to the width of the flange. For shear, the effective width b' is the width of the stem, or 12 in. Web reinforcement will be provided as necessary, and the maximum allowable unit shearing stress is $0.06 \times 2000 = 120$ p.s.i. The effective depth required for shear is

$$d = \frac{V}{vjb'} = \frac{32{,}250}{120 \times \frac{7}{8} \times 12} = 25.6 \text{ in., or } 26 \text{ in.}$$

Allowing for two rows of bars, the overall depth must be $26 + 3 = 29$ in. and the depth below the flange is $29 - 10 = 19$ in., as assumed. With an effective depth of 26 in., the flange width required for moment is

$$b = \frac{M}{Kd^2} = \frac{1{,}387{,}000}{139 \times 26^2} = 15 \text{ in.}$$

The assumed width of 18 in. will be maintained, and no revisions are necessary.

Assuming $j = \frac{7}{8}$, the approximate required steel area is

$$A_s = \frac{M}{f_s jd} = \frac{1{,}387{,}000}{18{,}000 \times 0.875 \times 26} = 3.39 \text{ sq. in.}$$

Eight $\frac{3}{4}$-in. round bars, with an area of 3.53 sq. in. will be assumed.

Investigation at Center. The actual steel ratio p is $\frac{3.53}{18 \times 26} =$ 0.0076, and the corresponding values of k and j (Table 7) are, 0.377 and 0.874, respectively.

Since $kd = 0.377 \times 26 = 9.81$ in. is less than the flange thickness, the neutral axis is in the flange and the rectangular-beam formulas apply, as assumed above. The actual compression stress in the concrete is

$$f_c = \frac{2M}{kjbd^2} = \frac{2 \times 1{,}387{,}000}{0.377 \times 0.874 \times 18 \times 26^2} = 690 \text{ p.s.i.}$$

This is well below the allowable value, as was to be expected, since the flange width furnished is 3 in. greater than required.

The revised required steel area is

$$A_s = \frac{1,387,000}{18,000 \times 0.874 \times 26} = 3.39 \text{ sq. in.}$$

The assumed bars are satisfactory.

Investigation at Support. Four bars will be bent up and continued across the support to the point of inflection in the adjoining girder, which will be assumed at the quarter-point of the clear span of that girder. The other four bars will be run straight through the support for a distance of 1 ft. beyond the center of the column, which will be enough to form a proper splice with the straight bars from the adjoining span. The effective cross-section of the girder at the support is then a doubly reinforced rectangular beam, with $b = 12$ in., $d = 26$ in., $d' = 3$ in., $A_s = 3.53$ sq. in., and $A'_s = \frac{3.53}{2} = 1.76$ sq. in. The moment at the support is assumed the same as that at the center. The stresses in the concrete and in the steel are 755 and 17,300 p.s.i., which are less than the allowable values of 900 and 18,000 lb., respectively.

Investigation of Bond Resistance. The critical section for bond in the negative-moment steel is at the face of the column, where the shear is 32,250 lb., and the sum of the perimeters of the bars is $8 \times 2.356 = 18.85$ in. The unit bond stress is

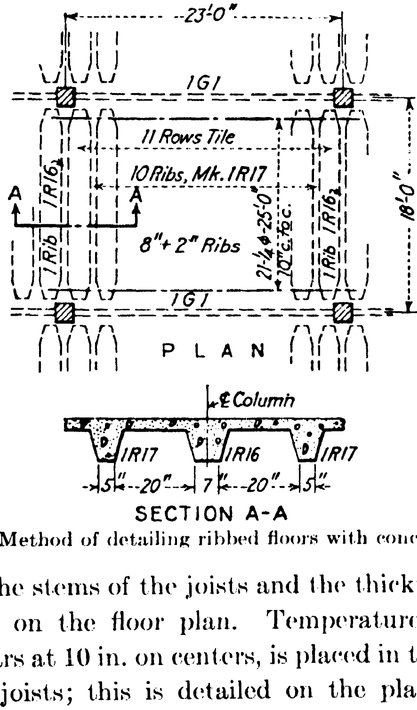
$$u = \frac{32,250}{18.85 \times 7/8 \times 26} = 76 \text{ p.s.i.}$$

This is well below the allowable value of $0.05 \times 2000 = 100$ p.s.i.

Design of Web Reinforcement. Resistance to diagonal tension stresses will be provided by bending up the four bars in pairs and by placing vertical stirrups in the proper positions. The computations are the same as for any uniformly loaded, continuous beam; a typical example is given in Art. 197.

203. Detail Drawings. Each floor plan must show the number of ribs, or joists, in each panel, the number of rows of tiles, and the arrangement of the joists. All identical joists are given the same

mark, which consists of a numeral indicating the floor, the letter *R* or *J* indicating ribs or joists, and a following numeral which is the mark of identity. Thus, 1*R*22 would designate first-floor joist number 22. The outlines of the rows of tile are shown by dotted lines. Where there are several adjacent identical rows, forming identical joists, only the end rows and joists in the group are shown, and the numbers of rows and joists are placed on dimension lines with arrows pointing to the boundaries of the group.



SECTION A-A

FIG. 109.—Method of detailing ribbed floors with concrete girders.

The depth of the stems of the joists and the thickness of the slab are also given on the floor plan. Temperature steel, usually ¼-in. round bars at 10 in. on centers, is placed in the slab at right angles to the joists; this is detailed on the plan in the usual manner.

A detail floor plan for the typical interior panel which was designed in the preceding articles is shown in Fig. 109. The girders are marked on the plan in the usual manner. At least one typical cross-section through the joists and through the girders should be given on the drawing. The joists 1*J*16 between the columns of Fig. 109 are 7 in. wide at the bottom instead of 5 in., in order to fill up the panel. It is always desirable to have a

joist such as 1J16 between the columns, in order to stiffen the columns in this direction. In tall structures subject to wind pressure, a deeper beam would be used in place of the joist 1J16, in order to add to the rigidity of the entire structure.

Ribbed-floor construction is also used in conjunction with structural-steel beams in steel-frame buildings. In such constructions steel beams *a* and *b*, Fig. 110, are placed between the columns, parallel to the floor joists, in addition to the beams *c* and *d* which

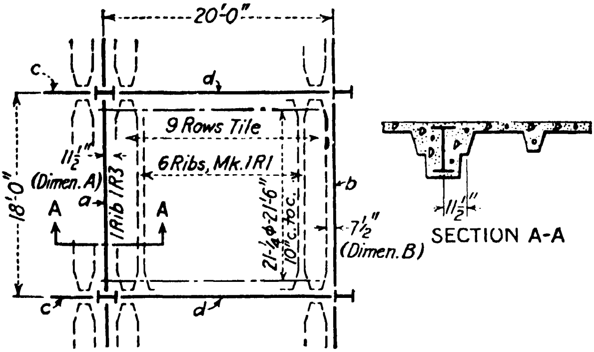


FIG. 110.—Method of detailing ribbed floors with steel beams.

support the joists, in order to furnish the necessary rigidity to the entire frame. It is then necessary to give on the floor plan the distances from the edges of the rows of tiles nearest these beams to the center lines of the beams, such as dimensions *A* and *B*, Fig. 110. If these dimensions are 8 in. or less, no reinforcement is used in the joist which is formed between the web of the steel beam and the adjacent row of tiles. If one of the dimensions is more than 8 in. that joist will be reinforced with at least one bar. In the former case, no joist mark is used, but if the joist is reinforced a mark is given to it and it is later detailed as explained in the following paragraph. The preceding statements also apply to joists or parts of joists which are parallel to and monolithic with a concrete beam or wall in a reinforced concrete structure.

The dimensions of the joists, the reinforcement, and the number of identical joists, are shown in a separate schedule, which is similar to the beam schedule in Fig. 138. The depth of the joists is listed as the depth of the stem below the slab plus the thickness

of the slab; the width is the width at the bottom. Thus, for the joist 1R17 in Fig. 109, the depth would be indicated as 8 + 2 and the width as 5 in.; for the joists 1R16 the depth would be 8 + 2 and the width 7 in. A separate schedule is made for the girders, in which are given the width and thickness of the flange, the width of the stem, the overall depth, and the reinforcement. Bending details must be given for all bent bars, either on the drawings or on separate sheets.

Design of a Ribbed Floor with Clay-tile Fillers

204. Dead Loads. The dead weight of the floor includes the weight of the concrete in the joists and in the slab, and the weight of the tiles. The following table gives average weights of clay-tile-and-concrete floors in pounds per square foot, for 4-in. ribs spaced 16 in. on centers.

Weights of Clay-tile-and-concrete Ribbed Floors

Depth of joist below slab, in.	Thickness of slab, in.	Average weight of floor, lb. per sq. ft.
4	2 2½	48 54
6	2 2½	59 64
8	2 2½	71 76
10	2 2½ 3	80 86 92
12	2 2½ 3	90 96 102

The values given above do not include the weight of an extra floor finish or plastered ceiling. Allowances for these items and for partitions should be made in computing design loads, as explained in Art. 199.

205. Design of Joists. The general method of designing the joists is the same as that explained in Art. 201 for joists which are formed by the use of steel-tile cores. If the joists are 4 in. wide and the tiles 12 in. wide, the distance center to center of joists is 16 in., and each joist supports $16/12 = 1.33$ sq. ft. of floor per linear foot of joist. Both dead and live loads must be included in the computations for moment and shear.

Because of the comparatively great strength of the walls of structural-clay tiles and the thorough bonding to the concrete which is obtained by the projections on the tile blocks, part of the side walls can be assumed as an effective part of the joists in resisting bending and shearing stresses. As a usual rule, the effective width of the joist is assumed as 1 in. or sometimes 1½ in. greater than the actual width of concrete between the surfaces of adjacent rows of tile. Thus, if 12-in. tile blocks are laid in rows which are 16 in. on centers, the normal width of the joist is 4 in., but the width that would be used in the design of the joist is 5 or 5½ in. The former value is preferred by conservative designers, but the latter has proved thoroughly safe.[1]

If the maximum unit shearing stress in a joist of given dimensions is greater than that allowable for concrete without web reinforcement, this stress may be reduced by placing one or more pieces of tile 8 in. in width at each end of each row, thus increasing the width of the joists at the point of maximum shear by 4 in. This is somewhat analogous to the use of tapered steel tiles in ribbed floors with steel-tile fillers.

206. Design of Girders. The method of designing the girders which support the joists is exactly the same as that used in the design of similar members in ribbed floors with steel-tile fillers (see Art. 202). The methods of detailing the joists and girders are also the same in the two types of construction.

[1] The Joint Code states that, if the fillers are so placed that the joints in alternate rows are staggered, the shells of the fillers in contact with the joists may be included in the calculations involving shear or negative bending moment; no other portions of the fillers may be included in the design calculations.

Design of Steel-joist Floors

207. Dead Load. The dead load which is supported by each joist will vary, among other things, with the spacing of the joists. The load per foot on each joist is equal to the load per square foot multiplied by the spacing of joists in feet, plus the weight of the joist in pounds per linear foot. Spacings of from 1 to $2\frac{1}{2}$ ft. are commonly used. Joist weights vary with the size of the joist from 5 to 10 lb. per lin. ft. The total dead load per square foot, exclusive of the weight of the joist, will include 25 lb. for the 2-in. concrete slab, from 5 to 15 lb. for an extra wearing surface, if

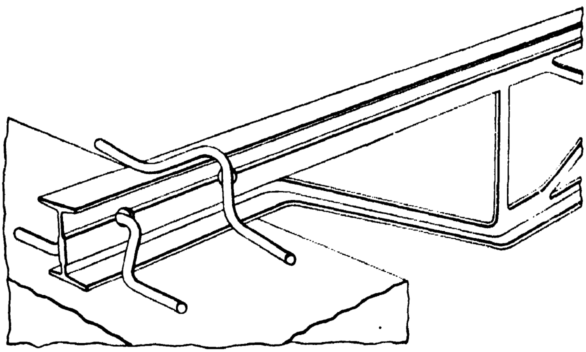


Fig. 111.—Method of anchoring steel joists in concrete walls.

required, 10 lb. for lath-and-plaster ceiling, and from 10 to 20 lb. for partitions, if such allowance is deemed necessary.

208. Selection of Joists. The total load that can be carried safely by each joist will vary with the span of the joist and the dimensions of the joist section. The manufacturers have prepared tables, giving the total load that the standard joist sections can support with various spans. These tables also give the total load per square foot of floor that can be supported by each joist for various spans and for spacings varying from 12 to 20 in. The tables are based usually on a unit tensile stress of 18,000 lb. per sq. in., and on a maximum deflection of $\frac{1}{360}$ of the span. They assume that adequate lateral bracing or bridging will be furnished.

The designer's problem is, therefore, merely one of selection. When the total load per square foot is known, by consulting the

manufacturers' tables, the designer can select several suitable combinations of joists and spacings, from which list the most economical combination is chosen. In computing the total load per square foot, the weight of the joist can be assumed as about

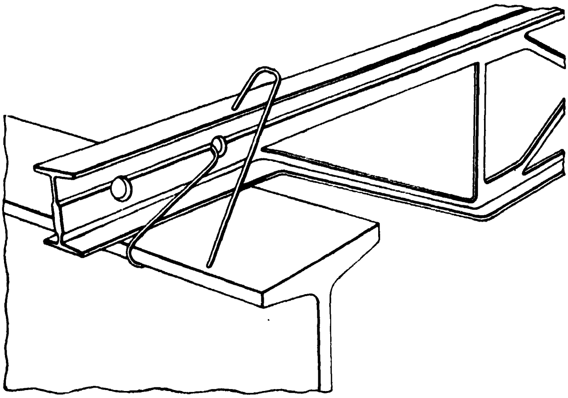


FIG. 112.—Method of anchoring steel joists on steel beams.

5 lb. per sq. ft. of floor area. This will simplify the computations without appreciable effect on the ultimate result.

209. Bearing Details. Anchorage of joists at the bearings is a most important detail in steel-joist construction. Where the

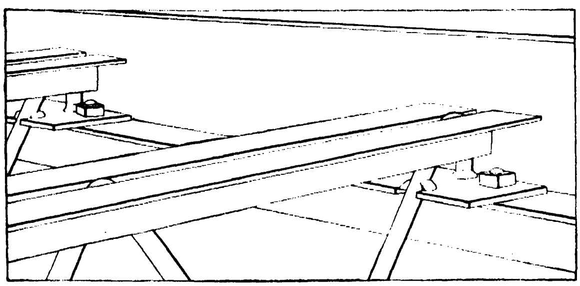


FIG. 113.—Method of anchoring steel joists on shelf angles.

bearings are on masonry walls, short straight or bent $\frac{3}{8}$-in. rods are run through holes in the ends of the joists, as shown in Fig. 111, and allowed to rest on the masonry. When the wall is carried up, the bars are embedded in the masonry, and serve as anchors to prevent displacement of the joists. The same bearing

anchorage is used when the joists rest on concrete beams; the beams are built up to the floor level as the concrete topping is poured.

For attaching joists to structural-steel beams, when welding is not specified, beam anchors are used. These are usually ¼-in. rods, with a hook on one end. The hook is placed through a hole in the end of the joist, as shown in Fig. 112, and the other end is hammered down around the beam flange. When the bearing is on shelf or seat angles, bolted connections are used, for which proper holes are provided in the end-bearing plates and in the shelf or seat angles, as shown in Fig. 113.

Flat-slab Floors

210. Description of General Type. A flat-slab floor, as its name implies, is one consisting of a reinforced concrete floor slab built monolithically with the columns and supported directly by the columns without the aid of beams and girders. The slab may be of uniform thickness throughout the entire floor area, or a part of it, symmetrical about the column, may be made somewhat thicker than the rest of the slab, the thickened portion of the slab thus formed constituting what is known as a dropped panel, or drop (see Fig. 116).

Dropped panels are used to reduce the shearing stresses in the slab within the area of the drop. The increase in the effective slab thickness which is provided by the drop also decreases the compression stresses in the concrete and reduces the amount of steel which is required over the column heads. In general the use of dropped panels is not economical for live loads less than 150 lb. per sq. ft. Dropped panels are usually square, the width being approximately equal to one-third of the panel length. The thickness of the drop varies in practice from $0.25t_2$ to $0.50t_2$, in which t_2 is the thickness of the slab outside of the drop. The general preference is for ratios approaching the upper limit. When drops are used over interior columns they are normally used over the wall columns also. Such drops should have a width parallel to the wall equal to the corresponding width of the interior-panel drops; at right angles to the wall they should project

beyond the center of the column a distance equal to one-half of the corresponding total width of the interior drops.

The columns in practically all cases flare out toward the top, forming a capital of a shape somewhat similar to an inverted truncated cone. This capital gives a wider support for the floor slab, which results in a decrease in the bending movement which the slab is called upon to resist, and a decrease in the shearing stresses around the perimeter of the column, and tends toward a more rigid structure. The effective diameter of the capital should be taken as the diameter of the circle at the point where a 45-degree line from the base of the capital intersects the bottom of the slab or dropped panel. In ornamental caps, the 45-degree line must fall within the concrete of the cap at all points. In practice, the effective diameter c of the cap is usually equal to $0.225l$, in which l is the average span of the panel.

At the wall columns, a bracket is often used in place of a regular half-capital. The width of the bracket is equal to the width of the column parallel to the wall; the sloping side of the bracket forms an angle of 45 degrees with the horizontal; the projection of the bracket beyond the face of the column is the same as, or somewhat less than, the projection of the interior-column capital.

211. Advantages of Flat-slab Floors. Structurally, a flat-slab floor has many advantages over the ordinary beam-and-girder floor. The most important of these may be enumerated as follows:

1. For ordinary spans with heavy loads, under average conditions, the flat-slab floor is more economical than the beam-and-girder floor.

2. In a multi-storied building, the same number of stories of a given clear height may be obtained with a smaller total building height, because of the smaller floor thickness.

3. The slab formwork is much simplified.

4. The flat-slab floor, owing to the lack of many sharp corners, is better able to resist continued exposure to fire than the beam-and-girder floor. It has been found by actual experience that the worst damage caused to reinforced concrete by severe fires

has occurred at places where there may be spalling, that is, at exposed edges and sharp corners.

5. Automatic sprinkler protection may be made more complete under a flat-slab floor since the nozzles may be placed well up near the under side of the slab without obstruction to the path of the spray.

6. More light may be admitted into the building if desired, by placing the wall beams above the floor level, and thus allowing the

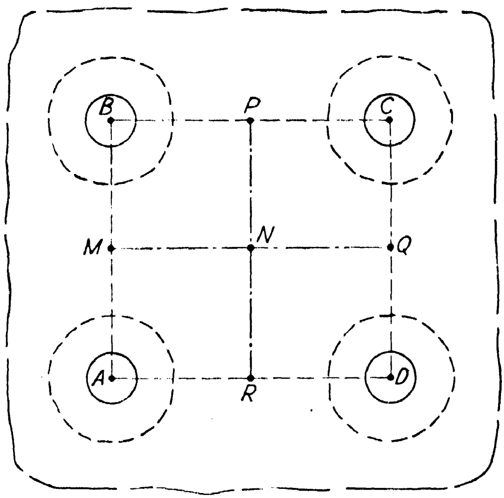


FIG. 114.

windows to be extended to the under side of the slab. The absence of deep beams and girders also removes the obstruction to the passage of light within the building.

7. Owing to the large number of smaller bars extending in several directions over the entire area of the floor, the danger of sudden failure or collapse is less than in the beam-and-girder type of floor. The relatively large breadth of structure also makes the effect of local variations in the concrete less than would be the case for narrow members like beams.

8. The opportunity for inspecting the position of the reinforcement is excellent, and the conditions attending deposition and placing of the concrete are favorable to securing uniformity and soundness in the concrete.

212. Bending Moments in Flat-slab Floors. Figure 114 represents a portion of a flat-slab floor including four column supports, the load on the floor being uniformly distributed. The full circles represent the column heads underneath the slab. It is evident that, considering any radial line from the column center, the curvature of the slab along this line will be convex upward for a certain distance, then concave upward, then convex upward again. This implies that at some point along each radial line there is a point of inflection where the radial bending moment changes from positive to negative. The locus of all these points may be represented by the dotted approximate circles centered about the column capitals.

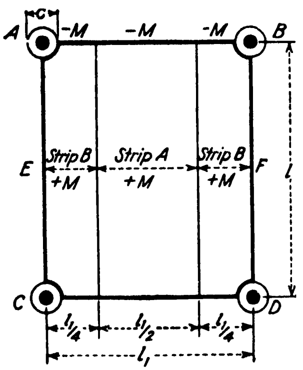


FIG. 115.

As the slab is loaded, deflection occurs. The point N at the midpoint of the panel, being the farthest away from the support, will deflect more than a point M, P, Q, or R midway between any two adjacent columns. The points M, P, Q, R will therefore be higher than point N but lower than the supports. This results in a negative moment along the line MQ at M, and a positive moment at N. The condition is similar along line PR.

The analysis of a flat-slab floor is a statically indeterminate problem and the elastic properties of the slab and the relative stiffness of its various parts must be considered in determining accurately the moments and shears in the slab. An arbitrary but satisfactory method of obtaining the theoretical bending moments is to divide each panel into rectangular strips, a middle strip A (Fig. 115) with a width equal to $\frac{l_1}{2}$ and two equal column strips B which occupy the outer portions of the panel. Similar strips A' and B' are assumed at right angles to those shown in Fig. 115. A system of imaginary beams is thus established and the load on the panel is transferred to the columns by these beams.

Consider strip A and the two strips B to form one wide but shallow continuous beam, rigidly supported at the four corners by the columns, and partially supported between the columns along the lines AB and CD by the perpendicular strips B'. In the resulting beam, negative moments exist along the lines AB and CD, and positive moments along the line EF. Strips B are obviously stiffer than strip A, and hence both the positive and negative moments in strips B (combined) will be greater than those in strip A.

It is a well-known fact that, in any continuous beam, the sum of the maximum positive moment in any span of the beam and the average of the negative moments at the adjacent supports are equal to the maximum moment in a corresponding simply supported beam. In a paper entitled "Statical Limitations upon the Steel Requirement in Reinforced-Concrete Flat-slab Floors," John R. Nichols[1] recommends that, in flat-slab analysis, the span of the corresponding simply supported beam be taken as $l - \frac{2}{3}c$, in which l is the panel length center to center of columns and c is the diameter of the capital. Thus, in Fig. 115.

$$M_{AB} + M_{EF} = \frac{1}{8}wl_1\left(l - \frac{2}{3}c\right)^2 = \frac{1}{8}Wl\left(1 - \frac{2c}{3l}\right)^2$$

in which w is the dead and live load on the slab, per unit of area, W is the total load on one panel, l is the distance center to center of columns parallel to the strips under consideration, l_1 is the distance center to center of columns perpendicular to the strips under consideration, and c is the diameter of the column capital.

The distribution of this total moment to the positive- and negative-moment sections EF and AB, respectively, must be determined and then the total positive moment in the section EF and the negative moment in the section AB must be apportioned to the strips A and B. Messrs H. M. Westergaard and W. A. Slater presented a solution of this problem of distribution in a paper entitled "Moments and Stresses in Slabs" which was published in the *Proceedings* of the American Concrete Institute, volume 17. The average percentages of the total moment

[1] *Trans.* Am. Soc. C. E., vol. 77.

$M_{AB} + M_{EF}$ [equation (75)] which they recommend for each of the various moment sections are as follows:

Column strips, negative moment	48 per cent
Column strips, positive moment	21 per cent
Middle strip, negative moment	17 per cent
Middle strip, positive moment	14 per cent
Total	100 per cent

The effect of a dropped panel is to stiffen the negative-moment portion of the column strip. This causes an increase in the negative moment in strip B and a corresponding decrease in the positive moment at the middle of the strip. The use of four-way reinforcement (see Art. 216) also stiffens the negative-moment portions of the column strip and at the same time reduces the stiffness of the negative-moment portions of the middle strip. The negative-moment coefficients specified for the column strips are therefore greater for four-way systems than for two-way systems, and the negative-moment coefficients for the middle strips are smaller for four-way than for two-way systems.

The preceding equation can be applied to the determination of moments in square panels and in rectangular panels in which the longer side of the panel is not more than 1.33 times the shorter side. In rectangular panels, separate moments must obviously be computed for the various sections of the two rectangular directions, using for l the length of the side of the panel in the direction parallel to the strip under consideration.

213. Moments in Interior Panels. Any theoretical analysis, such as the one outlined above, gives only approximate results, because of the many assumptions which are necessary. Tests of full-sized panels show that the actual stresses in the steel are less than would be obtained from the usual rectangular-beam equations with moments as indicated in the preceding article. By comparing the stresses determined by a sound theoretical analysis with those obtained by actual tests, moment coefficients can be obtained which will give rational and safe results. This process has resulted in the development of flat-slab regulations which are a part of practically all municipal building codes and technical society standards.

In addition to specifying moment coefficients, these codes give minimum requirements for slab thickness, cap and drop dimensions, and column sizes. They specify the arrangement of the reinforcement, and outline the method of computing stresses due to shear and bending. The formulas and methods of one code may differ from those of another, but there is sufficient similarity between them so that the designer who is familiar with the application of the provisions of one code has little difficulty in following those of any other code.

MOMENTS IN INTERIOR PANELS—STANDARD CODE

Strip	Moments in slabs without drops		Moments in slabs with drops	
	Negative	Positive	Negative	Positive
Slabs with two-way reinforcement				
Column strip	$-0.46M_0$	$+0.22M_0$	$-0.50M_0$	$+0.20M_0$
Middle strip	$-0.16M_0$	$+0.16M_0$	$-0.15M_0$	$+0.15M_0$
Slabs with four-way reinforcement				
Column strip	$-0.50M_0$	$+0.20M_0$	$-0.54M_0$	$+0.19M_0$
Middle strip	$-0.10M_0$	$+0.20M_0$	$-0.08M_0$	$+0.19M_0$

The code which is to govern a particular design depends largely on the location of the proposed structure. Practically every city of any size has its own flat-slab regulations which form a part of the complete building code. The provisions of these regulations must be adhered to (or exceeded) if the design is to be approved by the building department. In the absence of a governing municipal code, any recognized code may be followed.

The recommendations of the Joint Code (1936) are given in Appendix C.[1] These recommendations apply to flat-slab floors in which there are three or more rows of panels in each direction and in which the length of any one panel is not greater than 1.33 times the width of the panel. In this Code, the total moment M_{AB} + M_{EF} (Fig. 115) is specified as somewhat less than the value given

[1] The Standard Building Committee is proposing (1940) several minor changes in Flat Slab Regulations.

in the equation on page 332, and the distribution varies slightly from the percentages tabulated in Art. 212. For convenience in specifying moment coefficients and to simplify the subsequent detailing of the reinforcement, it is desirable to consider two adjoining column strips in adjacent panels as one band. This band is centered about the column line and has a width equal to

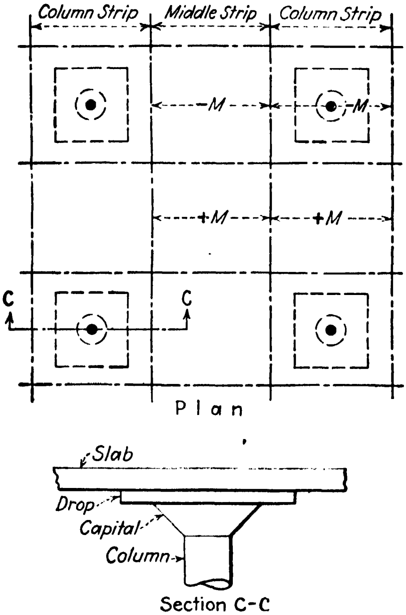


FIG. 116.

one-half of the panel width. In all of the following discussions, any reference to the "column strip" will imply the combined strips or band described above, as indicated in Fig. 116.

The moment distribution for interior panels, as given in the Joint Code, is as follows:

Let $M_0 = M_{AB} + M_{EF}$ (Fig. 115) in foot-pounds.

W = total load on the panel, including the weight of the drop, in pounds.

l = span of the strip under consideration, center to center of columns, in feet.

c = diameter of the column capital in feet.

$$M_0 = 0.09Wl\left(1 - \frac{2c}{3l}\right)^2 \qquad \text{(general case)} \qquad (a)$$

$$M_0 = 0.065Wl \qquad \text{(special case, } c = 0.225l_{av}) \qquad (b)$$

Coefficients of Wl for the specific case in which $M_0 = 0.065Wl$ [equation (b)] are given in Appendix C. Moment coefficients by bands for four-way reinforcement (see Art. 216) are also given in Appendix C.

214. Moments in Exterior Panels. In wall panels, the strips which are perpendicular to the wall are analogous to the end spans of a beam which is continuous over a number of supports. In such a beam, if the spans are all equal, the load uniform, and if the ends of the beam rest freely on the end supports, the maximum positive moment in the end spans is greater than in the intermediate spans; the negative moment at the second support is greater than the negative moment at any other interior support; the moment at the first support is equal to zero. If the ends of the beam are restrained instead of being freely supported, moments are induced at the end supports, and the maximum positive moments in the end spans as well as the negative moment at the second support are also affected. The latter two values will normally be greater than the corresponding moments in a typical interior span.

In flat-slab construction, the restraint which is offered at the end supports may vary considerably. The wall panels may rest freely on brick walls, they may be built monolithically with the wall columns and with marginal beams (wall beams) along the outer edge of the panels which are capable of resisting torsional stresses, or they may be built monolithically with the columns but without the rigid marginal beams. Obviously, the moment coefficients for sections parallel to the wall (strips perpendicular to the wall) would differ materially for these various conditions. Most flat-slab regulations do not take the condition of end restraint into consideration, but specify merely the coefficients that shall be used for the moment sections in all exterior panels.

The coefficients specified in the Joint Code are as follows: In wall panels and other panels in which the slab is non-continuous

at one edge, the maximum positive moments on the principal design sections parallel to the discontinuous edge (strips perpendicular to that edge) shall be increased 25 per cent over those specified for interior panels. At the wall or discontinuous edge the negative moment in the column strip shall be taken as not less than 90 per cent and in the middle strip not less than 66⅔ per cent of the corresponding moments specified for a normal interior panel. A tabulation of the resulting coefficients is given in Appendix C.

The value of c which shall be used in computing M_0 for the strips perpendicular to the wall in exterior panels is defined in Appendix C. Since brackets are normally used at exterior columns, and since the sloping face of these brackets usually projects beyond the columns a sufficient amount to make the distance from the center of the column to the extremity of the bracket equal to one-half of the diameter of the interior-column capital, the values of c, and hence M_0, for the exterior-panel strips perpendicular to the wall are generally the same as for an interior panel. The moments at the various sections of these strips can therefore usually be obtained by multiplying the moments at the corresponding sections in the interior panel by the percentages given in the preceding paragraph.

The moments to be used in the design of the middle strip parallel to the wall in an exterior panel are the same as for the corresponding sections in an interior panel. The moments to be used in the design of the half column strip adjacent and parallel to the wall depend upon the relative size of the marginal or wall beam. If the depth of this beam is 1½ times the thickness of the slab, or less, the moments in the half column strip, both positive and negative, are taken as one-half of the corresponding moments specified for a typical interior-panel column strip; but if the depth of the marginal beam is greater than 1½ times the depth of the slab, the moments in the half column strip are one-quarter of the interior-panel moments. In the latter case the comparative stiffness of the marginal beam relieves the half column strip of some of its load and thus reduces the moments in this strip.

According to the Joint Code, the marginal beam must then be designed to carry one-quarter of the total live load and dead load on the panel, in addition to the load directly superimposed on it. Where the depth of the marginal beam is $1\frac{1}{2}$ times the slab thickness, or less, the beam need be designed only for the superimposed load (*i.e.*, the wall load).

215. Thickness of Slab. In order to prevent undue deflection, certain limitations are placed on the minimum slab thickness that can be used in a given floor panel. Inasmuch as the actual deflection of a flat slab cannot be computed with any appreciable degree of accuracy, these limitations were developed from a study of the observed deflections in actual structures. The Joint Code specifies that, for a 2000-lb. concrete, the slab thickness (not including the thickness of the drop) shall in no case be less than $\frac{1}{32}$ of the longer dimension of the panel for floor slabs, and not less than $\frac{1}{40}$ of the same dimension for roof slabs. If the concrete has an ultimate compression strength f'_c greater than 2000 lb. per sq. in., these limiting thicknesses may be reduced by multiplying by the factor $\sqrt[3]{\dfrac{2000}{f'_c}}$.

The codes also give formulas for the minimum thickness of slab which is required in order to keep the unit compression stress in the concrete within the specified limit. These formulas are derived by analyzing the negative-moment section of the column strip as an ordinary rectangular beam. The Joint Code formulas for minimum total slab thicknesses (exclusive of the thickness of the drop) for 2000-lb. concretes are as follows:

For slabs without drops,

$$t_1 = 0.038\left(1 - 1.44\frac{c}{l}\right)l\sqrt{w'} + 1\frac{1}{2}$$

For slabs with drops,

$$t_2 = 0.02l\sqrt{w'} + 1$$

in which w' is the uniformly distributed dead and live load in pounds per square foot, t_1 is the thickness in inches of a slab with-

out drops, t_2 is the thickness in inches of the slab beyond the drop in panels with drops, and l is the panel length center to center of columns on the long side of the panel, in feet. For concretes with an ultimate compressive strength f'_c greater than 2000 lb. per sq. in., the values of t as given in these equations may be reduced by multiplying by the factor $\sqrt[3]{\frac{2000}{f'_c}}$.

216. Methods of Reinforcing Flat-slab Floors. There are, in the main, four different methods or systems of reinforcing the slab in this type of floor: (1) *Two-way system,* (2) *four-way system,* (3) *three-way system,* (4) *circumferential system.*

In the two-way system small bars are placed parallel to the lines of columns over the entire area of the floor at small intervals. The maximum spacing allowable varies in the different codes and specifications, but is seldom greater than one and one-half times the thickness of the slab, or greater than 12 in.

The four-way system consists of two main bands of steel running parallel to the lines of columns, each band centered about the column lines, and two diagonal bands of sufficient width to fill up the floor area left uncovered by the direct bands. In some cases short bars are placed near the top of the slab at right angles to the direct bands over the middle portion of the band to resist the negative moment over that portion; these latter bars constitute what are known as the across-direct bands.

The three-way system involves a special arrangement of columns, such that the lines connecting their center lines form a series of equilateral triangles. The reinforcement then follows the sides of these triangles, each band being centered about one of the panel sides. The three-way system is peculiarly adapted to such structures as car ba ns, garages, etc., on account of the large radius of curvature permitted by the arrangement of columns.

In the circumferential system, radial steel emanating from the column head, and circumferential steel in the form of concentric rings symmetrical about the column head are used. Concentric rings are also placed about the mid-point of the slab, and about the mid-points of the four edges of each panel. The three-way, four-way, and circumferential systems are not in such general use

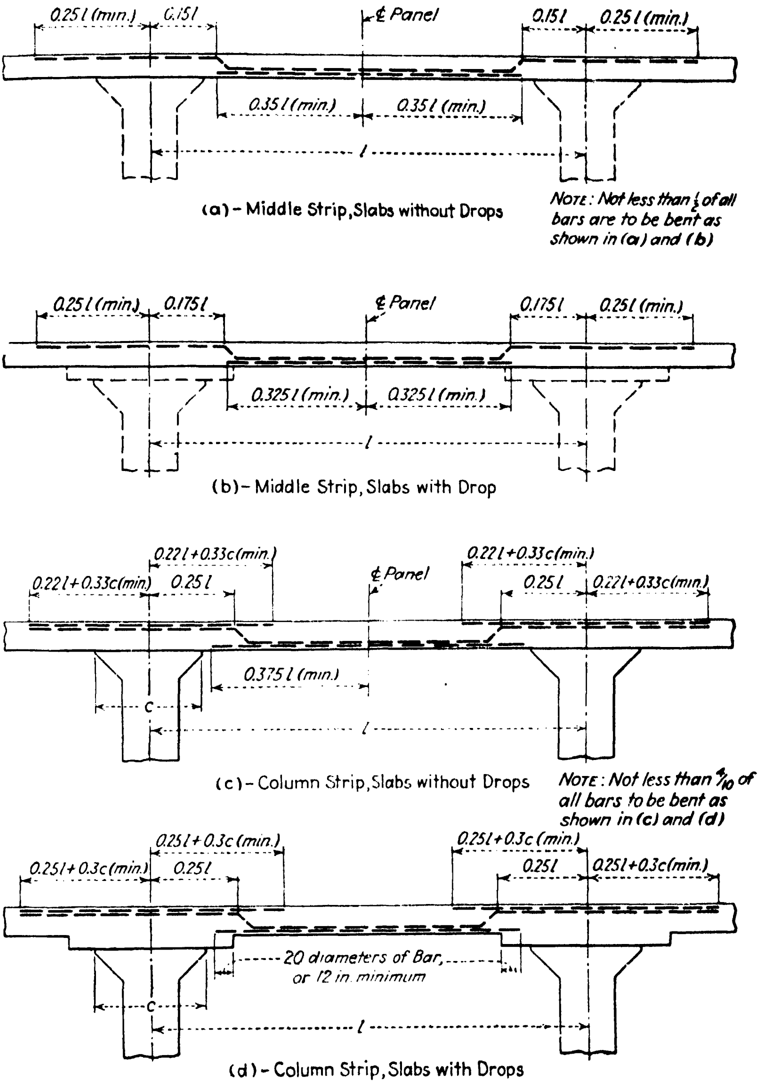


FIG. 117.—Joint Code specifications for placing reinforcement in two-way flat slabs.

as the two-way system, since the last is simpler in design and construction and has proved to be thoroughly satisfactory.

In both the two-way and four-way systems, the area of steel required at each section is obtained from the equation $M_s = A_s f_s jd$, in which M_s is the moment specified for the particular section under consideration and d is the distance from the compression face to the center of the tension steel (the effective depth)

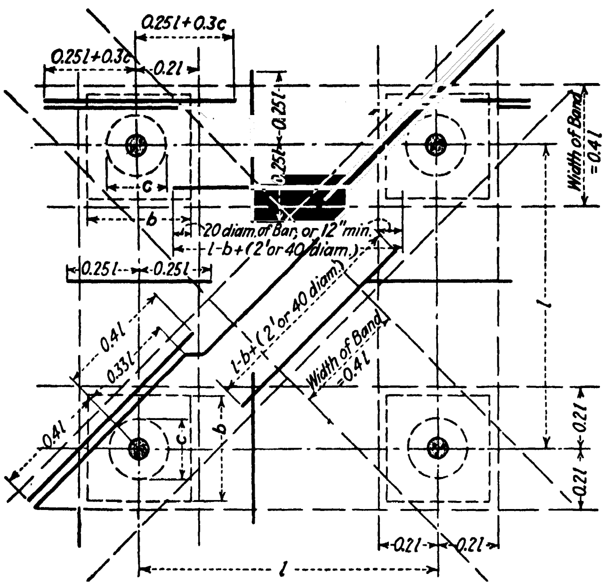


FIG. 118.—Specifications for placing reinforcement in four-way flat slabs with drops.

at the section. Part of the positive-moment reinforcement is bent up to furnish part or all of the negative-moment steel, in accordance with the provisions of the governing code. In Appendix C, Rule 5 and Table E apply to the placing of the reinforcement; these specifications (for two-way and four-way panels) are illustrated graphically in Figs. 117 and 118. The application of these specifications to a definite two-way design is illustrated in Arts. 219 and 220. The determination of the effective area of the reinforcement in diagonal bands in four-way panels is explained in Rule 3(*d*); moment coefficients by bands are given in Table C, Appendix C.

217. Factors to Be Considered in the Design of Flat-slab Buildings. Flat-slab floors are ordinarily designed to carry only a uniform load over the entire surface, the assumption being that no breaks in the continuity occur. Where heavy concentrated loads are to be sustained in addition to the uniform load, beams should be introduced in such positions as will enable them to carry the weight of the concentrations. Where openings in the slab occur, they should be framed by beams which will have the effect of restoring continuity to the slab. These beams should be designed to carry a portion of the floor load in addition to any concentrated loads that may rest upon them.

The columns should be designed to provide for bending stresses such as might be caused by unequally loaded panels. This is especially important in the exterior columns where both the dead and live loads cause continual bending, and where the direct loads are relatively small. The interior roof columns are not likely to be subjected to such eccentric loading, and the ratio of the possible bending stress to the direct load stress decreases as the number of floors to be supported increases. Hence, bending in the interior columns is not so important as in the exterior columns. It should be investigated, however, especially in the upper stories. The amount of bending moment to be assumed is usually stated in the various regulations governing flat-slab design. One method of analysis is outlined in Art. 224 for interior columns and in Art. 225 for exterior columns. The spacing of columns is governed by practically the same factors as in the case of the beam-and-girder type.

To provide proper drainage, a slight pitch may be given to the roof slab without any change in the theoretical computations. Sudden changes in slope, or steps, on the other hand, require special attention.

It should be remembered at all times that careful compliance with the building code pertaining to the place of construction is not only necessary to the acceptance of the design, but is also conducive to safety. As in all other types of construction, failures of flat-slab buildings have occurred. The causes of such failures may in most cases be traced to one or more of the following:

1. Strong commercial competition leading to the tendency to use thinner sections than good design dictates, especially in the absence of good building codes.

2. Faulty construction, such as poor mixing of concrete, inaccurate placing of the steel, too early removal of forms, or placing of concrete in freezing weather without adequate precaution.

3. Overloading of the floors beyond the load allowed in the design.

4. Faulty design, due to a lack of knowledge on the part of the designer.

The first essential of a safe and economical design is the removal of all agencies such as are stated above, that might lead to failure, or on the other hand, to needless waste.

Design of a Flat-slab Building

218. Data and Specifications. The method of design of flat-slab floors and other details involved in a reinforced concrete building are illustrated in the following articles which contain a complete design of a building 66 by 105 ft. in plan, consisting of two upper stories and a basement. The height of the upper stories, floor to floor, is 12 ft.-0 in. and that of the basement is 10 ft.-0 in. The floor plan is shown in Fig. 130. The live load to be supported by the floors is 200 lb. per sq. ft., and by the roof 40 lb. per sq. ft. An additional load of 40 lb. per sq. ft. is to be considered in the dead weight of the roof, to provide for a cinder concrete fill and for the roof covering. Adequate drainage will be provided by inclining the exterior slabs in the short direction of the building and by varying the thickness of the surfacing over the middle panel. A 2500-lb. concrete will be used for all of the construction except the footings, for which a 2000-lb. concrete will be assumed. The Joint Code (Appendixes B and C) will be used in the design of the floor and roof slabs, beams, columns, and footings. Structural grade steel will be used in all beams and slabs, and intermediate grade steel in the columns and footings.

219. Design of Interior Floor Panel. *Slab.* Table A, Appendix C. Two-way reinforcement with dropped panel. Assuming

$t_2 = 8$ in., the weight of the slab is 100 lb. per sq. ft. The total load on the slab $= w' = 300$ lb. per sq. ft.

$$t_2 = \sqrt[3]{\frac{2000}{2500}}(0.02l\sqrt{w'} + 1)$$

$$= 0.93(0.02 \times 22\sqrt{300} + 1) = 7.98 \text{ in.}$$

$$t_2 = \sqrt[3]{\frac{2000}{2500}} \times 0.375 \times 22 = 7.68 \text{ in.}$$

The assumed thickness of 8 in. is satisfactory.

Capital. Metal cap forms will be used, standard diameters being multiples of 6 in. The usual diameter c of cap is approximately equal to 0.225 times the average span of the panel.

$$0.225 \times 21.5 \times 12 = 58 \text{ in.} \qquad \text{Use 5 ft.-0 in.}$$

Drop. Table A, Appendix C. The minimum allowable thickness (t_1) through the dropped panel is $1.25t_2$ and the maximum $1.5t_2$. The upper limit is usually preferred, since the greater thickness reduces the amount of steel required at the column head and adds to the shearing resistance at the perimeter of the capital.

$$t_1 = 1.5 \times 8 = 12 \text{ in.}$$
$$b_1 = 0.35 \times 22 \times 12 = 92.4 \text{ in.}$$

The dropped panel will be made 4 in. thicker than the remainder of the slab, and 7 ft.-9 in. square.

Shearing Stresses. Rule 1, Appendix C. The unit shearing stress $\left(v = \frac{V}{\frac{7}{8}bd}\right)$ on a vertical section $t_1 - 1\frac{1}{2}$ in. $(= d_1)$ from the edge of the capital, and parallel with it, shall not exceed $0.03f'_c = 75$ p.s.i.

$$t_1 - 1\tfrac{1}{2} = 10\tfrac{1}{2} \text{ in.}$$
$$5 \text{ ft.-0 in.} + 2 \times 10\tfrac{1}{2} \text{ in.} = 6 \text{ ft.-9 in.} = 6.75 \text{ ft.}$$

The total load on the panel $= 300 \times 21 \times 22 + \frac{4}{12} \times 150 \times (7.75)^2 = 141{,}600$ lb.

At the section described above,

$$V = 141{,}600 - 350 \times \frac{\pi \times (6.75)^2}{4} = 129{,}100 \text{ lb.}$$

$$v = \frac{129{,}100}{\frac{7}{8} \times (\pi \times 6.75 \times 12) \times 10.5} = 55 \text{ p.s.i.}$$

The unit shearing stress on a vertical section $t_2 - 1\frac{1}{2}$ in. ($= d_2$) from the edge of the dropped panel, and parallel with it, shall not exceed $0.03f'_c = 75$ p.s.i.

$$t_2 - 1\tfrac{1}{2} = 6\tfrac{1}{2} \text{ in.}$$

$$7 \text{ ft.-9 in.} + 2 \times 6\tfrac{1}{2} \text{ in.} = 8 \text{ ft.-10 in.} = 8.83 \text{ ft.}$$

$$V = [(21 \times 22) - (8.83)^2] \times 300 = 115{,}200 \text{ lb.}$$

$$v = \frac{115{,}200}{\frac{7}{8} \times (4 \times 8.83 \times 12) \times 6.5} = 48 \text{ p.s.i.}$$

The assumed dimensions of the capital and drop need no revision.

Bending Moments. Table B, Appendix C. Since the panel is nearly square, moments and steel areas will be computed for the long direction and the same steel will be placed in the short direction. $W = 141{,}600$ lb.; $l = 22.0$ ft.; $c = 5.0$ ft.

$$M_0 = 0.09Wl\left(1 - \frac{2c}{3l}\right)^2 = 0.09 \times 141{,}600$$

$$\times 22.0\left(1 - \frac{2 \times 5.0}{3 \times 22.0}\right)^2 = 201{,}700 \text{ ft.-lb.} = 2{,}420{,}000 \text{ in.-lb.}$$

Column strip, positive $M = +0.20 \times 2{,}420{,}000$
$= +484{,}000$ in.-lb.

Column strip, negative $M = -0.50 \times 2{,}420{,}000$
$= -1{,}210{,}000$ in.-lb.

Middle strip, positive $M = +0.15 \times 2{,}420{,}000$
$= +363{,}000$ in.-lb.

Middle strip, negative $M = -0.15 \times 2{,}420{,}000$
$= -363{,}000$ in.-lb.

Steel Areas. In computing steel areas required, for those moment sections in which bars are placed in two directions (*i.e.*, the positive-moment section of the middle strip and the negative-moment section of the column strip) the governing row will be assumed as that row which is farthest from the tension face of the

slab. This removes all restrictions as to the order of placing the steel in the two directions. Values of j are obtained from Table 6, Appendix D. $f_s = 18{,}000$; $f_c = 0.40 \times 2500 = 1000$ p.s.i. for positive-moment sections, and $0.45 \times 2500 = 1125$ p.s.i. for negative-moment sections; $n = 12$. Equation (5), page 56, is used to compute the steel areas required at each of the moment sections. Bars which are parallel to a section (*i.e.*, perpendicular to the strip under consideration) obviously cannot be considered effective at that section.

Column Strip, Positive-moment Section. Assuming $\frac{5}{8}$-in. bars and $\frac{3}{4}$-in. clear insulation, $d = 8 - 1\frac{1}{16} = 6\frac{15}{16}$ in.

$$A_s = \frac{484{,}000}{18{,}000 \times 0.867 \times 6.93} = 4.48 \text{ sq. in.}$$

Fifteen $\frac{5}{8}$-in. round bars furnish 4.61 sq. in.

Column Strip, Negative-moment Section. Assuming $\frac{5}{8}$-in. bars and $\frac{3}{4}$-in. clear insulation, $d = 12 - 1\frac{11}{16} = 10\frac{5}{16}$ in.

$$A_s = \frac{1{,}210{,}000}{18{,}000 \times 0.857 \times 10.31} = 7.60 \text{ sq. in.}$$

Twenty-five $\frac{5}{8}$-in. round bars furnish 7.68 sq. in.

According to Table E, Appendix C, not less than four-tenths of the steel required in the positive-moment section of the column strip shall be straight bars. The remaining bars (in any event not less than four-tenths of the total steel) may be bent up to reinforce the negative-moment section at either end of the panel. Hence at each negative-moment section a total of 18 bars is furnished from the two adjacent positive-moment sections. Seven straight bars in the top of the slab over the column head are sufficient to complete the steel area required at the negative-moment section (see Fig. 119).

Middle Strip, Positive-moment Section. Assuming $\frac{1}{2}$-in. bars and $\frac{3}{4}$-in. clear insulation, $d = 8 - 1\frac{1}{2} = 6\frac{1}{2}$ in.

$$A_s = \frac{363{,}000}{18{,}000 \times 0.867 \times 6.5} = 3.58 \text{ sq. in.}$$

Eighteen $\frac{1}{2}$-in. round bars furnish 3.54 sq. in.

MIDDLE STRIP, NEGATIVE-MOMENT SECTION. Assuming ½-in. bars and ¾-in. clear insulation, $d = 8 - 1 = 7$ in.

$$A_s = \frac{363,000}{18,000 \times 0.857 \times 7} = 3.36 \text{ sq. in.}$$

According to Table E not less than five-tenths of the steel required in the positive-moment section of the middle strip must

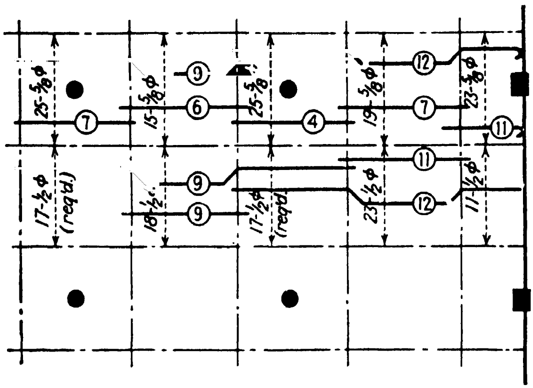


FIG. 119.—Arrangement of reinforcement in floor slab.

be bent up at both ends so as to reinforce the two adjacent negative-moment sections. Bending nine of the 18 bars required in the positive-moment section provides a total of 18 bars in each negative-moment section, which is more than ample in the present case.[1]

Fiber Stress in Concrete. The critical section is in the negative-moment portion of the column strip. According to Rule 3(*e*), Appendix C, the effective width of the column-head section, for

[1] Since the area required at both the positive- and negative-moment sections of the middle strip are approximately the same ($M = 0.15M_0$ for each), the negative-moment steel could be furnished by bending all of the positive-moment steel at one end only, placing the bars so that alternate bars are bent up at alternate ends. The bent-up portion of each bar would continue into the adjacent panel to the point of inflection, and the other end of the bar would be placed at a point $0.325l$ from the center of the panel, as required by Table E. While a strict interpretation of the Joint Code will not permit this detail, many designers prefer it to that shown in the text above. The main advantages are: (1) the bent bars are shorter and hence easier to handle, (2) no loose straight bars are used.

compression, shall be taken as the width of the dropped panel, 93 in.

$$p = \frac{25 \times 0.3068}{93 \times 10.31} = 0.0080$$

From Table 7, Appendix D, $k = 0.353$ and $j = 0.882$.

$$f_c = 1.2 \times \frac{2 \times 1{,}210{,}000}{0.353 \times 0.882 \times 93 \times (10.31)^2} = 940 \text{ p.s.i.*}$$

The allowable stress $= 0.45 \times 2500 = 1125$ p.s.i.

220. Design of Exterior Floor Panel. A comparison of Tables B and C, Appendix C, shows that the column strip negative moment at the wall is 90 per cent of that specified for the interior panel; the column strip positive moment in the exterior panel and the middle strip positive moment in that panel are each 125 per cent of the corresponding interior-panel moments; the middle strip negative moment at the wall is 66⅔ per cent of that specified for the interior panel. Hence the steel areas required at the various sections in the exterior panel (for bands perpendicular to the wall) may be obtained by multiplying the corresponding interior-panel required areas by the above percentages, as follows:

Middle Strip, Positive-moment Section.

$$A_s = 1.25 \times 3.58 = 4.48 \text{ sq. in.}$$

Twenty-three ½-in. round bars furnish 4.51 sq. in.

Middle Strip, Negative-moment Section at Wall.

$$A_s = 0.667 \times 3.36 = 2.24 \text{ sq. in.}$$

Eleven ½-in. round bars furnish 2.16 sq. in.

Column Strip, Positive-moment Section.

$$A_s = 1.25 \times 4.48 = 5.60 \text{ sq. in.}$$

Nineteen ⅝-in. round bars furnish 5.83 sq. in.

Column Strip, Negative-moment Section at Wall.

$$A_s = 0.9 \times 7.60 = 6.84 \text{ sq. in.}$$

Twenty-three ⅝-in. round bars furnish 7.06 sq. in.

* The maximum stress at the center of the column is greater than the average stress by about 20 per cent.

221. Arrangement of Reinforcement. The proposed method of placing and bending the steel so as to furnish the necessary areas at the various sections in both the interior and exterior panels is shown diagrammatically in Fig. 119.[1] The points at which the bars are bent and the percentages of bars which may or must be bent are obtained from Table E, Appendix C. The provisions of these rules are illustrated graphically in Fig. 117. According to Rule 4(*e*), Appendix C, the maximum allowable spacing of bars is $1\frac{1}{2} \times 8 = 12$ in. The minimum spacing is governed by the necessity of allowing space for the concrete to be deposited conveniently and effectively. A minimum spacing of about 3 in. center to center of bars should be maintained if possible. Bars less than ½ in. in diameter are difficult to handle because of their lack of stiffness, and bars greater than ¾ in. in diameter cannot be bent in place.

The complete steel schedule for the slab reinforcement is shown in Fig. 135. The location of the bands is shown in Fig. 130. In making up the placing plan shown in Fig. 130, bars running in one direction have been designated by the letter *P* and those in the other direction by the letter *M*. The additional straight bars in the top over the column heads are designated by the letter *T*. All bands in which the number, size, and bending details of the bars are identical are marked with the same number following the letter. The total number of identical bands is obtained by counting the similarly marked bands in Fig. 130.

222. Design of Roof Slab. The design of the interior and exterior panels of the roof slab is carried out in a manner similar to that used in the design of the floor slab. The total load on the roof includes the live load (40 lb. per sq. ft.), the weight of the roofing material (40 lb. per sq. ft.), and the dead weight of the slab itself. The thickness of slab beyond the drop is 6¼ in. and the total thickness through the dropped panel is 9¼ in. The diameter of the capital is 5 ft.-0 in., and the drop is 7 ft.-9 in. square in plan. The unit shearing stress on a vertical section at a distance $t_1 - 1\frac{1}{2}$ in. from the capital is 43 lb. per sq. in. and the corresponding stress at a distance $t_2 - 1\frac{1}{2}$ in. from the edge of the

[1] See also footnote 1, p. 346.

dropped panel is 36 lb. per sq. in. At the column-head section the fiber stress in the concrete is 755 lb. per sq. in.

The number of ½-in. round bars required in the various sections of the slab, and the proposed method of placing and bending the bars so as to furnish the required steel areas at all sections are shown diagrammatically in Fig. 120. The complete bending details are shown in Fig. 136. The band designations are shown in the left half of Fig. 130. It has been assumed, for simplicity,

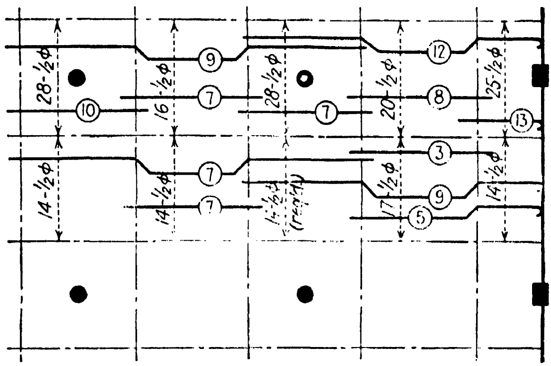


FIG. 120.—Arrangement of reinforcement in roof slab.

that the stairway and elevator shaft openings do not extend through the roof.

223. Design of Interior Columns. The interior columns are to be made of 2500-lb. concrete, with reinforcement of intermediate grade steel. They are to be round, with spirals, and designed in accordance with the Joint Code specifications. The fundamental principles involved in the design are explained in Art. 134. In flat-slab construction the columns are an important factor in adding to the rigidity of the slab, and most codes specify a minimum column size greater than the usual minimum for beam-and-girder floors. Ordinarily a minimum overall dimension of one-fifteenth of the average span of the panel is considered satisfactory. In the present case $\frac{1}{15} \times 21.5 \times 12 = 17.2$ or 18 in. will be taken as the minimum size.

According to the Joint Code, values of p_g may vary from 0.01 to 0.08 for columns with spirals, but values greater than 0.02 to 0.025 are normally inadvisable because of the crowding of the

steel which occurs just above the floor line where the bars are spliced by lapping. In general, greater overall economy is obtained by the use of the smaller values of p_g. Steel ratios approaching the maximum may be necessary, however, in heavily loaded columns, in order to keep the size of the columns within reasonable limits. In some cases architectural limitations may require a smaller column with the resulting greater steel ratio. Metal column forms, which are generally used for round columns, are available in multiples of 2 in., and column sizes should be selected accordingly.

The following table gives a summary of the loads (to the nearest thousand pounds) which are carried in each tier of columns. The weights of the columns were taken from Table 10. The sizes of the columns and the reinforcement required to support these loads were obtained from Table 8, and spiral details from Table 13. The excess strength of the top column cannot be avoided, since the minimum size columns and the minimum percentage of reinforcement have been used. The excess strength of the intermediate column is required to provide for bending stresses, as shown by the investigation in Art. 224.

INTERIOR COLUMNS, DESIGN FOR DIRECT LOAD ONLY

Column	Load from	Amount of load, lb.	Diameter and area of column	Vertical bars	Spirals (Table 13)	Load carried by concrete, lb.	Load carried by steel, lb.	Total load, lb.
Top	Roof	75,000	18 in.	6-$\frac{3}{4}\phi$				
	Column	3,000	255 sq. in.	2.65 sq. in.	$\frac{3}{8}\phi$-2$\frac{1}{4}$ in.	143,000	42,000	185,000
	Capital	2,000		$p_g = 0.0104$				
	Total	80,000						
Intermediate	Floor	142,000	22 in.	10-$\frac{3}{4}\phi$	$\frac{3}{8}\phi$-2 in.	214,000	71,000	285,000
	Column	4,000	380 sq. in.	4.42 sq. in.				
	Capital	2,000		$p_g = 0.0116$				
	Total	228,000						
Basement	Floor	142,000	26 in.	10-$\frac{7}{8}\phi$	$\frac{1}{2}\phi$-3 in.	299,000	96,000	395,000
	Column	5,000	531 sq. in.	6.01 sq. in.				
	Capital	2,000		$p_g = 0.0113$				
	Total	377,000						

The bars in the basement and first-floor columns are extended 2 ft.-3 in. above the first- and second-floor levels, respectively, in order to lap with the bars in the columns next above. A lap of 32 diameters is required by the Code when f'_c is less than 3000 p.s.i. In order to prevent too sharp a bend in the bars at the top of any column in getting them within the area of the column next above, wherever possible the diameters of two successive tiers of columns should not vary by more than about 6 in. The bend in each bar is usually made in a height of about 18 in., and the maximum slope of the bend is then 3 in. in 18 in., which is the maximum permitted by the Code.

224. Investigation for Bending Stresses. Bending stresses due to unequally loaded panels are not apt to affect the design of the interior columns, except possibly in the upper tiers. This is because the bending is usually caused by the live load only, and so the maximum unit stress due to bending is in most cases less than the allowable *increase* in fiber stress as compared with the stress permitted when no bending is considered.

The method of investigation for bending is explained in Chap. V; the average stress due to direct load $\left(\frac{N}{A_t}\right)$ is added to the extreme fiber stress due to bending $\left(\frac{Mc}{I}\right)$ in order to obtain the maximum combined stress f_c. The allowable unit combined stress as specified in the Joint Code for both spiral and tied columns is given in the equation

$$f_c = f_a\left(\frac{1 \times \frac{ec}{R^2}}{1 + C\frac{ec}{R^2}}\right)$$

in which f_a = average permissible stress on an equivalent axially loaded plain concrete column (*i.e.*, based on the area of the transformed section of the actual reinforced concrete column),

or $\quad f_a = \dfrac{0.225f'_c + f_s p_g}{1 + (n - 1)p_g}$ for spiral columns and 0.8 of this value for tied columns.

e = eccentricity of the resultant load N on the column $\left(e = \frac{M}{N}\right)$.

c = distance from the gravity axis of the column cross-section to the extreme compression fiber (*i.e.*, to the most highly stressed face of the column).

R = least radius of gyration of the column section.

C = ratio of f_a to the permissible fiber stress for members in flexure $\left(C = \frac{f_a}{0.45f'_c}\right)$.

Values of f_r may be taken directly from Diagrams 7 to 10 for spiral columns and from Diagrams 11 and 12 for tied columns, these diagrams being based on the preceding equation.

With unequally loaded panels the amount of bending moment transferred to the interior columns depends upon the relative stiffness of the slab on both sides of the columns and of that of the columns themselves. It is recommended that a moment[1] of $\frac{1}{40}W_1l$ shall be divided between the columns immediately above and below any floor, in direct proportion to their stiffness factors $\left(\text{values of } \frac{I}{h}\right)$, where W_1 = the total live load on one panel,

[1] The recommended moment of $\frac{1}{40}W_1l$ is based on the following analysis: Due to live load only, the negative moment in a column strip, for two-way slabs with drops and with $c = 0.225l$, is $0.0325W_1l$. This unbalanced moment (assuming live load to be placed only on one side of the columns under consideration) is resisted by the slab on the unloaded side and the columns above and below the floor, more or less in direct proportion to the stiffness factors $\left(\frac{I}{h}\right)$ of these members. Assuming that the stiffness factor of the slab is one-third of the sum of the stiffness factors of the two columns, the moment to be resisted by the two columns is $\frac{3}{4} \times 0.0325W_1l = 0.0244\,W_1l$ or approximately $\frac{1}{40}W_1l$.

In the investigation of the exterior columns (see Art. 225), since there is no slab beyond the wall the entire negative moment in the column strip at the wall is resisted by the two columns. This moment is $0.029Wl$, or approximately, $\frac{1}{35}Wl$. Here the full load (dead plus live) on the panel is used, because both the dead and live loads are unbalanced loads. The total moment of $\frac{1}{35}Wl$ is distributed to the columns in direct proportion to the stiffness factors.

l = the average span of the panel, I = the moment of inertia of the column, and h = its unsupported height.

In flat-slab construction, the unsupported height of a column is equal to the distance from the floor to the bottom of the capital. The moment of inertia of the column is equal to the sum of the moment of inertia of the overall cross-section of the column and $(n - 1)$ times that of the longitudinal steel. The longitudinal steel is placed directly inside of the spirals, making the diameter of the circle on which the steel is placed equal to the diameter out to out of spirals, less two diameters of spiral and one diameter of the longitudinal bars. With 1½ in. insulation to the spirals, as required by the Joint Code, this steel-circle diameter will vary from 4¼ to 5¼ in. less than the overall diameter of the column. The moment of inertia of the steel may be taken from Table 11 which is based on a steel-circle diameter 5 in. less than the overall column diameter.

The maximum combined stress occurs only when the panels at the floor line under consideration are unequally loaded, with a full live load on alternate panels and no load on the remaining panels. A full live load is assumed on all floors above the one considered. In the investigation for the combined stress at the top of any column, the total direct load is equal to the load in the preceding table (the full load) minus the weight of the column and minus one-half of a live panel load.

The tables on page 354 contain a summary of computations which are necessary to determine the unit stresses due to bending and direct stress in the interior columns. Diagrams 13 to 15 would give essentially the same results.

For the roof,

$$M = \tfrac{1}{40} \times 40 \times 21 \times 22 \times 21.5 \times 12 = 119{,}000 \text{ in.-lb.}$$

For the other floors,

$$M = \tfrac{1}{40} \times 200 \times 21 \times 22 \times 21.5 \times 12 = 596{,}000 \text{ in.-lb.}$$

In the present case, the investigation for bending and direct stress shows that no revision of the original columns, as designed for direct load only, is necessary. It should be noted, however,

that in the original selection, some excess strength was intentionally provided in the intermediate column. The amount there provided is shown to be more than is necessary for bending; but the minimum percentage of steel, approximately, has been used, and subsequent investigation indicated that a 20-in. column (the next smaller standard size) would require more than 3 per cent of steel in order to keep the combined stress within the allowable limits. The design as shown in the accompanying tables is therefore satisfactory.

INTERIOR COLUMNS. COMPUTATION OF MOMENTS OF INERTIA AND STIFFNESS FACTORS

Column	I_g (Table 10), in.^{4}	$(n-1)I_s$ (Table 11), in.^{4}	$I = I_g + (n-1)I_s$, in.^{4}	h, in.	$\frac{I}{h}$
Top	5,150	615	5,765	112	51.5
Intermediate	11,499	1,752	13,251	112	118.1
Basement	22,432	3,640	26,072	90	290.0

INTERIOR COLUMNS. COMPUTATION OF STRESSES DUE TO BENDING AND DIRECT LOAD

Column	Point	A_t, sq.in.	N (effective), lb.	f_c (direct), p.s.i.	M, in.-lb.	f_c (bending), p.s.i.	f_c (total), p.s.i.	$\frac{e}{d}$	f_c (allowable), p.s.i.
Top	Top	284.2	68,000	239	119,000	186	425	0.097	800
	Bottom		80,000	281	181,000	283	564	0.126	820
Intermediate	Top	428.6	178,000	415	415,000	345	760	0.106	810
	Bottom		228,000	531	173,000	143	674	0.034	720
Basement	Top	597.1	326,000	546	423,000	211	757	0.050	750
	Bottom		377,000	630	0	0	630	0	

225. Design of Exterior Columns. The exterior columns are to be made of 2500-lb. concrete, with longitudinal bars of

intermediate grade steel. They are to be rectangular in section, and the longitudinal bars are to be tied together by means of ties spaced 12 in. on centers, the diameter of tie being $\frac{3}{8}$ in. for the upper two tiers and $\frac{1}{2}$ in. for the lower tier, in accordance with the rule suggested in Art. 111.

In addition to the load from the floors, the exterior columns must support the weight of the walls enclosing the story next above. In estimating the weight of the enclosure walls, the

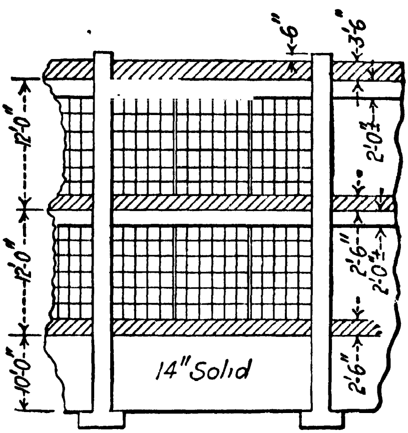


FIG. 121.

wall beams and brick spandrels underneath the windows are assumed 12 in. thick, the spandrel 2 ft.-6 in. deep, and the wall beam 2 ft.-0 in. deep. The brick parapet wall at the roof is assumed 12 in. thick and 3 ft.-6 in. deep. The weight of windows, including sash, is taken as 8 lb. per sq. ft. The weight of the brick masonry is assumed as 140 lb. per cu. ft. The general arrangement of a typical wall panel is shown in Fig. 121.

Bending stresses should always be considered in the design of the exterior columns, especially in the upper tiers. The direct loads on these columns are comparatively small and the bending moments due to unsymmetrical loading are large since both live load and dead load act together in causing these moments. According to footnote 1, page 352, the wall columns in flat-slab construction shall be designed to resist a bending moment from the slab of $\frac{1}{35}Wl$, in which W is the total load (dead and live)

on one panel and l is the average span of the panel. In the present design, at the first- and second-floor levels, $W = 141{,}600$ lb., $l = 21.5$ ft., and

$$M = \tfrac{1}{35}(141{,}600 \times 21.5 \times 12) = 1{,}042{,}000 \text{ in.-lb.}$$

At the roof level, $W = 75{,}300$ lb., and

$$M = \tfrac{1}{35}(75{,}300 \times 21.5 \times 12) = 555{,}000 \text{ in.-lb.}$$

Countermoments due to the weight of the structure that projects beyond the column center lines and countermoments due to the eccentricity of one column with respect to the column beneath may be deducted from the value of $\frac{1}{35}Wl$, and the resulting reduced moment is then divided between the two columns immediately above and below a given floor line in proportion to the stiffness factors $\left(\frac{I}{h}\right)$ of these columns.

Inasmuch as bending stresses constitute a large proportion of the total stresses in exterior columns, particularly in the upper tiers, it is of no use to design the columns first for direct load only. The general procedure is to assume column sizes and reinforcement, compute the combined extreme fiber stresses, including bending, and then revise the assumed sizes if necessary. Where the concrete is to be left exposed on the exterior face, the dimension along that face should be the same for all tiers of columns and of sufficient amount to give a satisfactory architectural appearance to the face. If this dimension is too small, the building will appear to be unstable or insecure. If it is too large, the building will appear squatty and the design will be needlessly uneconomical. An elevation drawn to scale, as in Fig. 127, will show whether or not an assumed face dimension is satisfactory. Better architectural appearance is obtained by allowing the columns to project 2 in. or more beyond the face of the wall. In the present case, a column width of 22 in. is selected. The minimum required dimension of 18 in. (see Art. 223) will be maintained in the direction perpendicular to the wall. Steel ratios may vary from 0.01 to 0.04. As stated

before, the smaller ratios of reinforcement result in more satisfactory designs. The column dimensions and steel areas ultimately selected for trial are shown in the table on page 358.

The countermoment at the top of the intermediate column is equal to the product of the wall load and the distance from the center of the wall beam to the center of the column. In the design under consideration, the wall beams are set back 2 in. from the faces of the columns, as shown in Fig. 122. At the top

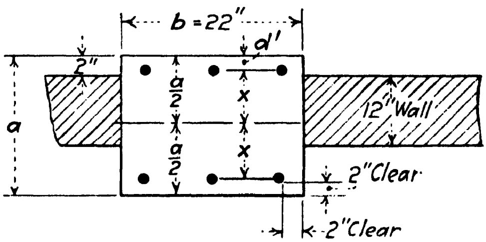


FIG. 122.

of the basement columns the countermoment due to column eccentricity is equal to the load on the intermediate column multiplied by the distance between the column centers, which is $\frac{1}{2}(20 - 18) = 1$ in.

Tables on pages 358 and 359 contain a summary of the design. Diagrams 17 and 18 have been used in computation of stresses (see also Chap. V). Figure 122 illustrates the terms used in the determination of moments of inertia. In computing the stress at the top of the basement column, the value of d' was assumed as the distance from the face of the column parallel to the wall to the center of the three bars parallel to that face, *i.e.*, $2\frac{1}{2}$ in. (see Fig. 123). Though not theoretically correct because of the two bars along the other sides of the columns, it simplifies the computations without introducing any appreciable error.

The allowable stress, where bending is included, is computed from the same formula as for the interior columns (Art. 224), except that the value of f_a is 0.8 of the value used for the interior columns, in accordance with the specifications of the Joint Code, as explained in Art. 111. Actually, the allowable stresses were taken from Diagram 11. The apparent excess strength of the top

and intermediate columns cannot be avoided, inasmuch as the minimum column size, and practically the minimum steel ratio, have been used.

The complete column details are shown in Fig. 123.

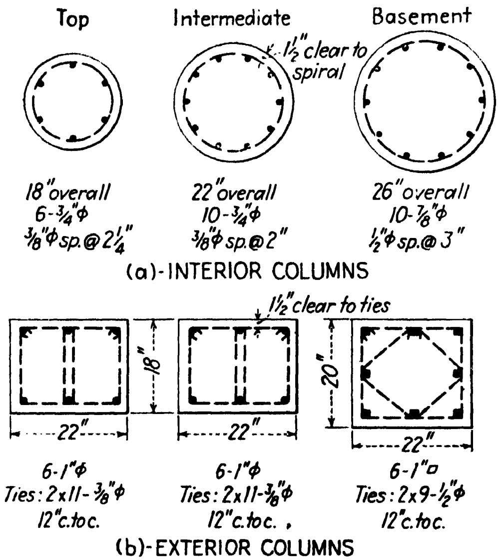


Fig. 123.—Details of columns.

EXTERIOR COLUMNS. COMPUTATION OF LOADS

Column	Load from	Amount of load, lb.	Size of column, in.	Vertical steel	A_t, sq. in.	$\frac{d'}{a}$
	Roof	38,000	22 × 18	6-1φ		
Top	Parapet†	14,000	A_g = 396 sq. in.	A_s = 4.71 sq. in.	447.8	0.139
	Column*	7,000		p_g = 0.0119		
	Total	59,000				
	Floor	71,000				
Intermediate	Wall	12,000	22 × 18	6-1φ	447.8	0.139
	Column	5,000	A_g = 396 sq. in.	A_s = 4.71 sq. in.		
	Total	147,000		p_g = 0.0119		
	Floor	71,000				
	Wall	12,000	22 × 20	8-1 in. sq.	528.0	0.125
Basement	Column	4,000	A_g = 440 sq. in.	A_s = 8.00 sq. in.		
	Total	234,000		p_g = 0.0182		

* This value includes the weight of the column above the roof (see Fig. 121). The weight of the bracket in each tier has been neglected, since it varies only from about 300 lb. for the basement column to 500 lb. for the top column.

† This value includes the weight of the stem of the wall beam.

EXTERIOR COLUMNS. COMPUTATION OF VALUES OF $\frac{I}{h}$

Column	$I_c = \frac{ba^3}{12}$, in.^{4}	$(n-1)I_s = 11(A_sx^2)$, in.^{4}	$I = I_c + (n-1)I_s$, in.^{4}	h, in.	$\frac{I}{h}$
Top	10,700	2,190	12,890	112	115.0
Intermediate	10,700	2,190	12,890	112	115.0
Basement	14,670	3,710*	18,380	91	202.0

* Here A_s is the area of six bars, since the I of the other two bars is zero.

EXTERIOR COLUMNS. COMPUTATION OF STRESSES

Column	Point	N, lb.	Counter-moment M_c, in.-lb.	$\frac{1}{16}Wl - M_c$, in.-lb.	M column, in.-lb.	$\frac{e}{a}$	np_g	K	$f_c = \frac{NK}{ba}$ actual, p.s.i.	f_c allowable, p.s.i.
Top	Top	54,000	14,000	541,000	541,000	0.556	0.143	4.62	693	900
	Bottom	59,000	12,000	1,030,000	515,000	0.484		4.04	653	880
Intermediate	Top	142,000			515,000	0.201	0.143	1.89	680	750
	Bottom	147,000	171,000*	871,000	316,000	0.119		1.47	548	695
Basement	Top	230,000			555,000	0.121	0.218	1.37	716	720
	Bottom	234,000			0	Taken by concrete (Table 9) = 198,000 lb. Taken by steel (Table 9) = 102,000 lb. Total = 300,000 lb				

* 171,000 = 147,000 × 1 + 12,000 × 2.

226. Design of Interior-column Footings. The interior-column footings are to be square, and reinforced in two directions. They will be made of 2000-lb. concrete, with intermediate grade steel. The allowable soil pressure is 2 tons per sq. ft.

The interior basement column is a 26-in. round column, with a total load of 377,000 lb. According to Art. 151, in the design of the footing the round column must be replaced with a square

column of the same area. The equivalent square has a side dimension equal to $\sqrt{531} = 23$ in. $= 1.92$ ft. The footing design from this point on is similar to the one in Art. 154. Only the essential computations are shown here.

The bearing area required is

$$\frac{377{,}000 + 30{,}000}{4000} = 102 \text{ sq. ft.}$$

A base 10 ft.-3 in. square (area = 105 sq. ft.) is selected. The net upward pressure is $\dfrac{377{,}000}{105} = 3590$ lb. per sq. ft.

$$M = 3590 \times 10.25 \times {}^{50}\!/_{12} \times 25 = 3{,}840{,}000 \text{ in.-lb.}$$

$$d = \sqrt{\frac{3{,}840{,}000}{10.25 \times 12 \times 131}} = 15.4 \text{ in.}$$

Assume that, for shear, an effective depth of 20 in. will be adequate. The width of the critical section for shear is then $23 + 2 \times 20 = 63$ in., and

$$V = \frac{5.25 + 10.25}{2} \times 2.5 \times 3590 = 69{,}500 \text{ lb.}$$

The allowable unit shear is 60 p.s.i.; hence,

$$60 = \frac{69{,}500}{63 \times 0.9 \times d}$$

Therefore,

$$d = 20.4 \text{ in.}$$

The assumed value of 20 in. may be considered satisfactory, in view of the many assumptions that are involved in the computations. The total thickness of the footing is then 24 in., and the weight 31,500 lb. The revised bearing area required is 102.1 sq. ft., and no change in the side dimensions is necessary.

$$A_s = \frac{0.85 \times 3{,}840{,}000}{20{,}000 \times 0.9 \times 20} = 9.1 \text{ sq. in.}$$

The allowable unit bond stress is $0.056 \times 2000 = 112$ p.s.i., and the maximum shear to be used in the equation for unit bond stress is $0.85 \times 3590 \times 10.25 \times 4.16 = 130{,}000$ lb.

$$\Sigma_0 \text{ (required)} = \frac{130{,}000}{112 \times 0.9 \times 20} = 64 \text{ in.}$$

Twenty-six $\frac{3}{4}$-in. round bars are selected; $A_s = 11.48$ sq. in., and $\Sigma_0 = 61.2$ in. The spacing of the bars is about $5\frac{3}{4}$ in., which is satisfactory.

Ten $\frac{7}{8}$-in. round dowels will be placed in the footing. The dowels must project into and above the footing a distance of 30 diameters. The length of the dowels is therefore $60 \times \frac{7}{8} = 52$ in. $= 4$ ft.-4 in.

A pedestal 6 in. thick is required to furnish, with the footing, 26 in. of embedment plus 4 in. of insulation for the dowels. The pedestal will be made 38 in. square (area 1444 sq. in.) which will furnish a 6-in. projection beyond the 26-in. round column. The unit stress on the gross area of the pedestal is $\frac{377{,}000}{1444} = 260$ p.s.i., which is well below the allowable value of $0.25 \times 2000 = 500$ p.s.i. (Art. 153). The actual loaded area of the pedestal is $\frac{\pi \times 26^2}{4} = 531$ sq. in., and the unit stress on this area is $\frac{377{,}000}{531} = 708$ p.s.i. The allowable unit stress (Art. 153) is

$$r_a = 0.25 \times 2000\sqrt[3]{\frac{1444}{531}} = 700 \text{ p.s.i.}$$

The slight excess stress (approximately 1 per cent) can be disregarded.

The complete details of the footing are shown in Fig. 124.

227. Design of Exterior-column Footings. Each exterior-column footing will consist of a solid block, rectangular in plan and reinforced in two directions. A 2000-lb. concrete will be used, with intermediate grade reinforcing steel. The basement

column is 22 × 20 in., and the total load on this column is 234,000 lb.

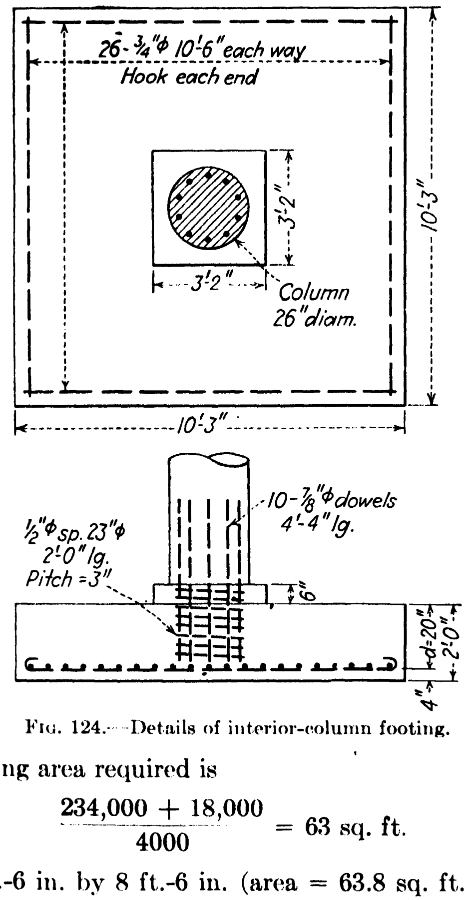


FIG. 124.—Details of interior-column footing.

The bearing area required is

$$\frac{234{,}000 + 18{,}000}{4000} = 63 \text{ sq. ft.}$$

A base 7 ft.-6 in. by 8 ft.-6 in. (area = 63.8 sq. ft.) is selected. The net upward pressure is $\frac{234\ 000}{63.8} = 3670$ lb. per sq. ft.

In the short direction (see Fig. 125*a*),

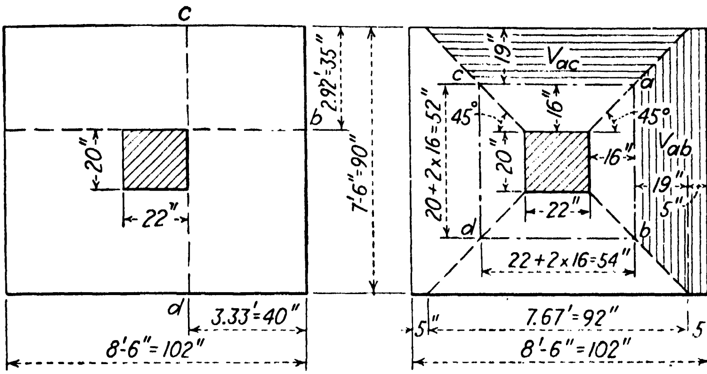
$$M_{ab} = 3670 \times 8.5 \times {}^{35}\!/_{12} \times 17.5 = 1{,}590{,}000 \text{ in.-lb.}$$

In the long direction,

$$M_{cd} = 3670 \times 7.5 \times {}^{40}\!/_{12} \times 20 = 1{,}830{,}000 \text{ in.-lb.}$$

$$d = \sqrt{\frac{1{,}830{,}000}{90 \times 131}} = 12.5 \text{ in.}$$

Assume that, for shear, an effective depth of 16 in. will be adequate. The width of the critical section *ab* (Fig. 125*b*) is



(a)-Critical sections for moment (b)-Critical sections for shear

FIG. 125.

then 20 + 2 × 16 = 52 in. = 4.33 ft., and *ac* is 22 + 2 × 16 = 54 in. = 4.5 ft.

$$V_{ab} = 3670\left(\frac{5}{12} \times 7.5 + \frac{7.5 + 4.33}{2} \times \frac{19}{12}\right) = 46{,}000 \text{ lb.}$$

$$V_{ac} = 3670\left(\frac{7.67 + 4.5}{2} \times \frac{19}{12}\right) = 35{,}400 \text{ lb.}$$

The depth required for shear will therefore obviously be governed by the critical section *ab*; hence, since the allowable unit shear is 60 p.s.i.,

$$60 = \frac{46{,}000}{52 \times 0.9 \times d}$$

Therefore,

$$d = 16.3 \text{ in.}$$

The assumed effective depth of 16 in. will be considered satisfactory. The total thickness of the footing is then 20 in., and the weight 16,000 lb. The revised required bearing area is 62.5 sq. ft., and no change in the assumed dimensions can be made if the usual practice of using 3-in. multiples for the lengths of the sides is followed.

In the long direction,

$$A_s = \frac{0.85 \times 1{,}830{,}000}{20{,}000 \times 0.9 \times 16} = 5.4 \text{ sq. in.}$$

In the short direction,

$$A_s = \frac{0.85 \times 1{,}590{,}000}{20{,}000 \times 0.9 \times 16} = 4.7 \text{ sq. in.}$$

Subsequent investigations for bond stresses (see Art. 150) show that twenty-four 5/8-in. round bars are required in each direction, furnishing an area of 7.76 sq. in. and a total perimeter of 47.2 in. Since the two sides of the footing are so nearly equal, the bars in each band can be spaced uniformly over the entire width of the footing, with about 4 in. insulation at the sides of the band. The requirements for placing the short bars, as given in Art. 148, would not materially alter this arrangement.

Eight 1-in. square dowels, 5 ft.-0 in. long, are required in each footing. A pedestal 14 in. deep is necessary in order to furnish, with the available footing thickness, the necessary 2 ft-6 in. embedment, with 4 in. insulation at the bottom. The size of the pedestal will be 30 × 28 in., which will allow a 4-in. projection on all sides of the column. The average unit compression on the gross area of the pedestal is $\frac{234{,}000}{30 \times 28} = 280$ p.s.i., and the stress on the loaded area is $\frac{234{,}000}{22 \times 20} = 530$ p.s.i. The allowable value of the latter stress is

$$r_a = 0.25 \times 2000\sqrt[3]{\frac{30 \times 28}{22 \times 20}} = 615 \text{ p.s.i.}$$

Complete details of the footing are shown in Fig. 126.

228. Design of Wall Beams. With the proposed framing as shown in Figs. 127 and 130, the unsupported spans of the end wall beams are 18 ft.-9 in. and 17 ft.-9 in., while those of the intermediate wall beams are 20 ft.-2 in. and 19 ft.-2 in. for the short

and long sides of the building, respectively. Since it is desirable to keep the depth of the wall beams constant throughout the building on account of architectural appearance, it is first neces-

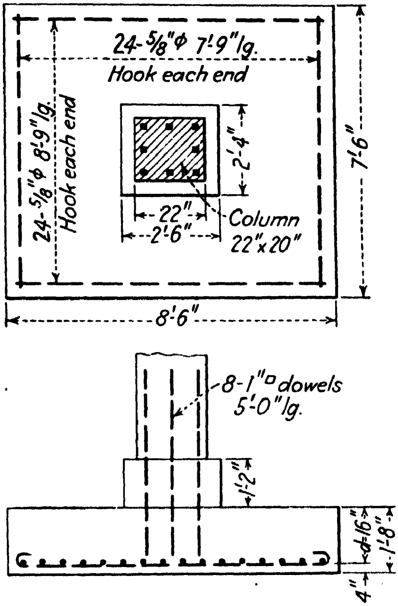
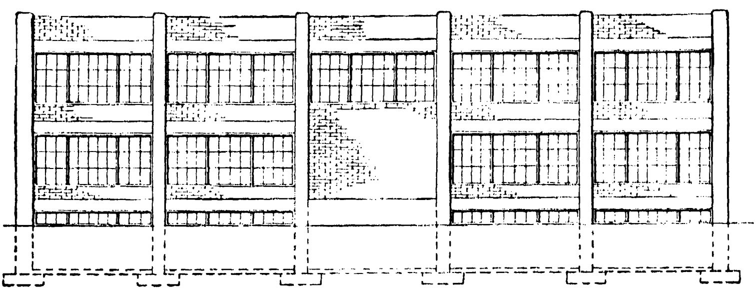


FIG. 126.—Details of exterior-column footing.

sary to determine the cross-section required for the maximum moment and shear. The width of the beams is taken as 12 in. in


REAR ELEVATION
FIG. 127.

all cases. As stated in Art. 218, $f'_c = 2500$ p.s.i., and $f_s = 18{,}000$ p.s.i.

The maximum shear occurs in the first- and second-floor intermediate beams along the short side of the building. According to Table D, Appendix C, a marginal beam which has a depth greater than $1\frac{1}{2}$ times the minimum slab thickness shall be designed to carry, in addition to the load superimposed directly upon it, a uniformly distributed load equal to at least one-quarter of the total live and dead load for which the adjacent panel is designed. The total load on the wall beam per linear foot is therefore,

$$
\begin{array}{lcr}
\text{Brick sill} = 2.5 \times 1 \times 140 & = & 350 \text{ lb.} \\
\text{Windows} = 7.5 \times 8 & = & 60 \\
\text{Stem of beam, assumed} & = & 165 \\
\text{Floor load} = \frac{1}{4} \times 300 \times 21 & = & \underline{1575} \\
 & & 2150 \text{ lb.}
\end{array}
$$

$$V = 2150 \times \frac{20.17}{2} = 21{,}600 \text{ lb.}$$

$$d \text{ (for shear)} = \frac{21{,}600}{150 \times \frac{7}{8} \times 12} = 13.7 \text{ in.}$$

The maximum moment occurs in the end spans on the short side of the building. These beams are continuous over one support only and are designed for a moment of $\frac{1}{10}wl^2$.

$$M = \frac{1}{10} \times 2150 \times (18.75)^2 \times 12 = 907{,}000 \text{ in.-lb.}$$

Since the beams are, in effect, T-beams with the tee on one side only, the effective width of flange, according to the Joint Code (see Art. 85), is

$$b = \frac{1}{12} \times 18.75 \times 12 + 12 = 30.75 \text{ in.}$$

On account of the relatively thick flange, it is apparent that the neutral axis will undoubtedly be in the flange. With this assumption, the beam must be designed as a rectangular beam 30.75 in. wide, and the required depth is obtained from equation (6), Art. 47.

$$d \text{ (for moment)} = \sqrt{\frac{907{,}000}{173 \times 30.75}} = 13.0 \text{ in.}$$

In order to give a desired architectural appearance, a total height of 21 in. is adopted for all of the wall beams. The effec-

tive depth at the center of the beam, allowing for one row of steel with 3 in. insulation, is 18 in., and with the proposed arrangement of the steel (Fig. 133) the effective depth at the support is 19 in. The weight of the stem per foot is then 165 lb., as assumed.

229. *Design of L-1 and L-2.* Since it has been determined above that a cross-section of 12 by 21 in. is satisfactory for all wall beams, it is now merely necessary to determine the area of steel required for each beam and to provide for shearing stresses. Assuming $j = 0.9$, for the end beam L-1,

$$A_s = \frac{907{,}000}{18{,}000 \times 0.9 \times 18} = 3.11 \text{ sq. in.}$$

Four 1-in. round bars furnish 3.14 sq. in.

Before investigating this beam over the support (first interior support) it is necessary to determine the amount of steel required in the intermediate beam. In L-2,

$$M = \tfrac{1}{12} \times 2150 \times (20.17)^2 \times 12 = 874{,}000 \text{ in.-lb.}$$

$$A_s = \frac{874{,}000}{18{,}000 \times 0.9 \times 18} = 3.04 \text{ sq. in.}$$

Four 1-in. round bars are selected.

Two bars from each beam are bent up to provide for the negative moment at the supports. Allowing 2 in. of insulation above the center of the steel at the top of the beam,[1] the effective depth at the support is 19 in.

$$\frac{d'}{d} = \frac{3}{19} = 0.158 \qquad np = np' = 12 \times \frac{3.14}{12 \times 19} = 0.165$$

From Diagrams 5 and 6, $k = 0.372$ and $j = 0.866$. At the first interior support,

$$f_s = \frac{907{,}000}{3.14 \times 0.866 \times 19} = 17{,}500 \text{ p.s.i.}$$

$$f_c = \frac{17{,}500 \times 0.372}{12(1 - 0.372)} = 865 \text{ p.s.i.}$$

both of which are satisfactory.

[1] Since the top steel is protected also by the brick sill, 2 in. of insulation is ample at the top of the beam.

The maximum unit bond stress is

$$u = \frac{21{,}600}{4 \times 3.14 \times \frac{7}{8} \times 19} = 104 \text{ p.s.i.}$$

The points at which the two bars (50 per cent of the steel) may be bent up are obtained from Diagram 1. In L-1 the bars may be bent $0.18 \times 18.75 = 3.4$ ft. from the edge of the column, and they are actually bent 3.33 ft. from the column. In L-2, the bars may be bent $0.21 \times 20.17 = 4.25$ ft. from the edge of the column, and they are bent 4.25 ft. from the column.

Assuming the point of inflection to be $\frac{1}{4}l$ from the edge of the column, in L-2 the bent bars must reach the top not closer to the edge of the support than $\frac{1}{2} \times \frac{1}{4} \times 20.17 = 2.5$ ft. The actual distance as shown in Fig. 133 is 2 ft.-11 in., which is satisfactory. The bent bars are continued along the top into the adjacent span to the point of inflection. The straight bars are continued 2 ft.-6 in. beyond the center of the column.

Since only two bars are bent up in each beam, and these at one place, their strength is disregarded in providing for diagonal tension; stirrups are placed at suitable intervals to furnish all of the web strength necessary. In L-2, the distance x_1 from the edge of the column is

$$x_1 = \frac{20.17}{2} - \frac{50 \times 12 \times \frac{7}{8} \times 18.0}{2150} = 5.7 \text{ ft.}$$

At the support, $V' = V - V_c = 21{,}600 - 50 \times 12 \times \frac{7}{8} \times 19 = 11{,}600$ lb. With $\frac{3}{8}$-in. round U-stirrups, and $f_v = 18{,}000$ p.s.i.,

$$s = \frac{2 \times 0.1104 \times 18{,}000 \times \frac{7}{8} \times 19}{11{,}600} = 5.6 \text{ in.}$$

Two feet from the edge of the column, $V' = 11{,}600 - 2 \times 2150 = 7300$ lb., and

$$s = 5.6 \times \frac{11{,}600}{7300} = 8.9 \text{ in.}$$

Four feet from the edge of the column, $V' = 7300 - 2 \times 2150 = 3000$ lb., and

$$s = 5.6 \times \frac{11{,}600}{3000} = 21.6 \text{ in.}$$

The maximum allowable spacing = $0.5 \times 19 = 9.5$ in. The first stirrup will be placed 2 in. from the edge of the column; the remainder spaced 5 at 5 in. and 5 at 8 in.

In L-1,

$$x_1 = \frac{18.75}{2} - \frac{50 \times 12 \times \frac{7}{8} \times 18.0}{2150} = 5.0 \text{ ft.}$$

At the support, $V' = \frac{18.75}{2} \times 2150 - 50 \times 12 \times \frac{7}{8} \times 19 =$ 10,200 lb., and

$$s = \frac{2 \times 0.1104 \times 18{,}000 \times \frac{7}{8} \times 19}{10{,}200} = 6.5 \text{ in.}$$

Two feet from the support, $V' = 10{,}200 - 2 \times 2150 = 5900$ lb., and

$$s = \frac{6.5 \times 10{,}200}{5900} = 11.2 \text{ in.}$$

The first stirrup will be placed 2 in. from the edge of the column; the remainder will be spaced 5 at 5 in. and 4 at 8 in. This maintains the same spacing as in the adjoining beam L-2, but uses two less stirrups.

The areas of steel required in the other wall beams, floor and roof, and the arrangement of the web reinforcement are determined in a manner similar to the above. The results are indicated in Fig. 133. The roof wall beams are made the same depth as those at the second-floor level for architectural reasons. In investigating the beams on the long side of the building it should be borne in mind that the first interior supports of the line are designed for a moment of $\frac{1}{10}wl^2$, while at the other interior supports a $\frac{1}{12}$ coefficient is used. The end supports at the corners of the building are arbitrarily reinforced for negative moment by bending up one-half of the longitudinal steel in the end beams. The necessary steel details are shown in Figs. 138 and 139.

230. Design of Stairway Slab. The stairway, which extends from the basement to the second-floor level, is located as shown in Fig. 130. The opening made by the stair well and the future elevator shaft is framed by a series of beams as shown. In order to keep the thickness of the stair slab down to a reasonable minimum, beam *B*-9 is placed at the edge of the floor-level landing slab. The stair slab is designed as a simple slab with a span equal to the horizontal distance from the middle of beam *B*-9 to the middle of the wall support of the intermediate landing slab. As stated in Art. 218, a 2500-lb. concrete is used and f_s = 18,000 p.s.i.

The dead load on the stair slab is made up of the weight of the intermediate landing slab, the weight of the inclined slab, and the weight of the treads. For purposes of computation it is sufficiently accurate to assume that this total dead load is uniformly distributed over the horizontal span. The live load is assumed as 100 lb. per sq. ft. of horizontal surface (see Art. 193).

The widths of the stair slabs and landing slabs as shown in Fig. 132 will prove satisfactory in the ordinary building of this size. The width of each tread, exclusive of the nosing, is made $10\frac{1}{4}$ in. and the rise about $7\frac{3}{16}$ in. These values satisfy the rules for convenient climbing (Art. 193) and also give a uniform rise of all treads between landing slabs.

Allowing 6-in. bearing on the brick exterior wall, the horizontal span of the stair slab is 12 ft.-1 in. The total length of slab between supports is $\sqrt{6^2 + (7.66)^2} + 3.83 = 13.5$ ft. Assuming the weight of the slab as 72 lb. per sq. ft., the total load on a 1-ft. strip of slab is

$$\begin{aligned}
\text{Treads} &= 9 \times \frac{7.19 \times 10.25}{2 \times 144} \times 150 = && 350 \text{ lb.} \\
\text{Slab} &= 13.5 \times 72 && = 970 \\
\text{Live load} &= 12 \times 100 && = \underline{1200} \\
& && \text{Total} = 2520 \text{ lb.}
\end{aligned}$$

$$M = \tfrac{1}{8} \times 2520 \times 12.1 \times 12 = 45{,}800 \text{ in.-lb.}$$

$$d = \sqrt{\frac{45{,}800}{173 \times 12}} = 4.7 \text{ in.}$$

An effective depth of $4\frac{3}{4}$ in. is selected; with 1-in. insulation the total slab thickness is $5\frac{3}{4}$ in. and the weight is 72 lb. per sq. ft. as assumed.

$$A_s = \frac{45{,}800}{18{,}000 \times 0.867 \times 4.75} = 0.62 \text{ sq. in. per ft. of width.}$$

This area is furnished by $\frac{1}{2}$-in. square bars, $4\frac{1}{2}$ in. center to center. One $\frac{1}{2}$-in. square bar, 3 ft.-8 in. long, is placed under each tread at right angles to the main reinforcement, as shown in Fig. 132.[1]

The floor-level landing slab is made 8 in. thick, and every other bar of the stair-slab reinforcement is continued across this slab. Two $\frac{1}{2}$-in. square bars 10 ft.-0 in. long are placed in the intermediate landing slab, at right angles to the main reinforcement, in order to assist in distributing the load on that slab and to provide for temperature stresses. Negative-moment stresses in the stair slabs across beams *B*-9 are provided for by means of short bent bars placed in the top of the slabs at these points as shown in Fig. 132.

231. Design of Beams Framing Stair Wall (see Figs. 128 and 130). *Beam B*-9. This beam supports a uniform load along its entire length, consisting of one-half of the stairway-slab load and one-half of the floor-level landing-slab load in addition to its own weight. The span of the beam is 9 ft.-6 in., the distance center to center of beams *B*-6 and *B*-7. The beam is designed as a T-beam with the tee on one side only; this is necessary because of the break between the up and down stair slabs at the landing.

Beam B-6. This beam is a simply supported rectangular beam, the load on which consists of the weight of a 4-in. hollow tile partition and the concentrated load from *B*-9 in addition to its own weight.

[1] The detailing shown in Fig. 132 assumes that the stairway slab will be poured with the structural frame. A more widely used method of construction places the stair slab after the main structure has been completed. In such cases, recesses must be left in the beams *B*-9 and in the brick wall in order to furnish support for the future stair slabs, and dowels must be placed in the beams *B*-9 so as to project into the stair slabs when they are poured.

Beam B-8. This beam carries the concentrated load from *B*-6, one-half of the load from the landing slab, and the weight of a 4-in. hollow tile partition in addition to its own weight. *B*-8 has been placed at the edge of the column strip. This fact, together with the improbability of having the full live load

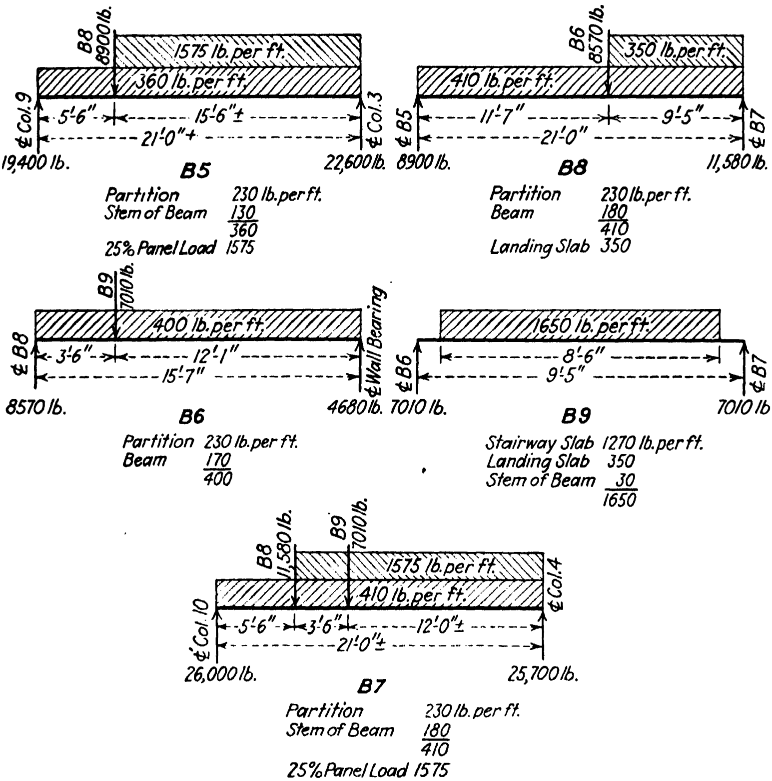


FIG. 128.—Loading diagrams for beams framing stair well.

on the portion of the floor near *B*-8, justifies the assumption that no floor load is supported by *B*-8.[1] Due to the open shaft on one side of the beam at the point of maximum moment, it must be designed as a T-beam with the tee on one side only. In

[1] Since beam *B*-8 and the slab adjacent to it are poured at the same time they cannot act independently of each other. A more conservative design would assume that a part of the adjacent slab load is supported by *B*-8.

order to illustrate the method of design where such irregular loads are involved, the design of *B*-8 is included below.

The loads on the beam (see Figs. 128 and 130) are as follows:

Uniform load over entire length of beam

Partition $= 20 \times 11.5$	$=$	230 lb. per lin. ft.
Weight of beam	$=$	180 lb. per lin. ft.
Total	$=$	410 lb. per lin. ft.

Additional uniform load from stairway landing slab,

$$\tfrac{1}{2} \times 3.5 \times 200 = 350 \text{ lb. per lin. ft.}$$

Concentrated load from *B*-6 = 8570 lb.

$$R_L = \frac{8570 \times 9.42 + 350 \times \dfrac{(9.42)^2}{2} + 410 \times \dfrac{21^2}{2}}{21} = 8900 \text{ lb.}$$

$R_R = 11{,}580$ lb.

The point of zero shear, which locates the point of maximum moment, occurs under the concentrated load. The maximum moment is

$$M = \left(8900 \times 11.5 - 410 \times \frac{(11.58)^2}{2}\right) \times 12 = 900{,}000 \text{ in.-lb.}$$

According to the Joint Code, the effective width of flange is equal to $10 + \frac{1}{12} \times 21 \times 12 = 31$ in. Because of the relatively large thickness of the flange it is reasonable to assume that the neutral axis is in the flange, in which case the equations for rectangular beams apply.

$$d = \sqrt{\frac{900{,}000}{31 \times 173}} = 12.9 \text{ in.}$$

An effective depth of 14 in. will be used, in order to avoid interference with the bars in *B*-6.

$$b' = \frac{11{,}580}{150 \times \frac{7}{8} \times 14} = 6.3 \text{ in.}$$

A width of stem of 10 in. will be used, in order to give a reasonable amount of resistance to torsional stresses. With 3 in. of insula-

tion, to provide for two rows of bars, the total height of the beam is 17 in. and the weight per foot is 180 lb., as assumed.

$$A_s = \frac{900{,}000}{18{,}000 \times 0.875 \times 14} = 4.07 \text{ sq. in.}$$

Four 1-in. square bars are selected (4.0 sq. in.). With $np = 12 \times \frac{4.0}{31 \times 14} = 0.011$ and $\frac{t}{d} = \frac{8}{14} = 0.57$, Diagram 2 shows that the neutral axis is in the flange, so that, from Table 7 with $p = \frac{4.0}{31 \times 14} = 0.0092$, $j = 0.876$, and the revised steel area required is 4.06 sq. in. The four 1-in. bars may be considered satisfactory.

Since this beam is poured monolithically with the beams supporting it, there will exist some negative moment at the supports. The amount of this bending moment is dependent upon too many factors to permit of an accurate determination. In order to provide for the stresses of negative moment, one-half of the steel is bent up and hooked over the support. Adequate bond resistance is furnished by the two deformed bars remaining. The points at which this steel may be bent up may best be found by determining the points where the bending moment is one-half of the maximum, by means of an equation involving the distances x from the supports to these points. For the left end of the beam as shown in Fig. 128, this equation is as follows:

$$8900x - 410\frac{x^2}{2} = 37{,}500$$

$$x = 4.7 \text{ ft.}$$

The two bars on this side are bent up 4 ft.-6 in. from the center of *B*-5. A similar equation for the right end of the beam locates the point where the bending moment is 37,500 ft.-lb., at a distance of 3.7 ft. from the center of *B*-7. The two bars are bent up 3 ft.-6 in. from that point.

Since only two bars are bent up, and since these are bent at a single point, they will not be depended upon for diagonal tension resistance, and vertical stirrups will be used wherever

web reinforcement is required. In computing the spacing of stirrups, it is assumed that the concrete can take care of the shear up to a unit value of 50 p.s.i. The total shear that the concrete can resist is, therefore,

$$V_c = 50 \times 10 \times \tfrac{7}{8} \times 14 = 6140 \text{ lb.}$$

The stirrups will be designed to resist all of the shear in excess of this amount.

With ¼-in. round U-stirrups, the maximum spacing which can be used at the left support (see Fig. 128) is obtained from equation (14), Art. 78.

$$s = \frac{2 \times 0.0491 \times 18{,}000 \times \tfrac{7}{8} \times 14}{8900 - 6140} = 7.8 \text{ in.}$$

The distance from the left support to the section where the unit shear is 50 p.s.i. is $\dfrac{8900 - 6140}{410} = 6.8$ ft. The maximum allowable spacing of stirrups is $0.5 \times 14 = 7$ in. The first stirrup will be placed 2 in. from the edge of beam *B*-5, then 11 at 7 in.

The maximum spacing which can be used at the right support (near *B*-7) is

$$s = \frac{2 \times 0.0491 \times 18{,}000 \times \tfrac{7}{8} \times 14}{11{,}580 - 6140} = 4 \text{ in.}$$

The distance from the right support to the section where the unit shear is 50 p.s.i. is $\dfrac{11{,}580 - 6140}{350 + 410} = 7.1$ ft. Two feet from *B*-7 the maximum allowable spacing is

$$s = \frac{2 \times 0.0491 \times 18{,}000 \times \tfrac{7}{8} \times 14}{11{,}580 - 6140 - 2(350 + 410)} = 5.5 \text{ in.}$$

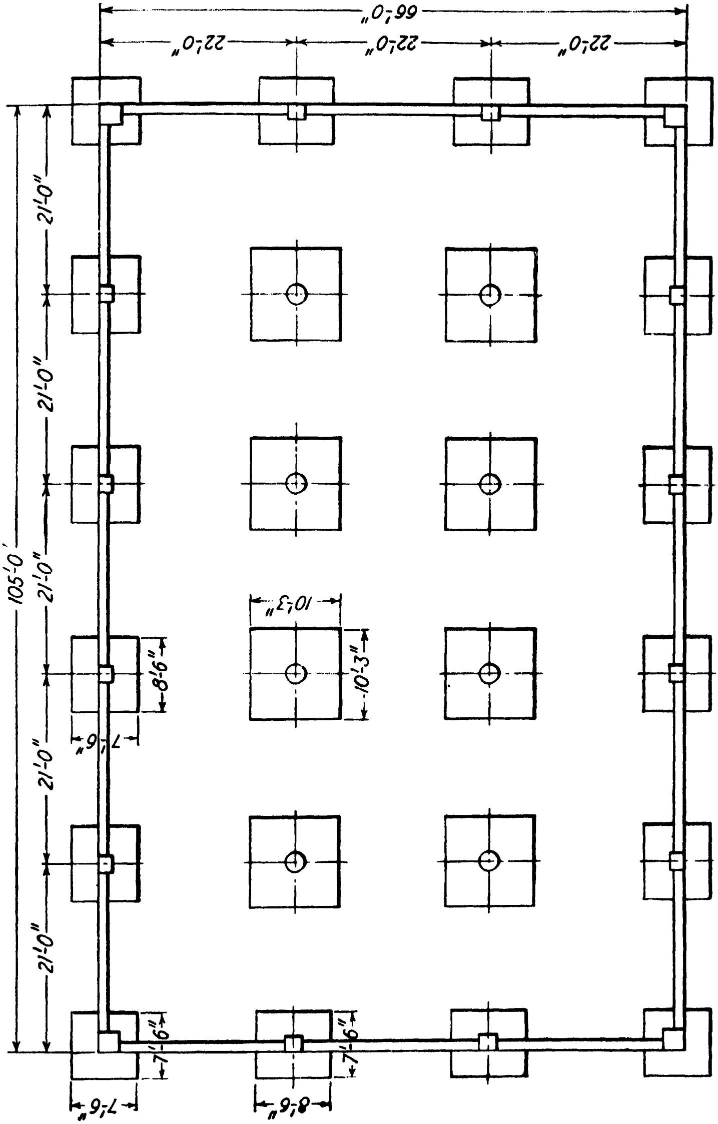
Four feet from *B*-7, $s = 9.0$ in. The first stirrup will be placed 2 in. from the edge of *B*-7, then 4 at 4 in., 4 at 5 in., and 5 at 7 in.

Beam B-5. In addition to supporting the concentration from beam *B*-8 and its own weight, beam *B*-5 must be designed to support the partition as shown and the required proportion of the floor load. The beam is a T-beam, with the tee on one side only.

Beam B-7. This beam supports loads as stated for *B*-5, and, in addition, the concentrated load from *B*-9. It is a T-beam, with the tee on one side only.

The complete framing details of the stairway beams are shown in Fig. 134, and the required steel details in Figs. 138 and 139.

232. Detail Drawings. Assembled drawings showing essential details of a building similar to the one designed in the preceding articles are shown in Figs. 140 and 141. Figure references on the assembled drawings refer to figures in this text, from which figures the necessary details may be noted.



FOUNDATION PLAN
Fig. 129.

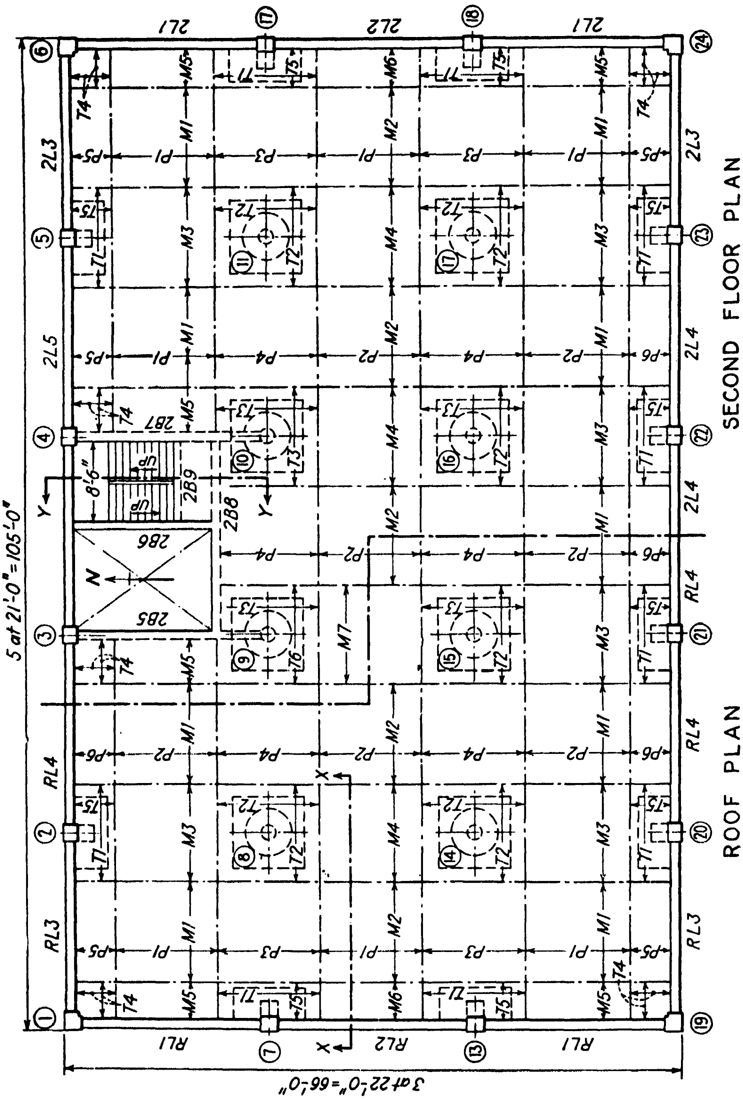
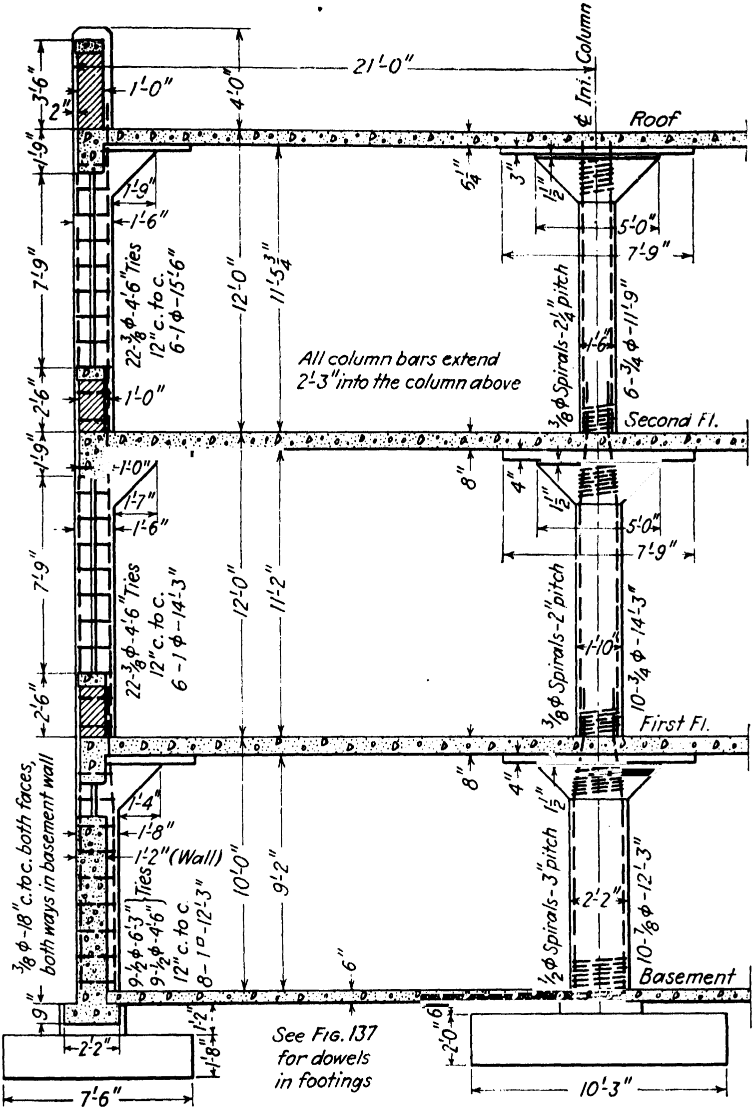
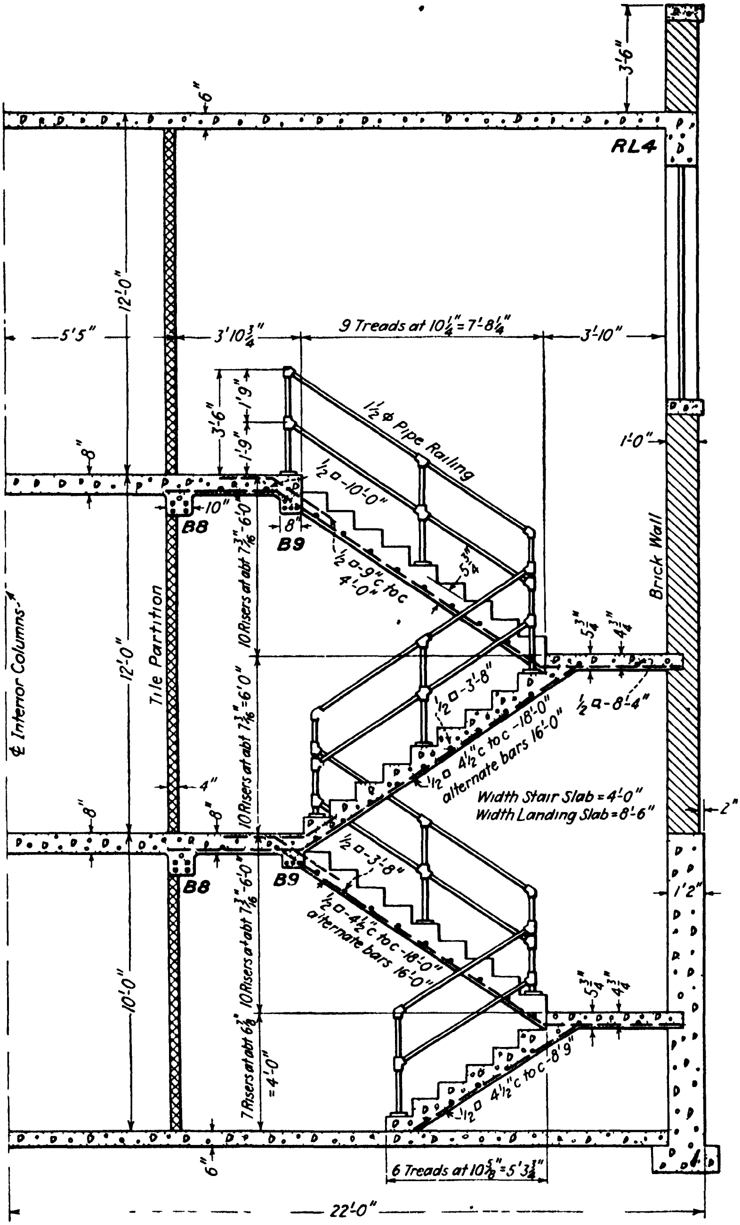


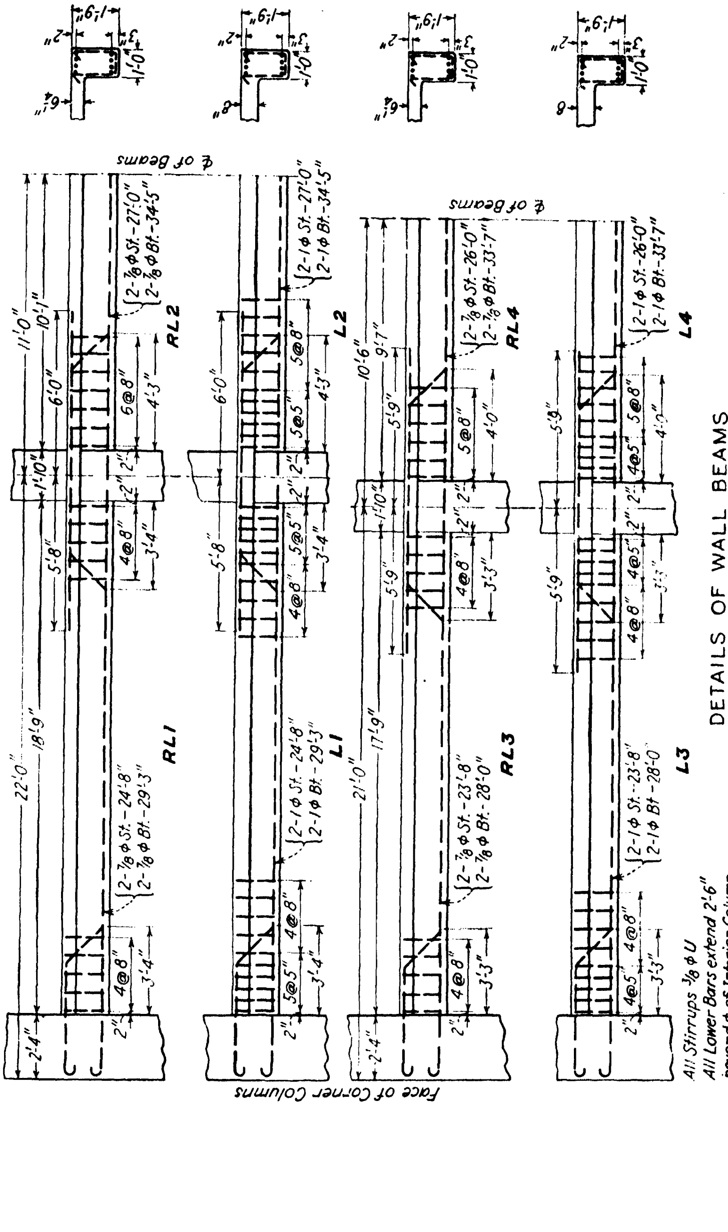
Fig. 130.



PARTIAL CROSS SECTION
SECTION X - X
Showing Column Steel
FIG. 131.

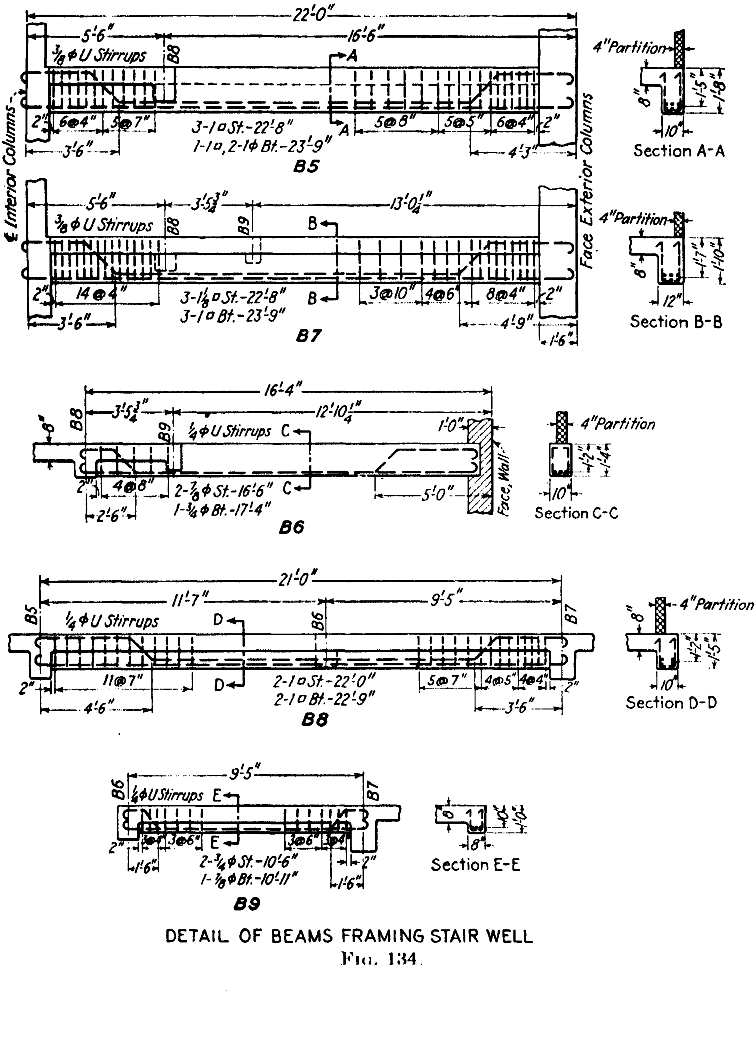


DETAIL OF STAIRWAY
SECTION Y-Y
Fig. 132.



DETAILS OF WALL BEAMS

FIG. 133.



DETAIL OF BEAMS FRAMING STAIR WELL

FIG. 134.

FLOOR BAND SCHEDULE				
Mk.	No. of Bands	Bars per Band		Bending Details
		Straight Bars	Bent Bars	
P 1	8	11-½φ-17'-0"	12-½φ-27'-0"	9'-0" 5'-8"
P 2	6	9-½φ-13'-8"	9-½φ-31'-10"	9'-0" 9'-0"
P 3	4	7-⅝φ-18'-0"	12-⅝φ-28'-6"	12'-0" 5'-8"
P 4	6	6-⅝φ-15'-4"	9-⅝φ-34'-10"	12'-0" 12'-0"
P 5	6	3-⅝φ-18'-0"	6-⅝φ-28'-6"	12'-0" 5'-8"
P 6	3	3-⅝φ-15'-4"	4-⅝φ-34'-10"	12'-0" 12'-0"
M 1	9	11-½φ-17'-9"	12-½φ-28'-3"	9'-5" 5'-11"
M 2	5	9-½φ-14'-4"	9-½φ-33'-4"	9'-5" 9'-5"
M 3	6	7-⅝φ-18'-9"	12-⅝φ-29'-9"	12'-6" 5'-11"
M 4	3	6-⅝φ-16'-4"	9-⅝φ-36'-4"	12'-6" 12'-6"
M 5	6	3-⅝φ-18'-9"	6-⅝φ-29'-9"	12'-5" 5'-11"
M 6	2	3-⅝φ-16'-4"	4-⅝φ-36'-4"	12'-6" 12'-6"
M 7	1	6-⅝φ-16'-4"	4-⅝φ-36'-4"	12'-6" 12'-6"
			5-⅝φ-35'-4"	12'-6" 11'-6"
T 1	10		11-⅝φ-7'-0"	6" hook
T 2	10	4-⅝φ-14'-0"		
T 3	5	7-⅝φ-14'-0"		
T 4	12		5-⅝φ-7'-0"	6" hook
T 5	10	5-⅝φ-14'-0"		
T 6	1	9-⅝φ-14'-0"		

FIG. 135.

ROOF BAND SCHEDULE

Mk.	No. of Bands	Bars per Band		Bending Details
		Straight Bars	Bent Bars	
P1	6	3-½φ-17'-0"	9-½φ-26'-10"	9'-0" 5'-8"
			5-½φ-17'-9"	5'-8"
P2	9	7-½φ-13'-8"	7-½φ-31'-8"	9'-0" 9'-0"
P3	4	8-½φ-18'-0"	12-½φ-28'-5"	12'-0" 5'-8"
P4	6	7-½φ-15'-3"	9-½φ-34'-8"	12'-0" 12'-0"
P5	4	4-½φ-18'-0"	6-½φ-28'-5"	12'-0" 5'-8"
P6	6	3-½φ-15'-3"	4-½φ-34'-8"	12'-0" 12'-0"
M1	10	3-½φ-17'-9"	9-½φ-28'-1"	9'-5" 5'-11"
			5-½φ-18'-6"	5'-11"
M2	5	7-½φ-14'-4"	7-½φ-33'-2"	9'-5" 9'-5"
M3	8	8-½φ-18-9"	12-½φ-29'-7"	12'-6" 5'-8"
M4	4	7-½φ-16'-3"	9-½φ-36'-2"	12'-6" 12'-6"
M5	4	4-½φ-18'-9"	6-½φ-29'-7"	12'-6" 5'-8"
M6	2	3-½φ-16'-3"	4-½φ-36'-2"	12'-6" 12'-6"
T1	12		13-½φ-7'-0"	6" hook
T2	12	7-½φ-14'-0"		
T3	4	10-½φ-14'-0"		
T4	8		6-½φ-7'-0"	6" hook
T5	12	5-½φ-14'-0"		

FIG. 136.

COLUMN SCHEDULE

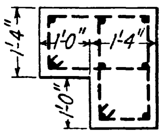
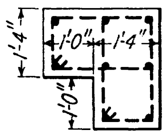
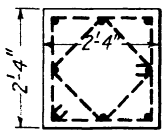
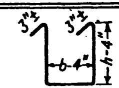
Column Mark	1, 6, 19, 24		2-5, 7, 12, 13, 18, 20-23	8-11, 14-17
No. of Columns	4		12	8
Second Floor	2'-4" x 2'-4" 8-$\frac{7}{8}\phi$ — 15'-6" 22-$\frac{3}{8}\phi$ Ties-6'-6" @ 12"		1'-6" x 1'-10" 6-1ϕ-15'-6" 22-$\frac{3}{8}\phi$ Ties-4'-6" @ 12" See Fig. 123 for detail of ties	18" Diameter 6-$\frac{3}{4}\phi$-11'-9" $\frac{3}{8}\phi$ Spirals @ $2\frac{1}{4}$"
First Floor	2'-4" x 2'-4" 8-$\frac{7}{8}\phi$-14'-3" 22-$\frac{3}{8}\phi$ Ties-6'-6" @ 12"		1'-6" x 1'-10" 6-1ϕ-14'-3" 22-$\frac{3}{8}\phi$ Ties-4'-6" @ 12"	22" Diameter 10-$\frac{3}{4}\phi$-14'-3" $\frac{3}{8}\phi$ Spirals @ 2"
Basement	2'-4" x 2'-4" 8-$\frac{7}{8}\phi$-12'-3" 9-$\frac{3}{8}\phi$ Ties-8'-6" @ 12" 9-$\frac{3}{8}\phi$ Ties-6'-3" @ 12"		1'-8" x 1'-10" 8-1□-12'-3" 9-$\frac{1}{2}\phi$ Ties-6'-3" @ 12" 9-$\frac{1}{2}\phi$ Ties-4'-6" @ 12"	26" Diameter 10-$\frac{7}{8}\phi$-12'-3" $\frac{1}{2}\phi$ Spirals @ 3"
Dowels	8-$\frac{7}{8}\phi$-4'-4"		8-1□-5'-0"	10-$\frac{7}{8}\phi$-4'-4"

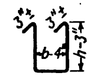
Fig. 137.

BEAM SCHEDULE

Mk.	Cross Section	No. of Beams	Longitudinal Steel per Beam		Stirrups per Beam	
			Bent Bars	Straight Bars	No. Size, Length	Spacing Each End
RL1	12"x21"	4	2-7/8 φ — 29'-3"	2-7/8 φ — 24'-8" ¶	10-3/8 φ — 4'-0"	4 @ 8"
RL2	12"x21"	2	2-7/8 φ — 34'-5"	2-7/8 φ — 27'-0"	14-3/8 φ — 4'-0"	6 @ 8"
RL3	12"x21"	4	2-7/8 φ — 28'-0"	2-7/8 φ — 23'-8" ¶	10-3/8 φ — 4'-0"	4 @ 8"
RL4	12"x21"	6	2-7/8 φ — 33'-7"	2-7/8 φ — 26'-0"	12-3/8 φ — 4'-0"	5 @ 8",
L1	12"x21"	8	2-1 φ — 29'-3"	2-1 φ — 24'-8" ¶	20-3/8 φ — 4'-0"	5 @ 5", 4 @ 8"
L2	12"x21"	4	2-1 φ — 34'-5"	2-1 φ — 27'-0"	22-3/8 φ — 4'-0"	5 @ 5", 5 @ 8"
L3	12"x21"	8	2-1 φ — 28'-0"	2-1 φ — 23'-8" ¶	18-3/8 φ — 4'-0"	4 @ 5", 4 @ 8"
L4	12"x21"	6	2-1 φ — 33'-7"	2-1 φ — 26'-0"	20-3/8 φ — 4'-0"	4 @ 5", 5 @ 8"
L5	12"x21"	4	2-1 φ — 28'-11"	2-1 φ — 24'-7" ¶	20-3/8 φ — 4'-0"	4 @ 5", 5 @ 8"
B5	10"x20"	2	2-1 φ — 23'-9" 1-1 □ — 23'-9"	3-1 □ — 22'-8" ✱	29-3/8 φ — 3'-10"	S. 6 @ 4", 5 @ 7" N. 6 @ 4", 5 @ 5", 5 @ 8"
B6	10"x16"	2	1-3/4 φ — 17'-4"	2-7/8 φ — 16'-6" ✱	5-1/4 φ — 3'-4"	S. 4 @ 8" N. None
B7	12"x22"	2	3-1 □ — 23'-9"	3-1⅛ □ — 22'-8" ✱	31-3/8 φ — 4'-4"	S. 14 @ 4" N. 8 @ 4", 4 @ 6", 3 @ 10"
B8	10"x17"	2	2-1 □ — 22'-9"	2-1 □ — 22'-0" ✱	26-1/4 φ — 3'-4"	W. 11 @ 7" E. 4 @ 4", 4 @ 5", 5 @ 7"
B9	8"x12	2	1-7/8 φ — 10'-11"	2-3/4 φ — 10'-6" ✱	14-1/4 φ — 2'-4"	3 @ 4", 3 @ 6"



Detail of Stirrups in Wall Beams



Detail for B5,B7,B8,B9

✱ *Straight bars so marked have 6" hooks each end*
¶ *Straight bars so marked have 6" hooks one end*

FIG. 138.

BEAM STEEL BENDING SCHEDULE

Mk.	Total No. of Bars	Size and Length		Add for Hook			H	45°		Add for Hook	H Out to Out
RL1	8	7/8 Φ	29'-3"	6"	4'-0"	1'-10½"	12'-1"	1'-10½"	8'-11"	—	1'-5"
RL2	4	7/8 Φ	34'-5"	—	9'-6"	1'-10½"	11'-8"	1-10½"	9'-6"	—	1'-5"
RL3	8	7/8 Φ	28'-0"	6"	3'-11"	1'-10½"	11'-3"	1'-10½"	8'-7"	—	1'-5"
RL4	12	7/8 Φ	33'-7"	—	9'-4"	1'-10½"	11'-2"	1'-10½"	9'-4"	—	1'-5"
L1	16	1 Φ	29'-3"	6"	4'-0"	1'-10½"	12'-1"	1'-10½"	8'-11"	—	1'-5"
L2	8	1 Φ	34'-5"	—	9'-5"	1'-10½"	11'-8"	1-10½"	9'-6"	—	1'-5"
L3	16	1 Φ	28'-0"	6"	3'-11"	1'-10½"	11'-3"	1'-10½"	8'-7"	—	1'-5"
L4	12	1 Φ	33'-7"	—	9'-4"	1'-10½"	11'-2"	1'-10½"	9'-4"	—	1'-5"
L5	8	1 Φ	28'-11"	6"	4'-2"	1'-10½"	11'-2"	1'-10½"	9'-4"	—	1'-5"
B5	4 2	1 Φ 1 □	23'-9" 23'-9"	6"	2'-5"	1'-8"	14'-3"	1'-8"	2'-9"	6"	1'-3" 1'-3"
B6	2	¾ Φ	17'-4"	6"	1'-6"	1' 5"	8'-10"	1'-5"	3'-2"	6"	1'-1"
B7	6	1 □	23'-9"	6"	2'-2'	1'-10½"	13'-9"	1'-10½"	3'-1"	6"	1'-5"
B8	4	1 □	22'-9"	6"	3'-7"	1'-3½"	13'-0"	1'-3½"	2'-7"	6"	1'-0"
B9	2	7/8 Φ	10'-11"	6"	10"	11"	6'-5"	11"	10"	6"	9"

FIG. 139.

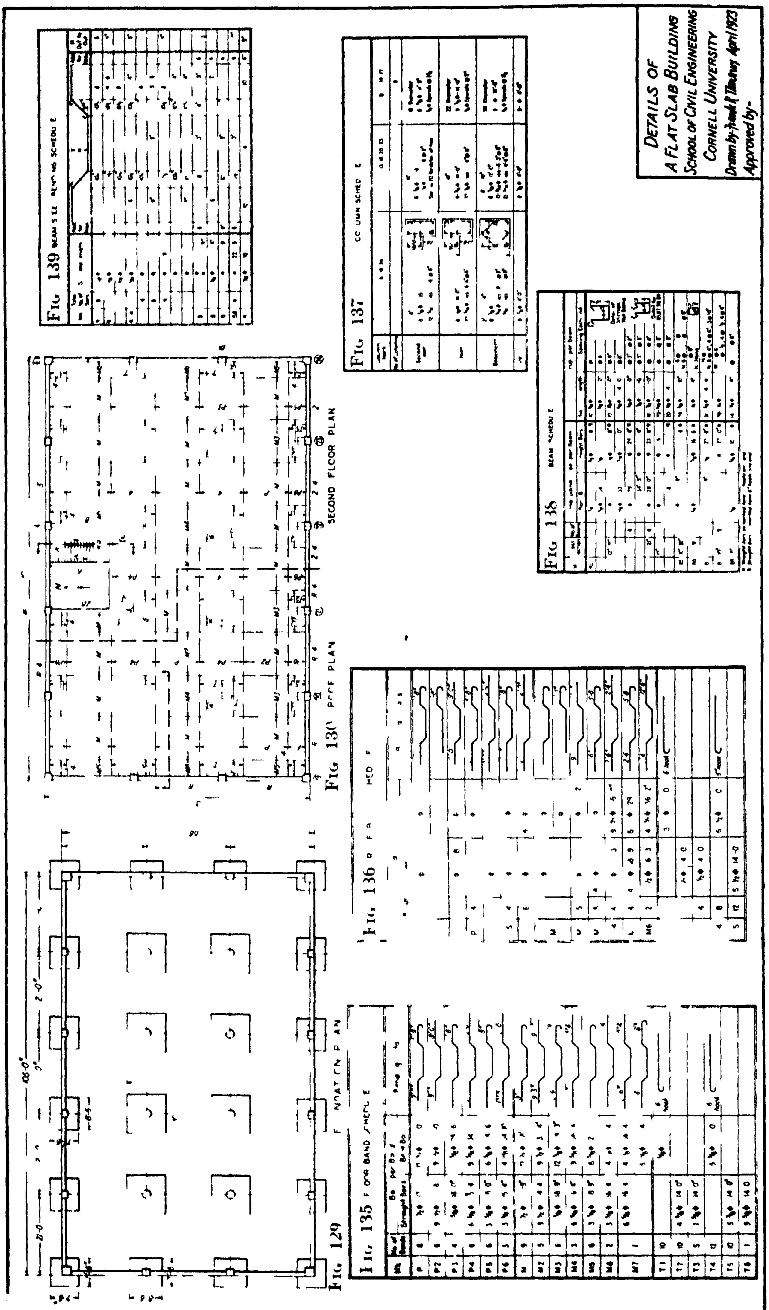


FIG. 140 Details of a flat-slab building

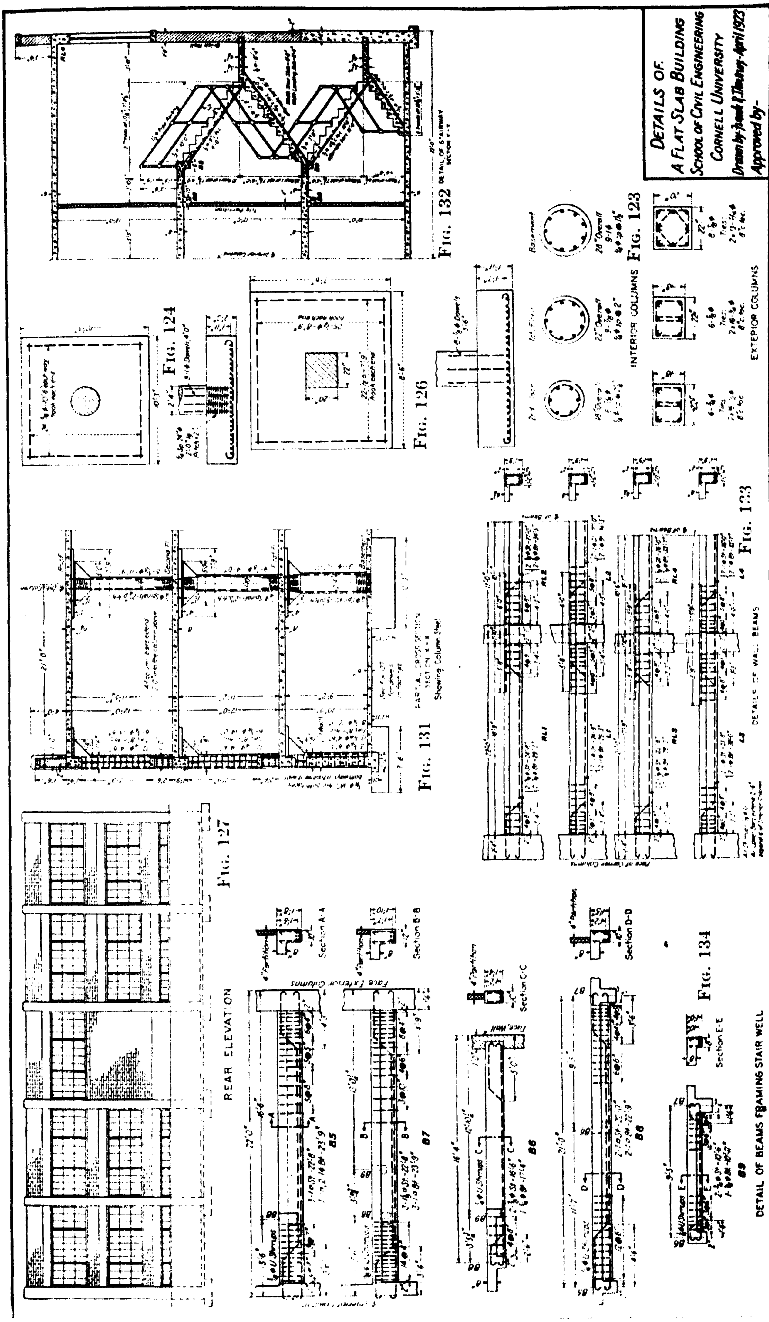


FIG. 141.—Details of a flat-slab building.

CHAPTER IX

RETAINING WALLS

233. Introductory. A pile of earth, cinders, or other material possessing more or less frictional stability, will, when deposited loosely in an unrestrained position, assume a definite slope, the steepness of which depends upon the internal friction of the material and other conditions, such as moisture content, etc. A mound of earth whose sides are permitted to assume this natural slope will, when thoroughly compacted, maintain its own integrity, and support external loads to a maximum amount which depends, among other things, upon the bearing qualities of the soil.

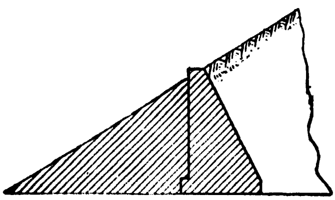


FIG. 142.

In engineering construction it frequently becomes necessary to prevent the sides of such a pile of earth from assuming this natural slope. Such a condition occurs when the width of a cut or embankment is limited either by restrictions of economy or right of ownership. The most common examples of the latter limitation are found in railway and highway construction where the width of the right of way is fixed. In such cases it is essential that the earth be held in position by means of a wall capable of resisting the lateral pressure caused by the conditions of restraint.

A wall whose express purpose is to hold in position a bank of earth or similar material is termed a retaining wall. The first step toward the design of a retaining wall is to determine its location. If the wall is to run along a fixed property line, such as a highway or a railroad, this provides definite placing. As is often the case, the amount of land available for the construction of a given cut or fill may be unlimited, but the cost of cutting

or filling sufficiently to allow the natural slope of the earth to obtain may be excessive. Wherever it is found that a retaining wall of the necessary height and section is cheaper than the additional cut or fill that it replaces, economy favors the construction. In Fig. 142, the wall replaces the shaded volume of fill. A few trials will show at what point the wall should be placed to obtain the minimum cost. Rough designs of the required wall sections are sufficient for making comparisons of costs for various positions of the wall.

234. Types of Retaining Walls. Masonry retaining walls may be divided into two general classes: (1) the gravity wall, which retains the bank of earth entirely by its own weight; (2) the reinforced concrete wall, which utilizes the weight of the earth behind it in resisting the overturning moment of the retained material. In this latter class are included the cantilever wall, a type of construction consisting of a vertical arm supported upon a horizontal base slab, the vertical arm acting as a free cantilever in overcoming the pressure from the earth; and the counterfort wall, the vertical slab of which is anchored or tied to the base slab by means of counterforts or buttresses—triangular cross walls extending from the top of the vertical slab to the extreme point of the base slab at regular intervals throughout the length of the wall. The vertical slab of the reinforced walls may be placed at the front, at the rear, or at any point along the base slab, the exact location depending upon limitations of economy and construction. Where conditions permit, a toe extension of from $\frac{1}{3}$ to $\frac{1}{2}l$ will produce a more economical design than would result if the vertical arm were placed at the front edge of the base slab. Paaswell[1] proves that for a given location of the resultant pressure, the most economical width of base occurs when the vertical arm is placed over the assumed point of application of the resultant pressure.

The back of a gravity-type wall may be vertical, or may slope toward or away from the filling. The most economical section is obtained when the back slopes toward the filling. On account

[1] Paaswell, "Retaining Walls," p. 82.

of difficulties of construction, however, the use of this section is restricted to comparatively isolated cases where unusual foundation conditions exist. In cold localities, where there is danger of upheaval by frost, economy may well be sacrificed to security, and the back be given a slight batter forward. Where this batter is of an appreciable amount, added stability may be obtained by constructing the back as a series of steps, thus utilizing more fully the relieving weight of the earth directly over the base of the wall.

The section of wall to be chosen will be determined by a consideration of economy, ease of construction, foundation requirements, and other factors imposed by existing conditions. In comparing the relative economy of gravity walls and reinforced concrete walls, the added cost of construction in the case of the latter must be included. A careful study of the different types leads to the conclusion that unless affected by unusual conditions, the gravity type will prove most economical for low walls, the cantilever type for walls of medium height, and the counterfort type for the higher walls. The critical height, or height of separation between the various types, is not clearly defined, since it depends upon too many economic as well as constructive conditions. In general it is found to be uneconomical to use the counterfort construction for walls that are less than 18 ft. in height.

235. Conditions of Loading. There are three general conditions of loading that need consideration: (1) walls with no surcharge, the top surface of the fill being horizontal and level with the top of the wall; (2) walls with an inclined surcharge, the top surface of the fill extending upward and back from the top of the back of the wall; (3) walls with a horizontal surcharge extending some distance above the top of the wall.

The angle of inclination δ in Case 2 is usually taken as the angle of repose ϕ of the retained material. For ordinary conditions, this may be assumed as 33 degrees 42 minutes, which corresponds to a slope of $1\frac{1}{2}:1$.

Case 3 includes loadings in which the actual surface of earth does not extend above the top of the wall, but supports an

external load such as a building, railroad tracks, etc. The loads are converted into an equivalent height of earth above the top of the wall by dividing the weight of the additional load per square foot by the weight of the earth per cubic foot, and the pressure estimated for this equivalent height of surcharge of earth.

When the additional load from the fill consists of the weight from one or more railroad tracks, it is best to follow the recom-

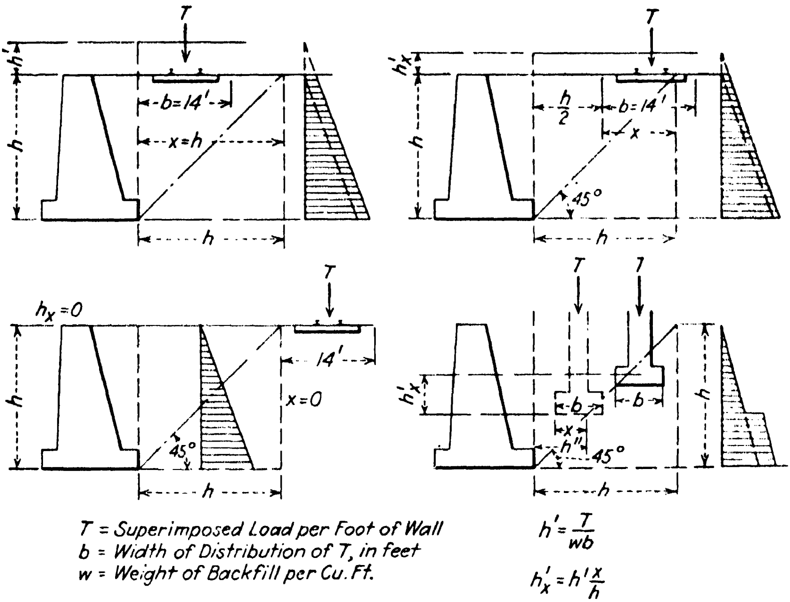


FIG. 143.

mendations of the American Railway Engineering Association as given below.

(*a*) In calculating the surcharge due to the track, the entire load shall be taken as uniformly distributed over a width of 14 ft. for a single track or tracks spaced more than 14 ft. centers, and the distance center to center of tracks where tracks are spaced less than 14 ft.

(*b*) In calculating the pressure on a retaining wall where the filling carries permanent tracks or structures, the full effect of the loaded surcharge shall be considered where the edge of the distributed load or structure is vertically above the back edge of the

heel of the wall. The effect of the loaded surcharge may be neglected where the edge of the distributed load or structure is at a distance from the vertical line through the back edge of the heel of the wall equal to h, the height of the wall. For intermediate positions the equivalent uniform surcharge load is to be taken as proportional. For example, for a track with the edge of the distributed load at a distance $\frac{h}{2}$ from the vertical line through the back edge of the heel of the wall, the equivalent uniform surcharge load is one-half of the normal distributed load distributed over the filling. Figure 143 explains the distribution. The height of surchage loading will be equal to the load per linear foot divided by bw (b = 14 ft. for a single-track railway). Where the edge of the distributed load cannot come nearer to the vertical line through the back edge of the heel of the wall than $h - x$, the equivalent uniformly distributed load in terms of the height is

$$h'_x = h'\frac{x}{h}$$

The terms of this equation are explained in Fig. 143.

236. Determination of Earth Thrust. The first essential in any design is the determination of the force to be resisted. The principal force governing the dimensions of a retaining wall is the pressure exerted by the retained material in its attempt to assume its natural slope. In order fully to determine the pressure of the filling against the wall, the resultant must be known in amount, in line of action, and in point of application.

Many theories have been advanced which lead to a purely academic determination of earth thrust. These mathematical discussions of the action of earth masses premise an ideal, incompressible, homogeneous material, without cohesion, possessing frictional resistance between its particles, and of indefinite extent in the mass. Such a fill is rarely found in practice. The degree of exactness of the thrust as determined by any of the theoretical methods will depend upon the difference between the actual conditions and the theoretical.

While refinements in the theory of earth pressure are, therefore, unwarranted from a practical standpoint, such academic thrust determinations, when modified in accordance with the conclusions drawn from actual tests and the results of engineering experience, may become the basis for a rational working formula. The subcommittee of the American Railway Engineering Association on Design of Plain and Reinforced Retaining Walls and Abutments, in its report of 1917, comments upon earth pressures as follows: "Actual tests on an extensive scale will be required to produce any results of real value. No such tests have yet been made, and in the absence of such definite information as they might be expected to produce, and believing that the intelligent use of theoretical formulas leads to economical and proper design, this committee therefore recommends that Rankine's formulas, which consider that the filling is a granular mass of indefinite extent, without cohesion, be used in the designing of retaining walls."

Rankine's development of earth pressure, which starts out with an infinitesimal prism and leads to an expression for the thrust of the entire earth mass upon a given surface, leads to the general equation,

$$P = C \times \frac{wh_1^2}{2} \qquad (76)$$

in which P is the total thrust upon the back of the wall, w the weight of the earth per cubic foot, h_1 the height of the earth column in feet, and C a constant depending upon the angle of inclination of the back of the wall, the conditions of loading, and the physical properties of the earth fill. The original development by Rankine includes formulas for vertical walls only. This theory has been expanded by Ketchum[1] to include walls leaning away from the filling and walls leaning toward the filling. These latter equations, in addition to the original vertical wall expressions, are given in the report of the Committee mentioned above, in substantially the form shown in Fig. 144.

[1] KETCHUM, "Walls, Bins, and Grain Elevators."

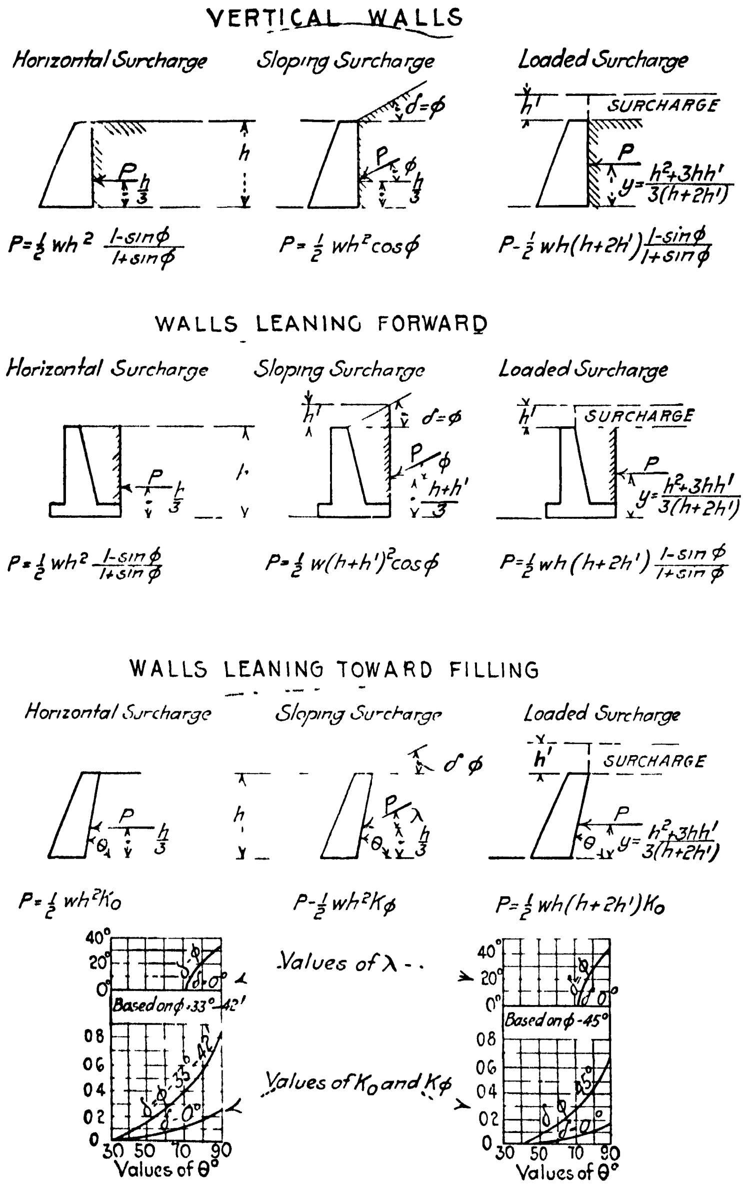


FIG. 144 Formulas for earth pressures on retaining walls.

The amount of the pressure on any given horizontal strip 1 ft. in height at a distance x ft. below the surface of the earth is given by the equation

$$P_1 = Cwx. \tag{77}$$

The pressure distribution along the back of the wall for Cases

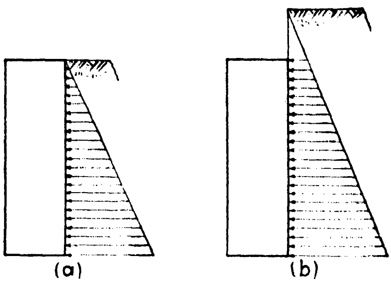


FIG. 145.

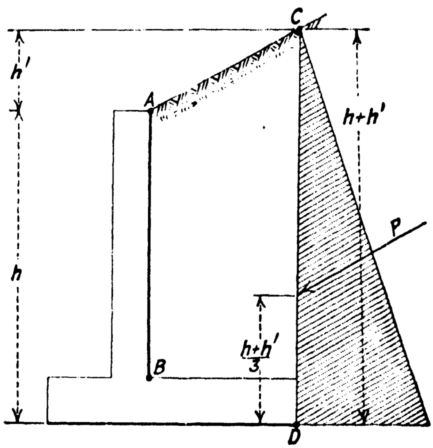


FIG. 146.

1 and 2 of Art. 235 is shown in Fig. 145*a*, and for Case 3 in Fig. 145*b*.

In determining foundation pressures in a wall of a cross-section as shown in Fig. 146, the earth fill vertically above the base slab is considered as a resisting pressure equivalent to the same weight of masonry, and the total overturning pressure is the total earth

thrust on a vertical plane at the heel of the wall. The height CD is therefore used in equation (76) for determining the earth thrust. This applies also to walls of gravity section in which the back slopes away from the fill. In determining the bending moment on the vertical arm AB (Fig. 146), the total thrust is due to a column of earth of a height equal to AB.

237. Line of Action and Point of Application of Earth Pressure. The line of action of the total thrust upon a wall with a vertical back exposed to the action of the earth is parallel to the top surface of the filling. In a wall whose back slopes away from the fill the total thrust upon a vertical plane through the heel of the wall acts parallel to the top surface of the earth. For walls leaning toward the filling, the resultant pressure P will be horizontal for a wall without surcharge or with a horizontal loaded surcharge, and will make an angle λ with the horizontal for a wall with a sloping surcharge. The values of λ will vary from the angle of surcharge, where the wall is vertical, to zero, where Rankine's theory shows that the resultant pressure is horizontal. Values of λ are given in Fig. 144.

For walls with no surcharge, or a sloping surcharge, the point of application of the total earth thrust is usually assumed at a point in the plane against which the earth is acting and at a distance of one-third its height, measured from the base of the plane. For walls with a loaded surcharge, the point of application is taken at the center of gravity of the pressure quadrilateral shown in Fig. 145*b*. The location of the point of application of the resultant thrust for the various conditions of loading is given in Fig. 144.

238. Factors Affecting the Design. Following a determination of the earth's thrust, an investigation must be made of all possible modes of failure, and each element of the construction so proportioned as to make such failures impossible. A gravity type of wall may fail by sliding along the plane of the base, by overturning, or by settlement at the toe, caused by crushing of the soil there. An extreme case of this will also cause overturning. A reinforced concrete wall may fail in any of the ways mentioned above. In addition, any of the thin sections which

together furnish the necessary strength and rigidity might yield in a manner similar to a corresponding element in other constructions.

239. Overturning and Crushing. When the overturning moment Py (Fig. 147) becomes equal to the stability moment Wg, the wall is at the point of incipient overturning. This condition exists when the resultant of the overturning and resisting loads passes through the toe of the wall. As long as the resultant load falls within the base, the wall is safe against overturning. The desirable location of the point of intersection of the resultant load and the base depends upon a consideration of foundation pressures.

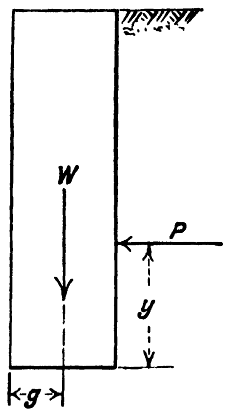


FIG. 147.

In Fig. 148*a*, let E represent the resultant of the total earth pressure and the resisting weight on a 1-ft. strip of wall, and let F represent its vertical component. The point of intersection of E with the base of the wall is located by the distance a. Under these conditions the column of earth directly under the base sustains an eccentric load F at a distance of $\frac{l}{2} - a$ from the gravity axis of the column. Applications of the principles of flexure and direct stress as outlined in Chap. V lead to the general expression for the values of p_1 and p_2, the unit pressures at the toe and heel of the wall, respectively, as follows:

$$p_1 = (4l - 6a)\frac{F}{l^2} \tag{78}$$

$$p_2 = (6a - 2l)\frac{F}{l^2} \tag{79}$$

Examination of equations (78) and (79) shows that uniform soil pressure occurs only when $a = \frac{l}{2}$, *i.e.*, when no eccentricity

of load exists. For this condition, $p_1 = p_2 = \frac{F}{l}$. Since an ideal foundation design demands a uniform distribution of upward pressure, the desired location of the point of intersection of the resultant E with the base is at the middle of the base. The

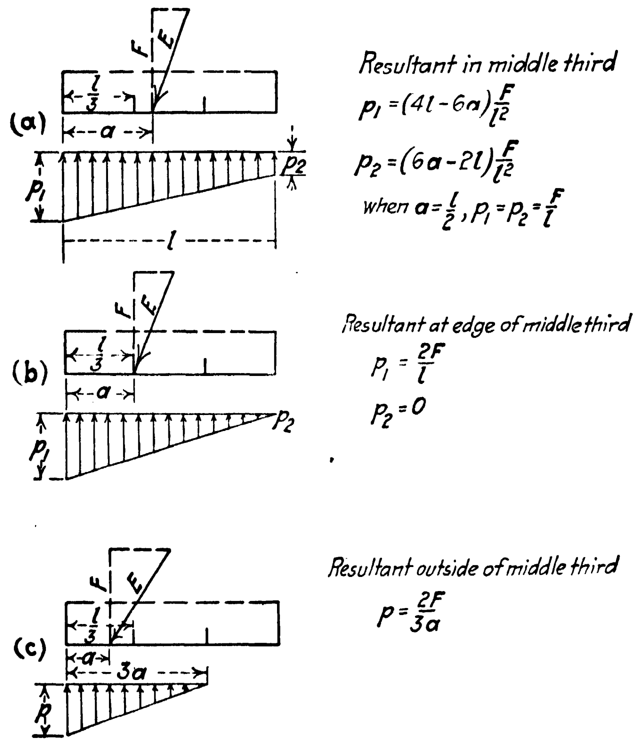


FIG. 148.—Formulas for pressures under foundations of retaining walls.

economics of retaining walls, however, usually forbids the fulfilment of this condition. Further examination of the equations indicates that when the intersection occurs within the middle third of the base, compression exists over the entire foundation—the footing is bearing on the soil along its entire length. If a is less than $\frac{l}{3}$, tension exists at the heel—the footing is not bearing

on the soil along its entire length. If the construction is such that this tension cannot be provided for, the entire load will have to be resisted by the compression under the forward portion of the wall, that is, over a distance $3a$ from the toe (see Fig. 148c). The expression for the unit pressure at the toe under this condition becomes

$$p_1 = \frac{2F}{3a} \tag{80}$$

A larger toe pressure results from the use of equation (80) than would be the case if equation (78) were applicable.

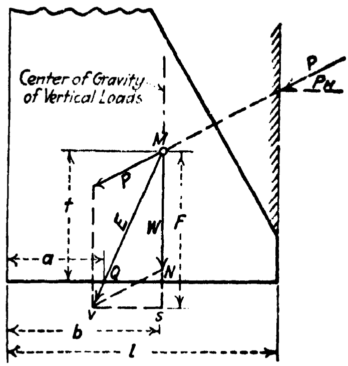


Fig. 149.

Analysis of the above discussion leads to the conclusion that, since it is economically undesirable under ordinary conditions to proportion the wall so as to cause a uniform distribution of the pressure on the foundation bed, a satisfactory design results when the line of action of the resultant pressure on the foundation bed intersects the base at any point within the middle third, provided the safe bearing pressure of the foundation material is not exceeded. When the wall rests upon a compressible material where settlement may be expected, the resultant thrust E should strike at the middle or back of the middle of the base so that the wall will settle toward the filling. Where the wall rests on solid rock, or is carried on piles, the resultant thrust E may strike slightly outside the middle third, provided the wall is sufficiently safe against overturning, and also provided the maximum allowable pressure is not exceeded.

The requirements of foundation pressures, therefore, require the wall to be safe against incipient overturning; a wall proportioned to distribute properly the load over the foundation will furnish a factor of safety, against the tendency to topple over, greater than 1. To allow for obvious exigencies, this condition

should always be investigated and a suitable factor of safety applied. Since the wall is designed for the greatest load that is anticipated, this factor need not be large. A value of 2 is satisfactory for ordinary design. An expression for the overturning safety factor may be derived as follows:

In Fig. 149, let F = vertical component of resultant E.
P_H = horizontal component of earth thrust.
l = length of base.
n = factor of safety against overturning.
a = distance from toe to point of intersection of resultant with base.

In the triangles MNQ and MSV,

$$\frac{t}{b - a} = \frac{F}{P_H}$$

$$\frac{F}{P_H \times t} = \frac{1}{b - a}$$

Multiplying each side by b,

$$\frac{Fb}{P_H \times t} = \frac{b}{b - a} = n \qquad (81)$$

An approximate value for n may be found by substituting for b in the above equation its near equivalent $\frac{l}{2}$.

$$n = \frac{l}{l - 2a} \qquad (82)$$

240. Sliding. In order to prevent sliding of the wall along the base, the frictional resistance of the base against the foundation material must be greater than the horizontal component of the thrust on the back of the wall. The frictional resistance of the base is equal to the resisting weight multiplied by the coefficient of friction of the masonry on the soil. The coefficient of friction of masonry on dry clay varies from 0.5 to 0.6; on wet clay 0.33; on sand 0.4; on gravel 0.6. A factor of safety of 1½ is usually considered satisfactory.

In case an adverse condition of sliding exists, the base may be widened, thus increasing the weight of the wall; narrow shallow trenches may be dug in the foundation, forming projections which will materially increase the resistance to sliding; the base may be inclined upward toward the toe; or the forward portion of the trench may be filled with masonry so that the wall butts directly against the original earth. Figure 150 illustrates the principles involved in sliding resistance.

241. Details of Construction. The front of the wall is usually built with a batter of from $\frac{1}{2}$ to 1 in. in 12 in. A coping, project-

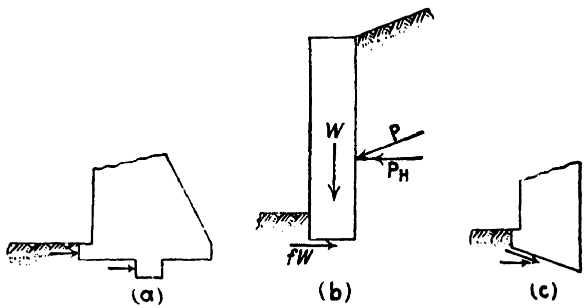


FIG. 150.

ing a short distance beyond the wall, adds to the architectural appearance and, to a certain extent, protects the masonry in the body of the wall from dripping water. The base of the foundation should be a sufficient distance below the surface of the ground to insure against the dangers of action by frost, a minimum of $2\frac{1}{2}$ ft. being sufficient in ordinary climates. Expansion joints should be provided at intervals along the wall, preferably not farther apart than 30 ft. In the reinforced walls where cracks would not only be unsightly but also detrimental to the integrity of the wall, additional steel should be placed at right angles to the main reinforcement to provide for temperature and shrinkage stresses. An amount varying from 0.1 to 0.33 per cent of the cross-section area is usually specified. Proper drainage of the fill behind the wall may be effected by inserting 4-in. drain tiles through the wall near the bottom at intervals of 10 to 15 ft., and piling crushed stone, gravel, or other coarse material around

these "weep holes" at the back. At least one drain should be provided for each pocket formed by counterforts.

242. Application of Fundamental Principles. It should be remembered that[1] "no theoretical formulas can be more than an aid to the judgment of the experienced designer. The main value of such formulas is in obtaining economical proportions, in obtaining a proper distribution of the stresses, and in making experience already gained more valuable." A careful study should be made of the conditions in the design of each wall and modifications of the above discussion made wherever required by the peculiarities of the problem under consideration.

The foregoing fundamental considerations will be elaborated upon, and the additional computations required to proportion properly the elements composing the various types of reinforced concrete walls will be explained in the following typical designs.

243. Design of Gravity Wall. A gravity wall 16 ft.-0 in. high is to sustain a bank of earth with a loaded horizontal surcharge equivalent to 4 ft. of filling above the top of the wall. The safe bearing pressure on the clay foundation bed is 2 tons per sq. ft. The weight of the retained fill is 100 lb. per cu. ft., and the angle of repose 33 degrees 42 minutes. Determine the required section of wall.

The ordinary procedure in the design of a gravity-type wall is to select a tentative section, the dimensions of which are governed by the judgment and experience of the designer. This tentative section is then analyzed in accordance with the principles outlined above, and modifications in the assumed dimensions made where necessary.

In the present case, the tentative dimensions are shown in Fig. 151. Investigation must be made with and without the portion of surcharge directly over the base included in the resisting weight W_3. Assuming that the former condition obtains, the analysis is as follows:

From Fig. 144, the total pressure per foot of wall against the vertical plane through the heel of the wall is

[1] Report of Committee on Masonry, *Bull.* A.R.E.A., February, 1917.

$$P = \frac{1}{2}wh(h + 2h') \times \frac{1 - \sin\phi}{1 + \sin\phi}$$
$$= \frac{1}{2} \times 100 \times 16(16 + 8)(0.286) = 5500 \text{ lb.}$$

The distance of the point of application of P above the bottom of the plane is

$$y = \frac{h^2 + 3h'h}{3(h + 2h')}$$
$$= \frac{16^2 + 3 \times 4 \times 16}{3(16 + 8)} = 6.24 \text{ ft.}$$
$$W_1 = \frac{8.5 + 1.5}{2} \times 13 \times 150 = 9750 \text{ lb.}$$
$$W_2 = 9.5 \times 3 \times 150 = 4280 \text{ lb.}$$
$$W_3 = \frac{17 + 4}{2} \times 7.0 \times 100 = 7350 \text{ lb.}$$
$$W = 21{,}380 \text{ lb.}$$

By taking moments about the toe of the wall, the point of application of the total resisting load W is found to be 5.05 ft. from that point.

The resultant of P and W = 22,100 lb. and intersects the base 3.44 ft. from the toe, or 0.27 ft. inside the forward edge of the middle third.

$$p_1 = (4 \times 9.5 - 6 \times 3.44)\frac{21{,}380}{9.5^2} = 4100 \text{ lb. per sq. ft.}$$
$$p_2 = (6 \times 3.44 - 2 \times 9.5)\frac{21{,}380}{9.5^2} = 389 \text{ lb. per sq. ft.}$$

Investigation of the same section, assuming that the surcharge directly over the base is not included in the resisting load W_3, shows that the total load W = 18,580 lb., and its point of application is 4.90 ft. from the toe of the wall. The point of application of the resultant of P and W is 3.05 ft. from the toe. This is but

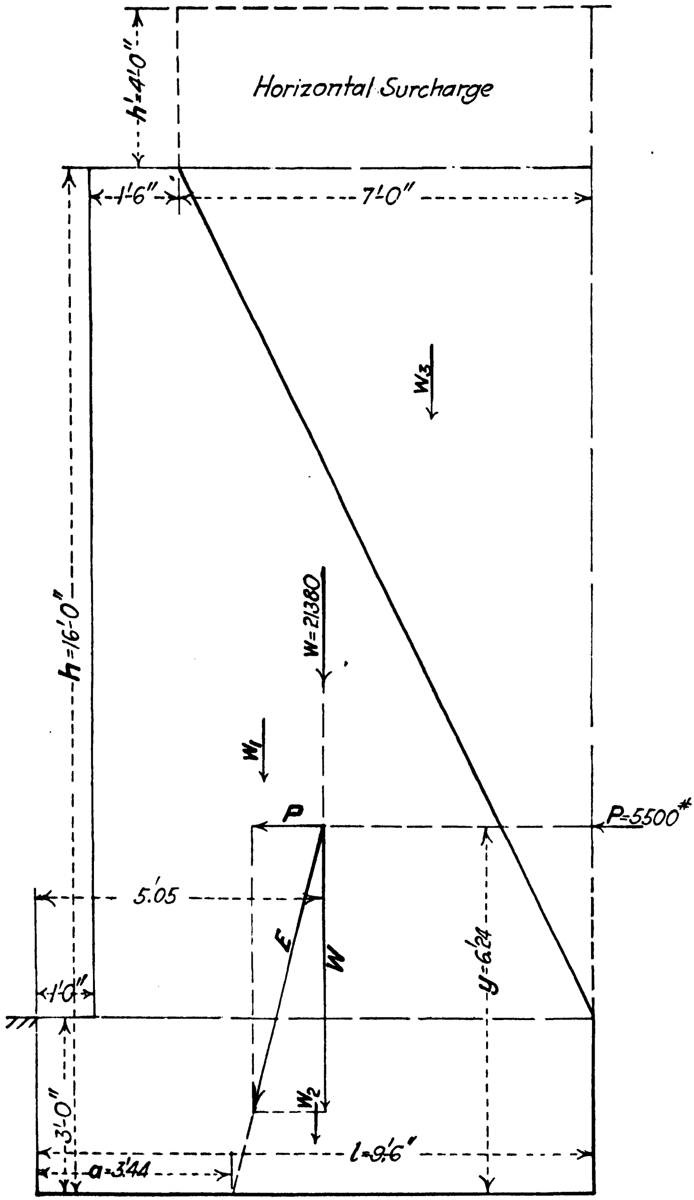


FIG. 151.—Details of gravity wall.

0.12 ft. outside the middle third, and will be assumed satisfactory. The toe pressure for this case is,

$$p_1 = \frac{2 \times 18{,}580}{3 \times 3.05} = 4050 \text{ lb. per sq. ft.}$$

For the latter loading condition, which is the most severe condition, the overturning moment is

$$5500 \times 6.24 = 34{,}300 \text{ ft.-lb.}$$

and the resisting moment is

$$18{,}580 \times 4.90 = 91{,}200 \text{ ft.-lb.}$$

$$\text{The factor of safety against overturning} = \frac{91{,}200}{34{,}300} = 2.66$$

The force producing sliding = 5500 lb., and the force resisting sliding, assuming the coefficient of friction as 0.5, is

$$0.5 \times 18{,}580 = 9290 \text{ lb.}$$

$$\text{The factor of safety against sliding} = \frac{9290}{5500} = 1.69$$

244. Design of Cantilever Wall. Design a reinforced concrete wall of the cantilever type 18 ft.-0 in. in height, to retain a bank of earth with a surcharge whose slope is $1\frac{1}{2}:1$. The wall is to be placed along the easement line, beyond which no encroachment is permissible. The soil is a firm clay with an allowable pressure of $3\frac{1}{2}$ tons per sq. ft. The weight of the retained fill is 100 lb. per cu. ft. The allowable unit stresses are as follows: $f_c = 800$, $f_s = 16{,}000$, $u = 100$, $v = 40$, and $n = 15$. The coefficient of friction between the concrete base and the subsoil equals 0.5.

Owing to the condition, an L-shaped wall with a vertical face is necessary. The tentative dimensions are shown in Fig. 152.

The overturning pressure on a vertical plane through the heel of the wall, from Fig. 144, is

$$\begin{aligned} P &= \tfrac{1}{2} \cos \phi \times w(h + h')^2 \\ &= \tfrac{1}{2} \times 0.832 \times 100 \times (26.6)^2 = 29{,}500 \text{ lb.} \end{aligned}$$

and its point of application is $\frac{26.6}{3} = 8.9$ ft. from the base of the plane.

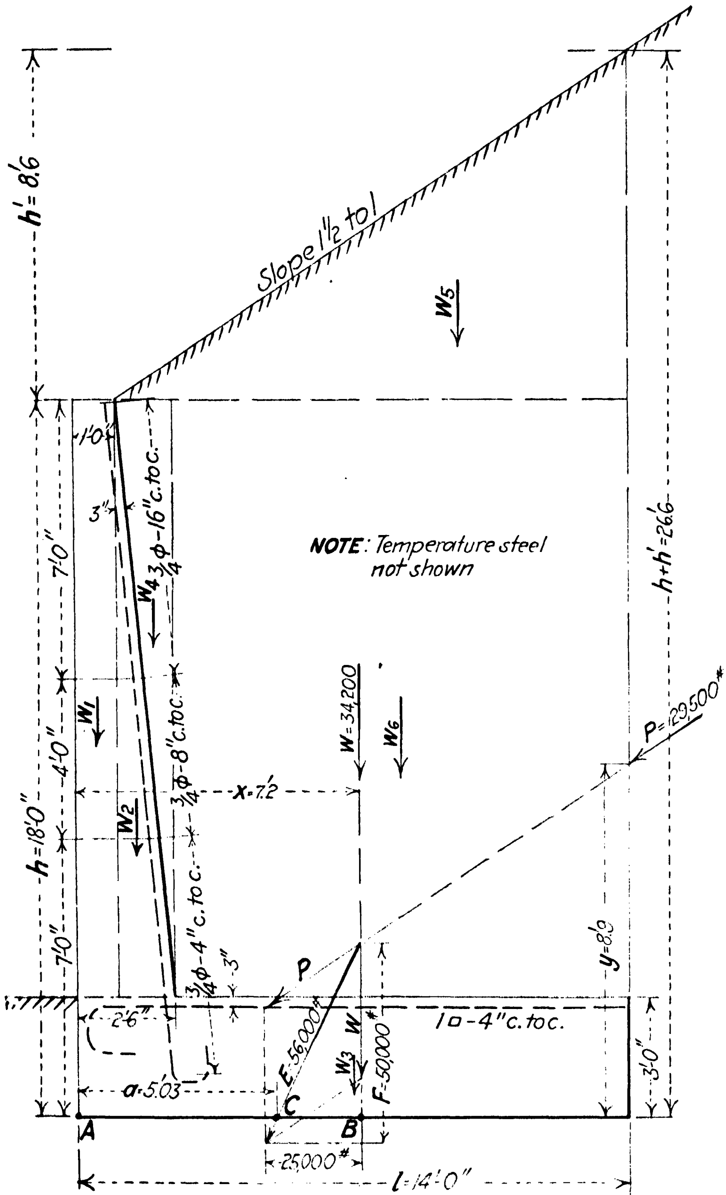


FIG. 152.—Details of cantilever wall.

The total relieving weight W, consisting of the weight of the wall and the earth directly over the base, is found as in the preceding example to be 34,200 lb., and its point of application 7.2 ft. from the front face of the wall. The resultant of P and W, found graphically, is 56,000 lb. and its point of intersection with the base is 5.03 ft. from the toe, or $4\frac{1}{4}$ in. inside the forward edge of the middle third.

$$p_1 = (4 \times 14 - 6 \times 5.03) \times \frac{50,000}{14^2} = 6600 \text{ lb. per sq. ft.}$$

$$p_2 = (6 \times 5.03 - 2 \times 14) \times \frac{50,000}{14^2} = 560 \text{ lb. per sq. ft.}$$

The factor of safety against overturning, by equation (81), Art. 239, equals

$$\frac{AB}{CB} = \frac{7.2}{7.2 - 5.03} = 3.3$$

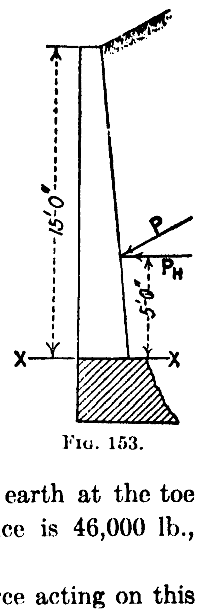


FIG. 153.

In order to furnish a sliding safety factor of $1\frac{1}{2}$, the friction force required equals 25,000 $\times 1\frac{1}{2}$ = 37,500 lb. The force furnished by the friction on the base equals $0.5 \times 50,000$ = 25,000 lb. Assuming that the concrete is poured directly against the original earth in front, the sliding resistance furnished by the earth at the toe is 3×7000 = 21,000 lb. The total resistance is 46,000 lb., which is ample.

Vertical Arm. Figure 153 represents the force acting on this member.

$$P = \frac{0.832}{2} \times 100 \times \overline{15}^2 = 9360 \text{ lb.}$$

$$P_H = 9360 \times 0.832 = 7800 \text{ lb.}$$

$$M_{x-x} = 7800 \times 5 \times 12 = 468,000 \text{ in.-lb.}$$

$$d = \sqrt{\frac{468,000}{146.7 \times 12}} = 16.3 \text{ in.}$$

$$V_{x-x} = P_H = 7800 \text{ lb.}$$

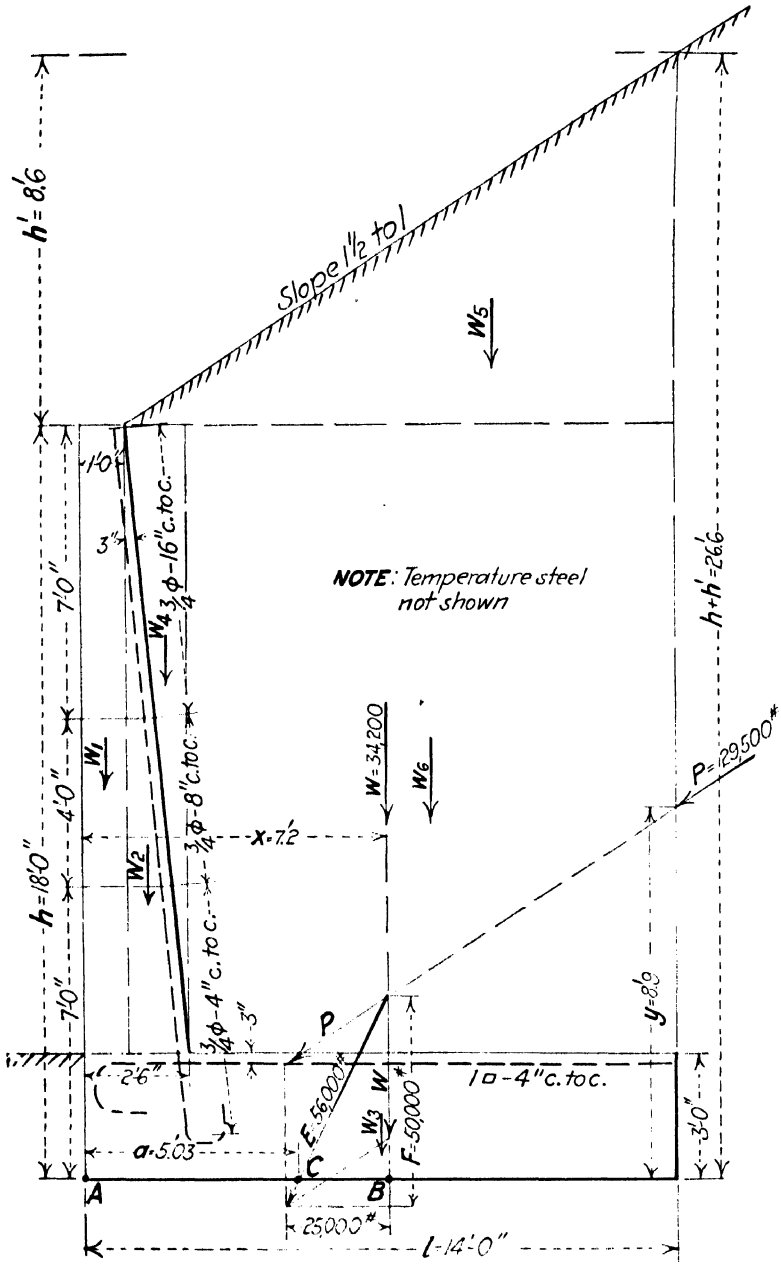


FIG. 152.—Details of cantilever wall.

The total relieving weight W, consisting of the weight of the wall and the earth directly over the base, is found as in the preceding example to be 34,200 lb., and its point of application 7.2 ft. from the front face of the wall. The resultant of P and W, found graphically, is 56,000 lb. and its point of intersection with the base is 5.03 ft. from the toe, or 4¼ in. inside the forward edge of the middle third.

$$p_1 = (4 \times 14 - 6 \times 5.03) \times \frac{50{,}000}{14^2} = 6600 \text{ lb. per sq. ft.}$$

$$p_2 = (6 \times 5.03 - 2 \times 14) \times \frac{50{,}000}{14^2} = 560 \text{ lb. per sq. ft.}$$

The factor of safety against overturning, by equation (81), Art. 239, equals

$$\frac{AB}{CB} = \frac{7.2}{7.2 - 5.03} = 3.3$$

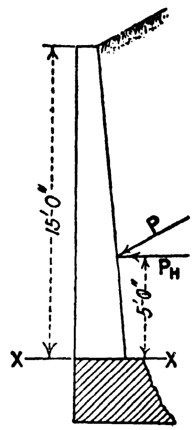


Fig. 153.

In order to furnish a sliding safety factor of 1½, the friction force required equals 25,000 × 1½ = 37,500 lb. The force furnished by the friction on the base equals 0.5 × 50,000 = 25,000 lb. Assuming that the concrete is poured directly against the original earth in front, the sliding resistance furnished by the earth at the toe is 3 × 7000 = 21,000 lb. The total resistance is 46,000 lb., which is ample.

Vertical Arm. Figure 153 represents the force acting on this member.

$$P = \frac{0.832}{2} \times 100 \times \overline{15}^2 = 9360 \text{ lb.}$$

$$P_H = 9360 \times 0.832 = 7800 \text{ lb.}$$

$$M_{x-x} = 7800 \times 5 \times 12 = 468{,}000 \text{ in.-lb.}$$

$$d = \sqrt{\frac{468{,}000}{146.7 \times 12}} = 16.3 \text{ in.}$$

$$V_{x-x} = P_H = 7800 \text{ lb.}$$

$$d = \frac{7800}{40 \times 12 \times \frac{7}{8}} = 18.6 \text{ in.}$$

The thickness of 30 in. selected gives a value of $d = 27$ in., allowing 3 in. insulation. A smaller value of d would require excessive reinforcement, which would add greatly to the cost of handling and placing. Assuming $j = 0.875$,

$$A_s = \frac{468{,}000}{16{,}000 \times 0.875 \times 27} = 1.24 \text{ sq. in. per ft. of wall.}$$

¾-in. round deformed bars 4 in. center to center are selected.

With this arrangement, $p = \frac{0.4418}{4 \times 27} = 0.0041$, and from Table 5, $j = 0.902$. The revised required steel area = 1.20 sq. in. per ft. of wall, and the 4-in. spacing is satisfactory.

$$u = \frac{7800}{12/4 \times 2.356 \times 0.9 \times 27} = 45 \text{ p.s.i.}$$

The bars must continue into the base a distance beyond the top of the base slab sufficient to develop their strength in bond, or

$$\frac{16{,}000}{4 \times 100} \times \frac{3}{4} = 30 \text{ in.}$$

Investigation must be made of the depth and area of steel required at intermediate planes in the height of the vertical arm. This may be accomplished by considering the wall above the plane in question as a free cantilever and analyzing in a manner similar to that followed above for the entire vertical arm. The actual effective depth furnished at the plane should be used in solving for the steel area. The depth required at any section should be less than that furnished.

On account of the decreasing pressure, the number of bars required per foot of wall at any horizontal plane decreases as the top of the vertical arm is approached. Therefore, alternate bars may be discontinued a safe distance beyond the points of theoretical cut-off. The points at which these bars are no longer required may best be found by computing the steel area required

at two or more intermediate heights, and plotting required steel areas against height of vertical arm. In the present case, every other bar is discontinued at a distance of 4 ft. from the top of the base slab, and every other remaining bar is cut off at a point 8 ft. from the top of the base slab, both of these being well above the theoretical points of cut-off. This arrangement gives a spacing of 16 in. for the bars near the top of the wall. This represents practically the maximum spacing desirable for these bars.

Base Slab. Figure 154 represents the forces acting on the base slab. The maximum moment occurs along the plane *B-B*, and is equal to the difference in moments of the upward foundation pressure and the downward load from the filling above the portion of the base slab behind the plane *B-B*. The vertical component of the resultant earth pressure on the wall, assumed as uniformly distributed over the rear portion of the base slab, should also be included in computing the downward moment. The total downward load of the filling and base slab itself to the rear of *B-B* is

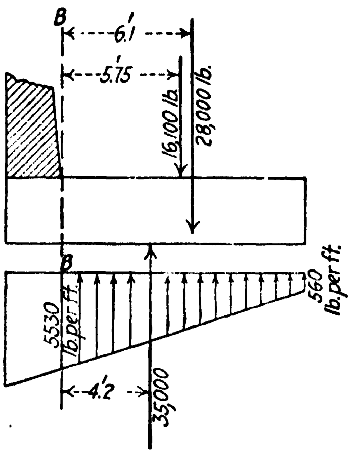


FIG. 154.

$$\frac{16 + 23.6}{2} \times 11.5 \times 100 + 3 \times 11.5 \times 150 = 28,000 \text{ lb.}$$

Its point of application is 6.1 ft. from *B*. The vertical component of the thrust $P = 29,500 \times 0.554 = 16,100$ lb., and is assumed as uniformly distributed over the base slab to the rear of plane *B-B*.

The total upward pressure from the foundation (see Fig. 154) is

$$\frac{560 + 5530}{2} \times 11.5 = 35,000 \text{ lb.}$$

and its center of gravity is

$$\frac{11.5(5530 + 2 \times 560)}{3(5530 + 560)} = 4.2 \text{ ft. from } B$$

$$\begin{aligned} M_{B-B} &= 28{,}000 \times 6.1 + 16{,}100 \times 5.75 - 35{,}000 \times 4.2 \\ &= 116{,}700 \text{ ft.-lb., or } 1{,}400{,}000 \text{ in.-lb.} \end{aligned}$$

$$V_{B-B} = 28{,}000 + 16{,}100 - 35{,}000 = 9100 \text{ lb.}$$

$$d \text{ for moment} = \sqrt{\frac{1{,}400{,}000}{146.7 \times 12}} = 28.2 \text{ in.}$$

$$d \text{ for shear} = \frac{9100}{40 \times \frac{7}{8} \times 12} = 21.6 \text{ in.}$$

Allowing 3 in. for insulation, the effective depth of the assumed section is 33 in. A smaller value would require excessive steel.

$$A_s = \frac{1{,}400{,}000}{16{,}000 \times 0.857 \times 33} = 3.0 \text{ sq. in. per ft. of wall.}$$

This is furnished by 1-in. square deformed bars 4 in. center to center.[1]

$$u = \frac{9100}{12/4 \times 4 \times 7/8 \times 33} = 26 \text{ p.s.i.}$$

$$l_1 = \frac{16{,}000}{4 \times 100} \times 1 = 40 \text{ in.}$$

A hook on the wall end of each bar provides for the deficiency in length of embedment.

Since the downward moment is greater than the upward moment, the tension face is uppermost, and the steel must be placed along that face. Alternate bars in the base slab could be discontinued as outlined in the design of the vertical slab.

In the vertical slab the principal temperature reinforcement is horizontal, and most of it is placed along the exposed face. On the front face, $\frac{1}{2}$-in. round horizontal bars, 12 in. center to center

[1] This amount of steel is rather large for efficient handling. A more satisfactory design would result by increasing the thickness of the base slab so that the spacing of the 1-in. square bars could be increased to approximately 6 in.

vertically, are used, and wired to ½-in. round vertical bars 36 in. center to center. On the rear face, ½-in. round horizontal bars 24 in. center to center are wired to the main slab reinforcement.

In the base slab, ½-in. round bars are wired to the main reinforcing steel, and placed 12 in. center to center to prevent the formation of cracks which would permit seepage of water into the slab, with the resulting damage to the reinforcing bars.

245. Design of Counterfort Wall. Design a reinforced concrete wall of the counterfort type, 24 ft.-0 in. in height, to support an earth fill, the upper surface of which is horizontal and level with the top of the wall. The weight of the filling is 100 lb. per cu. ft., and the angle of repose 33 degrees 42 minutes (slope 1½:1). The allowable pressure on the soil is 2 tons per sq. ft., and the coefficient of friction between the base and the subsoil is 0.4. A spacing of 10 ft. for the counterforts for walls of medium height will usually be economical. The ordinary range of counterfort spacing is from 8 ft. for the low walls to 12 ft. for the higher walls. The thickness of counterfort varies from 12 to 18 in. In the present case a thickness of 18 in. and a spacing of 10 ft. will be used. The remaining dimensions are assumed as shown in Fig. 155. In estimating foundation pressures, a length of wall of 10 ft. is considered to allow for the effect of the counterfort. Allowable unit stresses are as follows: $f_s = 650$, $f_s = 16{,}000$, $u = 100$, $v = 40$, and $n = 15$.

$$\text{From Fig. 144, } P = 0.143 \times 100 \times \overline{24}^2 \times 10 = 82{,}500 \text{ lb.}$$
$$y = \tfrac{1}{3} \times 24 = 8 \text{ ft.}$$

As in the preceding designs,

$$\begin{aligned}
W_1 &= 10 \times 1 \times 22 \times 150 &= \ \ 33{,}000 \text{ lb.}\\
W_2 &= 2 \times 11 \times 10 \times 150 &= \ \ 33{,}000 \text{ lb.}\\
W_3 &= \tfrac{1}{2} \times 7 \times 22 \times 1.5 \times 150 &= \ \ 17{,}300 \text{ lb.}\\
W_4 &= \tfrac{1}{2} \times 7 \times 22 \times 1.5 \times 100 &= \ \ 11{,}600 \text{ lb.}\\
W_5 &= 7 \times 8.5 \times 22 \times 100 &= 131{,}000 \text{ lb.}\\
\hline
W &= & 225{,}900 \text{ lb.}
\end{aligned}$$

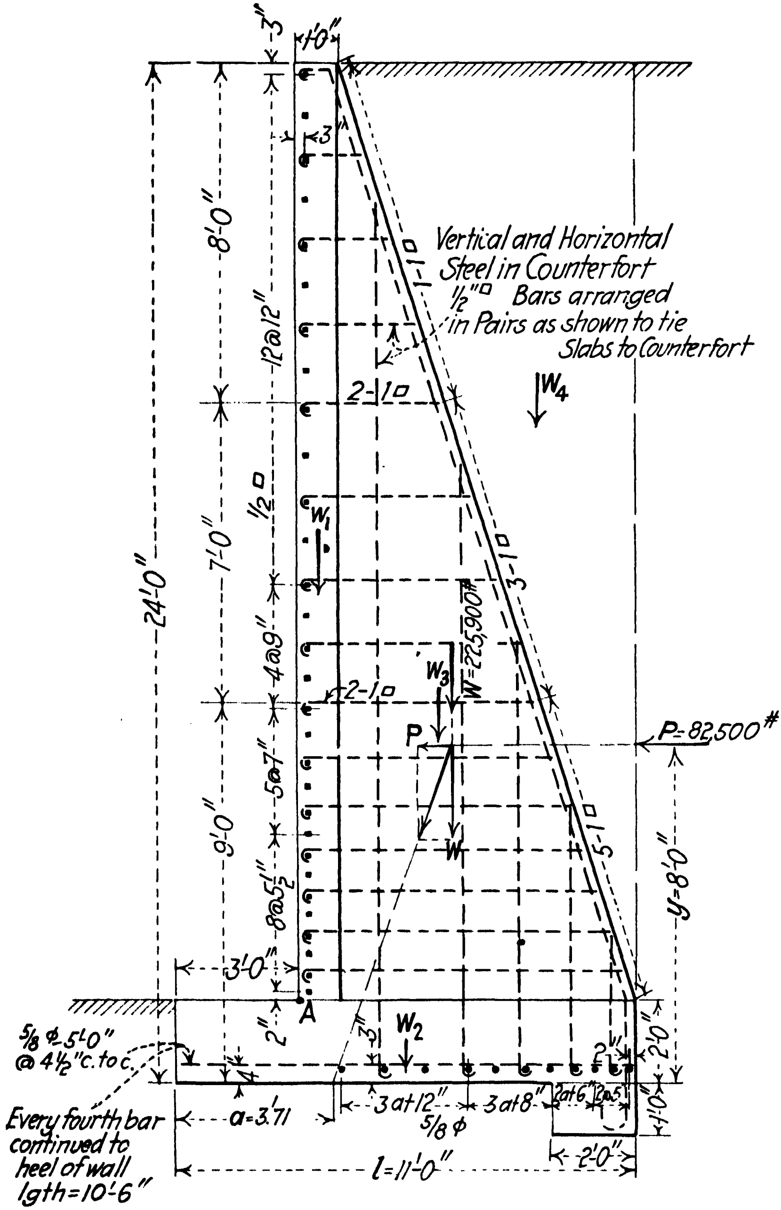


FIG. 155.—Details of counterfort wall.

The point of application of the total resisting weight W is found by taking moments about the toe of the base slab to be at a distance of 6.64 ft. from the toe. The resultant pressure on the base is

$$\sqrt{(225{,}900)^2 + (82{,}500)^2} = 240{,}000 \text{ lb.}$$

and its point of intersection with the base is determined analytically as follows: In Fig. 156 let x be the horizontal distance

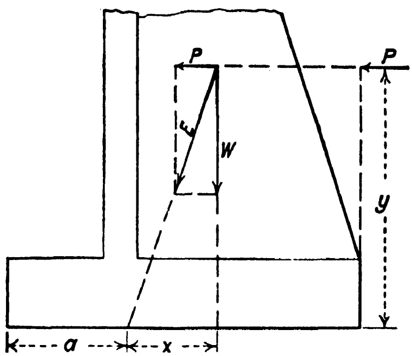


FIG. 156.

from the line of action of W to the point where the resultant E intersects the base. Then, by similar triangles,

$$\frac{x}{y} = \frac{P}{W}$$

$$x = \frac{82{,}500 \times 8}{225{,}900} = 2.93 \text{ ft.}$$

The distance $a = 6.64 - 2.93 = 3.71$ ft. The resultant pressure on the base intersects the base 0.04 ft. inside the middle third.

$$p_1 = \frac{1}{10}(4 \times 11 - 6 \times 3.71)\frac{225{,}900}{11^2} = 4050 \text{ lb. per sq. ft.}$$

$$p_2 = \frac{1}{10}(6 \times 3.71 - 2 \times 11)\frac{225{,}900}{11^2} = 50 \text{ lb. per sq. ft.}$$

The factor of safety against overturning is

$$\frac{225{,}900 \times 6.64}{82{,}500 \times 8} = 2.27$$

The factor of safety against sliding, including in the sliding resistance the resistance offered by the soil in front of the 12-in. key wall shown in Fig. 155, is

$$\frac{225{,}900 \times 0.4 + 10 \times 4000}{82{,}500} = 1.58$$

Vertical Slab. The vertical slab is designed as a simple slab supported by the counterforts. The thickness of slab required is governed by the pressure at the top of the base slab. For a horizontal strip of vertical slab 12 in. high, 22 ft. down from the top of the wall, the pressure per linear foot, from equation (77), Art. 236, is

$$P_1 = wxC = wx\frac{(1 - \sin\phi)}{(1 + \sin\phi)}$$
$$= 100 \times 22 \times 0.286 = 630 \text{ lb.}$$

The maximum bending moment in the strip, assuming the length as the clear distance between counterforts, is

$$M = \tfrac{1}{8} \times 630 \times 8.5^2 \times 12 = 68{,}400 \text{ in.-lb.}$$
$$d = \sqrt{\frac{68{,}400}{107.7 \times 12}} = 7.3 \text{ in.}$$
$$V = \tfrac{1}{2} \times 630 \times 8.5 = 2680 \text{ lb.}$$
$$d = \frac{2680}{40 \times \tfrac{7}{8} \times 12} = 6.4 \text{ in.}$$

The thickness of 12 in. adopted, furnishing an effective depth of 9 in., is satisfactory. A thinner section would be impractical because of the difficulty in placing the concrete in the forms

With $K = \dfrac{68{,}400}{12 \times 9^2} = 70.5$, Table 6 gives $j = 0.895$.

$$A_s = \frac{68{,}400}{16{,}000 \times 0.895 \times 9} = 0.53 \text{ sq. in. per ft.}$$

One-half-inch square bars, 5.5 in. center to center, are selected.

$$u = \frac{2680}{2 \times \frac{12}{5.5} \times 0.895 \times 9} = 76 \text{ p.s.i.}$$

A similar investigation for strips of the vertical arm at heights of 18, 12, and 6 ft. give spacings of ½-in. square bars required of 7, 12, and 24 in., respectively. The bars are placed as shown in Fig. 155.

Heel Slab. The rear portion of the base slab is designed as a simple beam supported by the counterforts. The load on the slab is equal to the difference between the upward soil pressure and the downward load from the earth above it. The load distribution is shown in Fig. 157. The slab is analyzed in strips 2 ft. wide. The strengthening action of its monolithic construction fully warrants such a procedure. The strip at the heel is subjected to a resultant load per foot of

$$(2200 + 300) \times 2 - \left(\frac{50 + 780}{2}\right) \times 2 = 4170 \text{ lb.}$$

$$M = \tfrac{1}{8} \times 4170 \times 8.5^2 \times 12 = 453{,}000 \text{ in.-lb.}$$

$$V = 4170 \times \frac{8.5}{2} = 17{,}700 \text{ lb.}$$

The depth required is governed in this case by the shear.

$$d = \frac{17{,}700}{40 \times \frac{7}{8} \times 24} = 21.0 \text{ in.}$$

Allowing 3 in. of insulation for the steel, an effective depth of 21 in. is furnished and the assumed slab thickness is satisfactory.

$$A_s = \frac{1}{2} \times \frac{453{,}000}{16{,}000 \times 0.9 \times 21} = 0.75 \text{ sq. in. per ft.}$$

Five-eighths-inch round bars 5 in. center to center furnish the necessary area.

$$u = \frac{17{,}700}{1.964 \times 2\tfrac{4}{5} \times \tfrac{7}{8} \times 21} = 101 \text{ p.s.i.}$$

With deformed bars, the allowable bond stress is 100 p.s.i.

$$p = \frac{0.3068}{5 \times 21} = 0.0029$$

From Table 7, $j = 0.914$ and the revised steel area = 0.74 sq. in. per ft.

Since the downward load exceeds the upward pressure, the steel at the heel must be placed near the bottom of the slab.

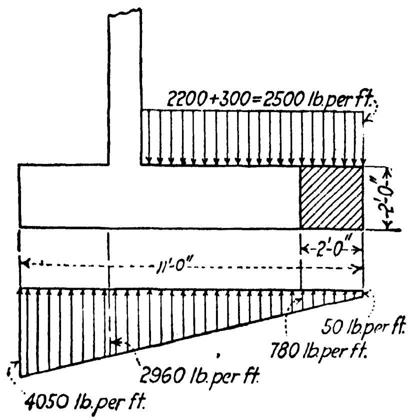


FIG. 157.

The point where the upward pressure becomes equal to the downward load is at a distance of x ft. from the heel, where

$$x = (2500 - 50) \times {}^{11}/_{4000} = 6.75 \text{ ft.}$$

The heel-slab steel in front of this point is required in the top of the slab. In the present case, the top steel is omitted, since the point of inflection is approximately under the rear face of the vertical slab.

Following an investigation of the other strips along the heel slab, the reinforcement is placed as shown in Fig. 155.

Toe Slab. The toe slab is designed as a free cantilever with a length of 3 ft.-0 in. The moment in the cantilever is due to an upward load as represented by the trapezoid of pressure underneath (Fig. 157) and a downward load equal to the weight

of the toe slab itself. The resultant moment equals 183,800 in.-lb., and the resultant shear 9600 lb. With an effective depth of 20 in., the unit shear is

$$v = \frac{9600}{12 \times \frac{7}{8} \times 20} = 45 \text{ p.s.i.}$$

which may be considered satisfactory.

$$A_s = \frac{183{,}800}{16{,}000 \times 0.9 \times 20} = 0.64 \text{ sq. in. per ft.}$$

To satisfy requirements of bond, $\frac{5}{8}$-in. round deformed bars are spaced at $4\frac{1}{2}$ in. center to center, and every fourth bar continued to the heel of the wall to furnish transverse reinforcement in the heel slab.

$$u = \frac{9600}{1.964 \times \frac{12}{4.5} \times \frac{7}{8} \times 20} = 103 \text{ p.s.i.}$$

Counterfort. The moment in the counterfort is due to the pressure of the earth on the vertical slab over a length of wall equal to the distance center to center of counterforts.

$$P = 0.143 \times 100 \times \overline{22}^2 \times 10 = 69{,}300 \text{ lb.}$$
$$y = \tfrac{1}{3} \times 22 = 7.33 \text{ ft.}$$
$$M = 69{,}300 \times 7.33 \times 12 = 6{,}110{,}000 \text{ in.-lb.}$$

The effective depth of the counterfort is the perpendicular distance from the point A (Fig. 155) to the reinforcing steel.

$$d = 8 \times \frac{22}{\sqrt{22^2 + 7^2}} \times 12 - 3 = 88 \text{ in.}$$

$$A_s = \frac{6{,}110{,}000}{16{,}000 \times 0.9 \times 88} = 4.83 \text{ sq. in.}$$

Five 1-in. square deformed bars are used.

$$p = \frac{5.0}{18 \times 88} = 0.0032 \qquad j = 0.912 \qquad A_s = 4.77$$

$$u = \frac{69{,}300}{4 \times 5 \times \frac{7}{8} \times 88} = 43 \text{ p.s.i.}$$

The effective depth to be used in determining the unit shear on the base of the counterfort is equal to the horizontal distance from A (Fig. 155) to the reinforcing steel; *i.e.*,

$$d = (8 \times 12) - 3 = 93 \text{ in.}$$

$$v = \frac{69{,}300}{18 \times \frac{7}{8} \times 93} = 46 \text{ p.s.i.}$$

This value of the unit shear is satisfactory because horizontal bars are provided to anchor the vertical slab and the counterfort together. These bars serve the added function of web reinforcement.

Following an investigation of the moment and shear at heights of 6, 12, and 18 ft., the bars are bent into the counterfort in pairs as shown in Fig. 155, and anchored to the vertical-slab steel.

To provide for the pull of the vertical slab, horizontal ½-in. square bars are placed in pairs in the counterfort, one on either side, so arranged as to hook over the vertical-slab reinforcement and extend to the rear of the counterfort. These bars are so spaced as to engage every other bar in the vertical slab. The amount of pull at the base of the vertical slab per foot of height is

$$0.286 \times 100 \times 22 \times 8.5 = 5360 \text{ lb.}$$

The area of steel required per foot of height is

$$\frac{5360}{16{,}000} = 0.334 \text{ sq. in.}$$

The ½-in. square bars, 11 in. center to center, furnish 0.56 sq. in. Similar investigations at various heights show that the arrangement described above is satisfactory. Vertical ½-in. square bars are placed at intervals in the counterfort and anchored to the base-slab reinforcement as shown in Fig. 155.

The main reinforcing bars must extend into the base slab a distance equal to

$$l_1 = \frac{16{,}000}{4 \times 100} \times 1 = 40 \text{ in.}$$

A key wall 1 ft.-6 in. deep and 2 ft.-0 in. wide is constructed under the heel of the wall to provide the necessary embedment. The main bars are hooked as a further precaution.

In the above design the vertical slab and rear portion of the base slab have been assumed as simply supported and no provision made for continuity over the supports (the counterforts). Some designers prefer to consider these portions as partially or fully continuous, and to provide for the negative moment at the supports by bending every second or third bar to the opposite face of the slab. Where such an assumption is made, a moment coefficient of $1/10$ or $1/12$ may, of course, be used.

To provide for the negative moment, some bars may be bent to the opposite face at the counterforts, or additional straight bars may be placed in that face across the counterforts. The length of such bars should be from 0.5 to 0.6 of the spacing of counterforts. In the higher walls some provision should be made for continuity even though the slabs are designed as simply supported.

CHAPTER X

ARCHES

246. Advantages and Forms of Reinforcement. An arch with a parabolic axis and loaded with a fixed uniform load would require no steel reinforcement, for the line of pressure would coincide with the axis of the arch, no moment would be produced in any section, and the stresses would be wholly compressive. In the principal adaptation of the concrete arch, namely the concrete arch bridge, the live load is not fixed, and the most desirable form of arch departs considerably from the parabola. Provided the ratio of live load to dead load is so small that under no loading conditions the line of the resultant pressure departs from the middle third, compressive stresses only exist. Often, however, in order to accomplish this result without the use of reinforcement, rather heavy sections are required. With the use of steel reinforcement it is not necessary to keep the line of pressure within the middle third as the steel can care for tensile stresses just as in a reinforced concrete beam.

As in all concrete members whose main stress is compression, there is no theoretical economy in the use of the reinforcement. With the line of pressure within the middle third, the stress in the steel is compression and the working values low. Even when the line of pressure is outside the middle third, the tensile stress is usually not great, as the compressive stress in the concrete on the other side of the section is the governing factor. As in columns, the use of the reinforcement makes the arch a more secure and reliable structure and aids in preventing cracks due to settlement.

The forms of reinforcement vary. The smaller arches are now generally constructed with ordinary reinforcing bars on both sides, these bars being usually tied together in some manner in order to be more effective in aiding the concrete in resisting the

compression. Some of the earlier arches used wire netting as reinforcement and some of the so-called "system" types use certain forms of structural steel. A few of the larger arches of the present day are built with heavy structural steel reinforcement, or a combination of structural steel and ordinary reinforcing bars.

247. The Curve of the Intrados. The form of the curve of the intrados (see Fig. 158) is one of the main considerations in the design. This curve is usually circular, multi-centered, elliptical or parabolic. The multi-centered arch is perhaps the most

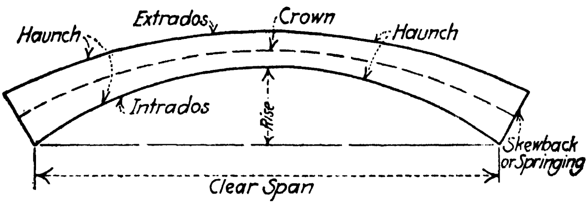


FIG. 158.

common, as it gives a pleasing appearance and generally an economical design. It is usually either three centered or five centered. The three-centered arch ordinarily gives the more economic design, but the five-centered arch has more graceful lines and is often necessary on account of clearance requirements. The formulas for the radii when the span and rise are known are as follows (see Fig. 159):

$$R_1 = \frac{x^2 + y^2}{2y}$$

$$R_2 = \frac{1}{2} \cdot \frac{\overline{BE}^2 + \overline{ED}^2}{ED \cos \theta_1 - BE \sin \theta_1}$$

$$R_3 = \frac{1}{2} \cdot \frac{\overline{AF}^2 + \overline{FB}^2}{FB \cos (\theta_1 + \theta_2) - AF \sin (\theta_1 + \theta_2)}$$

$$\sin \theta_1 = \frac{x}{R_1}$$

$$\sin \frac{1}{2}\theta_2 = \frac{\frac{1}{2}\sqrt{\overline{DE}^2 + \overline{BE}^2}}{R_2}$$

248. Spandrels. The space between the back of the arch (the curve of the extrados) and the roadway is known as the

spandrel. This space may be entirely filled with earth which is retained by side walls supported by the arch ring. This type of construction is known as the filled spandrel arch. The spandrel space may, on the other hand, be left more or less open, and the roadway supported on a series of transverse or longitudinal walls or on a complete superstructure of columns, girders, beams, and slabs. This is known as open spandrel construction. With filled spandrels, the side walls may be of the gravity type or they may be reinforced and tied together with cross walls. Solid filling increases the dead load, but open spandrel construction requires a much larger amount of form work. For short span arches and for arches where the ratio of rise to span is small, the filled spandrel type will usually be found to be the more economical.

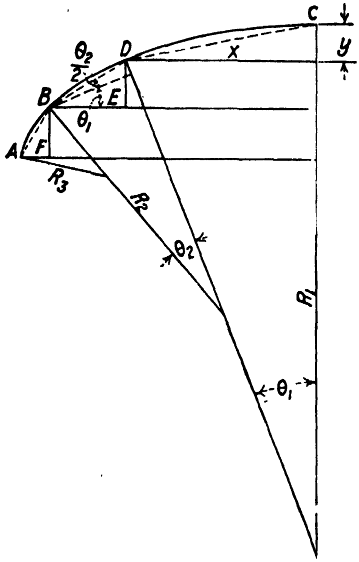


FIG. 159.

249. Loads. The principal load on an arch ring is the dead load, which consists of the weight of the arch ring itself, the spandrel construction, and the roadway. With open spandrel construction the dead loads (except that of the ring itself) and their points of application are definitely known. In such cases it is usual to assume the weight of the arch ring as concentrated at the point of application of the superimposed loads. While in filled arches the pressure from the filling is really inclined, it is on the side of safety to neglect the horizontal component of the inclined force and design for the vertical loads only. Except in arches of comparatively high rise, the error is not great.

The depth of filling at the crown depends on the type of load which the arch is to sustain. For highway bridges it is rarely advisable to use less than 1 ft. below the pavement, while for railroad bridges not less than 2 ft. below the ballast should be used

in order to distribute the load uniformly and absorb the shocks from rapidly moving traffic.

The weight of the earth filling is taken as either 100 or 120 lb. per cu. ft. The latter value is generally used where it is probable

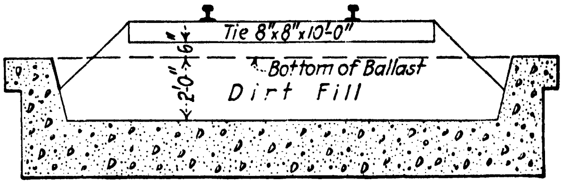


FIG. 160.

that at times the filling may be thoroughly saturated with water. Ballast is assumed to weigh 120 lb. per cu. ft. and pavement (except wood block) and concrete 150 lb. per cu. ft.

The live load used in a design should be the greatest that is likely to come upon the structure. For railroad bridges of 100-ft.

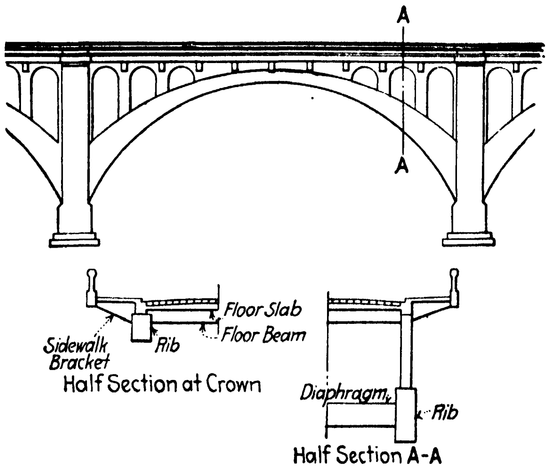


FIG. 161.

span or less, the weight of the locomotive properly distributed should furnish the basis for design. For longer spans a somewhat lighter load should be used. Where the filling is sufficient thoroughly to distribute the load, an equivalent uniform live load may be used. For highway bridges the same live loads are

used as for the design of slab, beam, and girder bridges (see Chap. XI) except that in filled arches no allowance for impact need be made.

250. The Arch Ring. The arch ring may have a width equal or nearly so to that of the roadway it supports, or a series of narrow rings or ribs may be constructed on which the roadway is supported. The former type is known as the barrel type and is illustrated in Fig. 160. The latter is known as the ribbed type

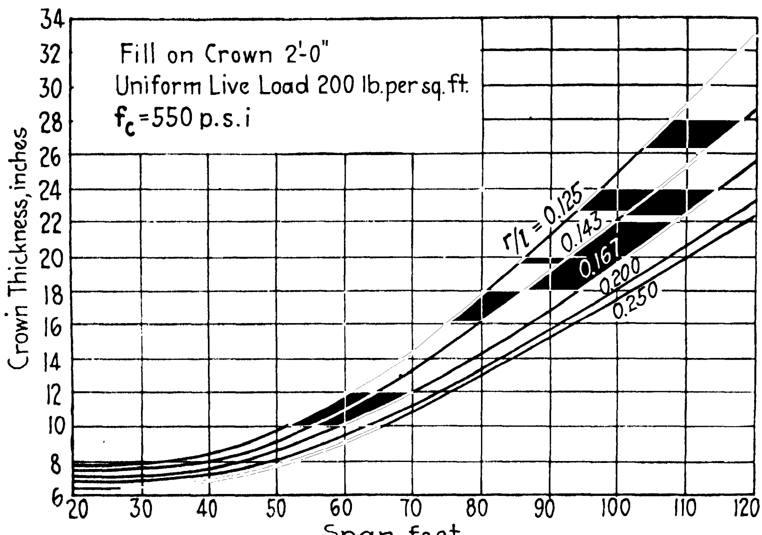


FIG. 162.

and is illustrated by Fig. 161. The former is always used with filled spandrel construction. It is usual to make such a design for a section of arch 1 ft. in width as in the design of slabs reinforced in one direction only. The weight of roadway, fill, and arch ring may be assumed concentrated at a number of selected points, without any great error. In the ribbed type, two ribs give the more simple design, and should preferably be used except for very wide bridges.

251. Crown Thickness. Various empirical formulas have been proposed for determining a trial thickness of arch ring at the crown. These depend upon the span, the rise, and the loads to

be sustained. They give variable results, and none of them seems to fit a great variety of conditions. In flat arches the temperature and arch shortening stresses are of great importance, and no formula so far developed, to the authors' knowledge, gives sufficiently definite consideration to these factors. Figures 162 and 163, taken from an article by Joseph P. Schwada in the *Engineering News* of Nov. 9, 1916, seem to give as satisfactory a final thickness as any other method. It should be noted that these curves are for barrel arches for definite unit stresses and

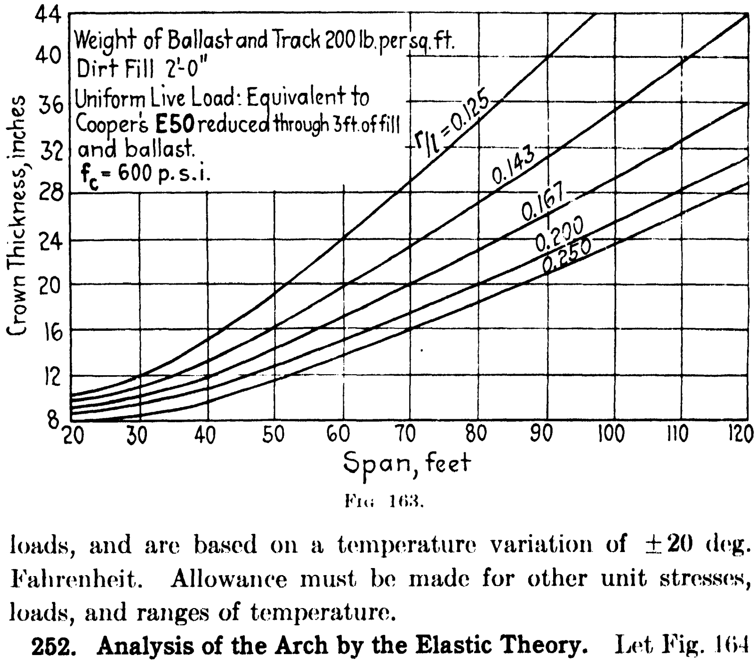


FIG. 163.

loads, and are based on a temperature variation of ± 20 deg. Fahrenheit. Allowance must be made for other unit stresses, loads, and ranges of temperature.

252. Analysis of the Arch by the Elastic Theory. Let Fig. 164 represent a curved beam, the curvature of the beam being small in proportion to its depth, so that the length of all fibers may be considered equal. Assuming ab as fixed and considering an elementary length Δs, the plane dc, in deflecting, rotates through an angle $\Delta\phi$ to the position $d'c'$. The change in length of a fiber at a distance e from the neutral axis is $e\Delta\phi$. The deformation per unit length is then $\dfrac{e\Delta\phi}{\Delta s} = \dfrac{f}{E}$. But $M = \dfrac{fI}{e} = \dfrac{E\Delta\phi I}{\Delta s}$

and

$$\Delta\phi = \frac{M\Delta s}{EI}.$$

In the rotation of bending, it is assumed that each small block successively changes, beginning from the left end. In Fig. 165, in which the curved line 1-0 represents the neutral axis from the

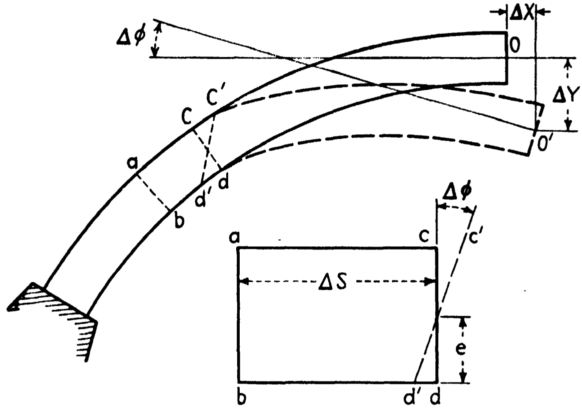
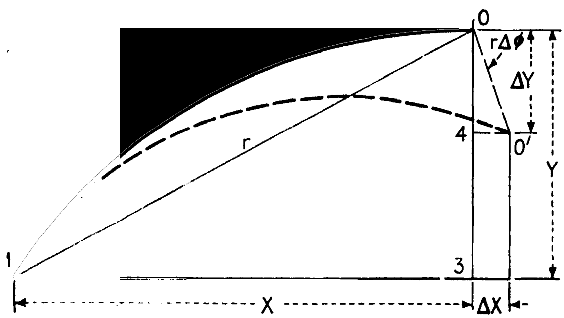


FIG. 164.



165.

center of any element Δs to the crown, from similar triangles 1-0-3 and 0-4-0', since $r\Delta\phi$ is practically perpendicular to r, $\frac{r\Delta\phi}{r} = \frac{\Delta y}{x}$ so that $\Delta y = x\Delta\phi$; and since for a positive or clockwise rotation Δy is negative, substituting the value of $\Delta\phi$,

$$\Delta y = -\frac{Mx\Delta s}{EI}$$

Similarly,

$$\Delta x = \frac{My\Delta s}{EI}$$

A concrete arch with fixed ends is statically indeterminate since there are six unknown quantities, three at each support, namely the vertical and horizontal components of the reaction and the bending moment. Since the ends are fixed, at the ends $\Delta x = 0$, $\Delta y = 0$, and $\Delta \phi = 0$. By considering the arch cut at the crown, the thrust, shear, and moment at that point may be determined,

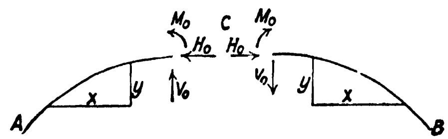


FIG. 166.

and these being known, each half of the arch becomes statically determinate.

With the origin of coordinates at the crown C (Fig. 166), the horizontal movement of C due to bending bears the same relation to each cantilever. Then, from the theory developed above,

$$\sum\nolimits_C^A \Delta x = -\sum\nolimits_C^B \Delta x$$

or

$$\sum\nolimits_C^A \frac{My\Delta s}{EI} = -\sum\nolimits_C^B \frac{My\Delta s}{EI} \tag{83}$$

The changes in Δy are equal and in the same direction, so that

$$\sum\nolimits_C^A \frac{Mx\Delta s}{EI} = \sum\nolimits_C^B \frac{Mx\Delta s}{EI} \tag{84}$$

Also the changes in direction of the tangent to the axis at C are equal but opposite in direction, hence,

$$\sum\nolimits_C^A \frac{M\Delta s}{EI} = -\sum\nolimits_C^B \frac{M\Delta s}{EI} \tag{85}$$

Denoting $\sum_C^A M$ as $\sum M_L$ and $\sum_C^B M$ as $\sum M_R$, dividing the arch ring into divisions such that Δs is a constant,[1] and eliminating the constant E,

$$\sum M_L \frac{y}{I} = -\sum M_R \frac{y}{I} \tag{86}$$

$$\sum M_L \frac{x}{I} = \sum M_R \frac{x}{I} \tag{87}$$

$$\sum \frac{M_L}{I} = -\sum \frac{M_R}{I} \tag{88}$$

Considering the left half of the arch as a free body (Fig. 167), for any section M_0 tends to produce counter-clockwise rotation, which will be taken as positive. Similarly V_0 is always positive.

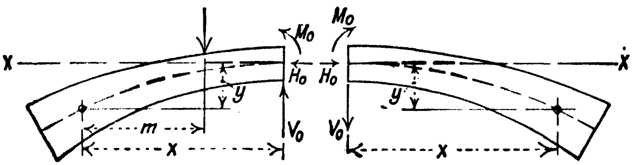


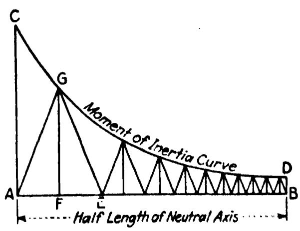
FIG. 167.

H_0 produces compression and acts towards the cut section at the crown. Hence for any section

$$M_L = M_0 + H_0 y + V_0 x - m_L \tag{89}$$

where m is the bending moment at the section due to the external loads.

[1] If the arch is divided so that $\frac{I}{\Delta s}$ is a constant, the equation for H_0, V_0, and M_0 are



$$H_0 = \frac{n\Sigma my - \Sigma m \Sigma y}{2[(\Sigma y)^2 - n\Sigma y^2]}$$

$$V_0 = \frac{\Sigma mx}{2\Sigma x^2}$$

$$M_0 = \frac{\Sigma m + 2H_0 \Sigma y}{2n}$$

In this case all ys are measured downward from the axis through the crown and are considered as negative; n equals the number of divisions in one-half of the arch.

Similarly for the right half

$$M_R = M_0 + H_0 y - V_0 x - m_R \tag{90}$$

Substituting in equations (86), (87), and (88), and combining the terms

$$2H_0\sum\frac{y^2}{I} + 2M_0\sum\frac{y}{I} - \sum\frac{m_L y}{I} - \sum\frac{m_R y}{I} = 0 \tag{91}$$

$$2V_0\sum\frac{x^2}{I} - \sum\frac{m_L x}{I} + \sum\frac{m_R x}{I} = 0 \tag{92}$$

$$2H_0\sum\frac{y}{I} + 2M_0\sum\frac{1}{I} - \sum\frac{m_L}{I} - \sum\frac{m_R}{I} = 0 \tag{93}$$

Considering the application of load on the left half of the arch only, the terms containing m_R disappear. Combining equations (91) and (93),

---

For temperature changes

$$H_0 = \mp\frac{EI}{\Delta s}\,\frac{\omega t l n}{2[n\Sigma y^2 - (\Sigma y)^2]}$$

$$M_0 = \frac{H_0\Sigma y}{n}$$

and

$$M = M_0 + H_0 y$$

For arch shortening

$$H_0 = \frac{I}{\Delta s}\cdot\frac{c_a l n}{2[n\Sigma y^2 - (\Sigma y)^2]}$$

The graphical method of dividing the half axis into divisions so that $\dfrac{I}{\Delta s}$ is constant is shown in the accompanying figure. AB is drawn equal to one-half the arch axis. The curve CD is then drawn through points whose ordinates are I and whose abscissas are the corresponding distances along the arch axis measured from the springing. A length AE is then assumed, a perpendicular FG erected at its center, and the lines AG and GE drawn. Starting from E, lines are drawn parallel alternately to AG and GE. Only two or three trials are usually necessary to divide the axis into the required number of divisions. For most arches ten divisions are sufficient. The base of each triangle thus formed corresponds to s and its altitude to I. Since all the triangles are similar, $\dfrac{I}{\Delta s}$ is constant throughout. The center of all these divisions is now located on a drawing of the arch axis and their coordinates with the crown as the origin determined.

$$H_0 = \frac{\sum\frac{my}{I}\sum\frac{1}{I} - \sum\frac{m}{I}\sum\frac{y}{I}}{2\left[\sum\frac{y^2}{I}\sum\frac{1}{I} - \left(\sum\frac{y}{I}\right)^2\right]} \tag{94}$$

$$M_0 = \frac{\sum\frac{m}{I} - 2H_0\sum\frac{y}{I}}{2\sum\frac{1}{I}} \tag{95}$$

while

$$V_0 = \frac{\sum\frac{mx}{I}}{2\sum\frac{x^2}{I}} \tag{96}$$

If the origin of coordinates[1] is shifted so that $\sum\frac{y}{I} = 0$, equations (94) and (95) become

$$H_0 = \frac{\sum\frac{my}{I}}{2\sum\frac{y^2}{I}} \tag{97}$$

[1] This position may be determined as follows:

Let y' = distance from axis at crown to axis taken so that $\sum\frac{y}{I} = 0$

y_c = distance from axis at crown to any division point

y = corresponding distance from new axis to the same division point

Then

$$y' = y_c - y \qquad \text{or} \qquad y = y_c - y'$$

and

$$\sum\left(\frac{y_c - y'}{I}\right) = 0$$

or

$$\sum\frac{y_c}{I} - \sum\frac{y'}{I} = 0$$

Therefore,

$$\sum\frac{y_c}{I} = \sum\frac{y'}{I}$$

and since y' is a constant

$$y' = \frac{\sum\frac{y_c}{I}}{\sum\frac{1}{I}}$$

$$M_0 = \frac{\sum \frac{m}{I}}{2\sum \frac{1}{I}} \tag{98}$$

Equation (97) will give the same value of H_0 as equation (94) while the value of M_0 from equation (98) is different by $\dfrac{H_0 \sum \frac{y}{I}}{\sum \frac{1}{I}} = H_0 y'$ from that determined from equation (95). In the general expression for moment at any section, $M = M_0 + H_0 y + V_0 x - m$, the sum of the first two terms is the same whichever position of the axis is taken as a basis for the summations.

The ordinate y when measured upward from the axis is taken as positive, when measured downward as negative.

For temperature stresses, Δx is equal to the change in length of the half span $= \frac{\omega t l}{2}$, where $\omega =$ the coefficient for 1 deg. of temperature change, t the number of degrees of temperature change, and l the span. Then, from equation (83),

$$\sum \frac{M_L y}{I} \cdot \frac{\Delta s}{E} = \frac{\omega t l}{2}$$

Also, since $\Delta \phi = 0$,

$$\sum \frac{M_L}{I} = 0$$

There being no external loads, $m = 0$, and from symmetry $V_0 = 0$; hence, $M = M_0 + H_0 y$. Substituting the value of M in the above equations,

$$M_0 \sum \frac{y}{I} + H_0 \sum \frac{y^2}{I} = \frac{\omega t l}{2} \cdot \frac{E}{\Delta s}$$

and

$$M_0 \sum \frac{1}{I} + H_0 \sum \frac{y}{I} = 0$$

But with
$$\sum \frac{y}{I} = 0$$
$$H_0 = \pm \frac{\omega t l}{2\sum \frac{y^2}{I}} \cdot \frac{E}{\Delta s} * \tag{99}$$
$$M_0 = 0$$
and
$$M = H_0 y \tag{100}$$

A thrust throughout the arch producing an average stress on the concrete equal to c_a p.s.i. would shorten the arch span an amount equal to $\frac{c_a l}{E}$ if the arch and the abutments were not fixed. Since the abutments are fixed, and the arch cannot shorten, there is a tensile stress developed. The action is similar to that of a fall in temperature. The resulting H_0 may be found by substituting $\frac{c_a l}{E}$ for $\omega t l$ of equation (99). There results

$$H_0 = + \frac{c_a l}{2\Delta s \sum \frac{y^2}{I}} \tag{101}$$
and similarly
$$M = H_0 y.$$

253. Approximate Methods of Analysis. Since the form and dimensions of an arch must be assumed before any calculations are made, it frequently happens that the first assumptions do not give an economical design, and all the calculations must be repeated. A complete analysis is a long and tedious operation and it is desirable to have a method for more nearly determining the dimensions in advance so that the final stresses will be close to the desired values. Victor A. Cochrane† has developed a series of approximate equations for determining stresses in an arch ring, which give values very close to those obtained by the exact method. While the authors' experience with these equa-

* + for a fall in temperature
 − for a rise in temperature.

† *Proc.*, Engineers' Society of Western Pennsylvania, vol. 32, p. 647.

TABLE OF FORMULAS FOR THRUSTS AND MOMENTS AT CROWN AND SPRINGING SECTIONS OF ARCHES

OPEN SPANDREL ARCHES

STRESSES PRODUCED BY	SECTION AT CROWN	SECTION AT SPRINGING
DEAD LOAD	$T_c = \frac{6+5r}{48r} w_c l$ M_c assumed $=0$	$V_s = \frac{3+5r}{6} w_c l$ $\qquad T_s = \sqrt{T_c^2 + V_s^2}$ M_s assumed $=0$
LIVE LOAD MAXIMUM POSITIVE MOMENT	$T_c = \frac{59.5+31.5r+5r^2}{1000r} wl$ $M_c = +\frac{67.5+80r+40r^2-(20+13r)u_s+3u_s^2}{10000} wl^2$	$T_s = \frac{93.3-42.5r+155r^2}{1000r} wl$ $T_c = (\frac{1}{2}T_{m_2}+T_{m_1}) - \frac{0.0063}{u_s r} wl$ $M_s = +\frac{322+27r-(18.1+13r)(3.79-0.6r-u_s)^2}{10000} wl^2$
LIVE LOAD MAXIMUM NEGATIVE MOMENT	$T_c = \frac{61+2u_s+(7+2u_s)r-70r^2}{1000r} wl$ $M_c = -\frac{55-9u_s+0.85u_s^2}{10000} wl$	$T_s = \frac{24.4+(211-2u_s)r+(120+30u_s)r^2}{1000r} wl$ $T_c = (\frac{1}{2}T_{m_2} + \frac{0.0063}{u_s r}) wl$ $M_s = \frac{289-172r-(17.8-28r)(3.36+2.4r-u_s)^2}{10000} wl^2$
TEMPERATURE (FALL OF $t°$)	$T_c = -[18.4u_s-7.5-(20.5u_s-15)r-(10u_s-2)r^2]\frac{wtEI_0}{h^2}$ $M_c = -[46.25-17.5u_s+2.44u_s^2+5r+(25.5-3.6u_s)r^2]\frac{hT_c}{100}$	$T_s = (1.09-1.75r)T_c$ $M_s = M_c + hT_c$

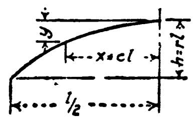
FILLED SPANDREL ARCHES

STRESSES PRODUCED BY	SECTION AT CROWN	SECTION AT SPRINGING
DEAD LOAD	$T_c = \frac{1+3r}{8r} w_c l$ M_c assumed $=0$	$V_s = \frac{2+15r}{4} w_c l$ $\qquad T_s = \sqrt{T_c^2 + V_s^2}$ M_s assumed $=0$
LIVE LOAD MAXIMUM POSITIVE MOMENT	$T_c = \frac{57.6+(189-8u_s)r-220r^2}{1000r} wl$ $M_c = +\frac{72+105r+220r^2-(17+10r)u_s+1.5u_s^2}{10000} wl^2$	$T_s = \frac{84+6.6u_s+(153-51u_s)r-(500-140u_s)r^2}{1000r} wl$ $T_c = [\frac{1}{2}T_{m_2}+T_{m_1}+\frac{0.0026(u_s-2)^2}{r}] wl$ $M_s = +\frac{316+1370r-2100r^2-(17-20r)(3.68+7.6r-u_s)^2}{10000} wl^2$
LIVE LOAD MAXIMUM NEGATIVE MOMENT	$T_c = \frac{57.8+2u_s+(10+30u_s)r-(380+30u_s)r^2}{1000r} wl$ $M_c = -\frac{49.4-100r-(3-26r)u_s-0.75u_s^2}{10000} wl^2$	$T_s = \frac{27.6+(125+6u_s)r+320r^2}{1000r} wl$ $T_c = [\frac{1}{2}T_{m_2} - \frac{0.0026(u_s-2)^2}{r}] wl$ $M_s = -\frac{283-480r-9(4.22-2.8r-u_s)^2}{10000} wl^2$
TEMPERATURE (FALL OF $t°$)	$T_c = -[19.4u_s-7.5+(17u_s-31)r-140(u_s-1)r^2]\frac{\omega tEI_0}{h^2}$ $M_c = -(38.5-12.8u_s+1.6u_s^2)\frac{hT_c}{100}$	$T_s = (1.13-2.55r)T_c$ $M_s = M_c + hT_c$

AVERAGE STRESSES

FOR DEAD LOAD	$f_a = [1.030+2.5(r+0.05)^2 - \frac{(20r+8)u_s-(u_s-1)^2}{100}] f_{ac}$
FOR LIVE LOAD PRODUCING MAX. POS. MOMENT AT CROWN.	$f_a = [0.920+2.6r^3-0.04u_s+\frac{(6.7+33r)(4-u_s)^2}{1000}] f_{ac}$
FOR LIVE LOAD PRODUCING MAX. NEG. MOMENT AT CROWN	$f_a = [0.940+1.1(r+0.1)^2-0.055u_s+\frac{(6+155r^2)(4-u_s)^2}{1000}] f_{ac}$
FOR LIVE LOAD PRODUCING MAX. POS. MOMENT AT SPRINGING	$f_a = [0.920+3.7r^3-0.04u_s+\frac{(7.5+37r)(4-u_s)^2}{1000}] f_{ac}$
FOR LIVE LOAD PRODUCING MAX. NEG. MOMENT AT SPRINGING	$f_a = [0.950+1.7r^2-0.05u_s+\frac{(4+48r)(4-u_s)^2}{1000}] f_{ac}$
FOR TEMPERATURE AND ARCH SHORTENING STRESSES	$f_a = [1.075-0.8r-(0.081-0.11r)u_s] f_{ac}$

These formulas are for arches having axes as determined by the following equations:



For Open Spandrel Arches:

$$y = \frac{8rl}{6+5r}(3c^2+10c^4r)$$

For Filled Spandrel Arches:

$$y = \frac{4rl}{1+3r}(c^2+24c^5r)$$

tions seems to indicate that the results obtained by their use are slightly smaller than those obtained by exact analysis, they are sufficiently accurate to warrant their use in preliminary calculations, and in less important structures might serve as a basis for the final designs. Some of the equations developed by

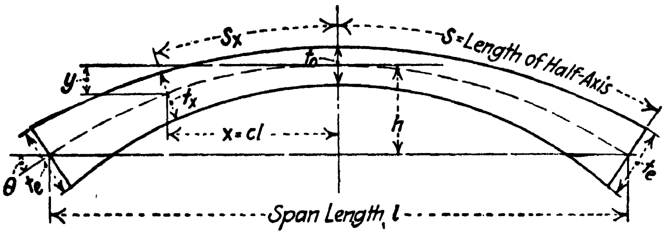


Fig. 168.

Mr. Cochrane are given on page 435. The following notation is used (see Fig. 168).

l = span of arch axis in feet.

h = rise of arch axis in feet.

r = rise ratio $\frac{h}{l}$.

y = ordinate of arch axis, the abscissa of which is cl.

s = length of half axis, measured along axis from crown to springing.

s_x = distance along axis from crown to a given section whose abscissa is $cl = x$.

t_0 = thickness of arch rib at crown.

t_e = thickness of arch rib at springing.

t_x = thickness of arch rib at point whose abscissa is x.

u_s = ratio of thickness of springing to thickness at crown $= \frac{t_e}{t_0}$.

M_c = moment at crown in foot-pounds.

T_c = thrust at crown in pounds.

M_s = moment at springing in foot-pounds.

T_s = thrust at springing in pounds.
T_{m_1} = coefficient for wl for thrust at crown corresponding to maximum positive moment at crown.
T_{m_2} = coefficient for wl for thrust at crown corresponding to maximum negative moment at crown.
V_s = approximate dead-load vertical end reaction, or one-half dead weight of span in pounds.
w_c = weight of arch at crown, plus average weight of superstructure at crown, in pounds per linear foot of span.
w = live load in pounds per linear foot of span (not necessarily the same for all positions of the live load).
ω = coefficient of linear expansion due to temperature changes.
t = change in temperature in degrees Fahrenheit.
E = modulus of elasticity of concrete in pounds per square foot.
I_0 = moment of inertia of arch rib at crown in biquadratic feet.
f_a = average direct stress throughout arch in pounds per square foot.
f_{ac} = direct stress at crown section in pounds per square foot.

For open spandrel arches,

$$y = \frac{8rl}{6 + 5r} \cdot (3c^2 + 10c^4r) \tag{102}$$

and

$$\tan \theta = \frac{8r}{6 + 5r} \cdot (3 + 5r) \tag{103}$$

For filled spandrel arches,

$$y = \frac{4rl}{1 + 3r} \cdot (c^2 + 24c^5r) \tag{104}$$

and

$$\tan \theta = \frac{4r}{1 + 3r} \cdot (1 + 7.5r) \tag{105}$$

If the half axis is divided into 10 equal sections, the ratio of the depth of the arch at the center of each section to the depth at the crown is given in the following table.

THICKNESSES OF TYPICAL ARCHES

Value of v	Values of u_s							
	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.05	1.007	1.006	1.005	1.004	1.003	1.002	1.001	1.000
0.15	1.021	1.018	1.015	1.012	1.009	1.006	1.003	1.000
0.25	1.035	1.030	1.025	1.020	1.015	1.010	1.005	1.000
0.35	1.049	1.042	1.035	1.028	1.023	1.021	1.023	1.030
0.45	1.063	1.054	1.048	1.048	1.057	1.070	1.083	1.101
0.55	1.077	1.072	1.085	1.105	1.133	1.165	1.193	1.231
0.65	1.095	1.125	1.168	1.215	1.269	1.328	1.385	1.455
0.75	1.145	1.223	1.311	1.403	1.508	1.625	1.737	1.865
0.85	1.245	1.393	1.547	1.700	1.862	2.025	2.185	2.355
0.95	1.406	1.621	1.837	2.055	2.277	2.495	2.709	2.932
1.00	1.500	1.750	2.000	2.250	2.500	2.750	3.000	3.250

The value of s, the length of the half axis, may be determined by scaling from the drawing or it may be taken by interpolation from the following table:

LENGTHS OF THE HALF ARCH AXIS s IN TERMS OF THE SPAN LENGTH l

Kinds of arches	Values of $\frac{s}{l}$ for rise-ratio r =				
	0.10	0.15	0.20	0.25	0.30
Open spandrel arches..........	0.513	0.529	0.551	0.577	0.607
Filled spandrel arches..........	0.515	0.534	0.559		

The formulas for moment, thrusts, and average stresses are given on page 435.

254. Form of Arch Axis. It is the usual practice to make the arch axis conform to the dead-load equilibrium polygon through the crown and the springing. The positive live-load moments (those producing compression in the upper fiber) are greater than the negative live-load moments at both the crown and springing sections. At certain sections in the haunch the negative live-load moments will be the greater. If the axis is made to conform to the equilibrium polygon for dead load plus

one-half live load over the whole span, the total maximum positive and negative moments due to live load and dead load will be equal. Unless however, the ratio of live to dead load is unusually large, there will be little difference between such an axis and the one conforming to the dead-load equilibrium polygon.

The effect of the shortening of the arch axis is to produce positive bending moments at the crown and larger negative bending moments at the springing. Also the fall in temperature is often specified as greater than the rise which tends to produce larger positive than negative moments at the crown and larger negative than positive moments at the springing. In the haunch of flat arches the stresses produced by arch shortening and temperature changes are not nearly so great as those produced at the springing.

It is of benefit at the springing to make the arch axis conform to the dead-load equilibrium polygon, since the arch shortening moments and the excess of fall over rise of temperature moments are the reverse of the larger live-load moments, thus making the total positive and negative moments more nearly equal. The reverse is true at the crown, but since the springing section is much the larger it seems desirable to favor it rather than the crown section. The method of laying out the arch axis for dead load is outlined in Art. 256.

255. Procedure in Arch Design.

1. Assume a crown thickness from Fig. 162 or Fig. 163, or by comparison with a previous design, and compute the total dead load per linear foot of span at the crown.

2. Assume a vertical springing thickness of from two to three times the crown thickness. For flat, heavily loaded arches the lower limit should be assumed, while the upper limit will give best results for arches of high rise. Assume the amount of reinforcement at the crown and at the springing.

3. Make approximate computations for the length of the span and rise of the arch axis and determine the rise ratio, $\frac{h}{l}$.

4. By the equations of Art. 253 calculate the extreme fiber stresses due to the proper combinations of moments and thrusts

using the methods and diagrams of Chap. V. If the stresses are too small or too great, change the thickness at the crown or at the springing or both, and repeat the operation. It is usually not necessary to make approximate calculations for maximum negative moment at the crown or for maximum positive moment at the springing, especially if the fall in temperature is taken as being appreciably greater than the rise in temperature.

5. Lay out the arch axis according to equation (102) or equation (104). In flat arches y should be computed at the quarter point of the span and for three or four intermediate points between quarter point and springing. In arches of high rise, one additional point between quarter point and crown should be determined. From the first table of Art. 253 determine the thickness of the arch at several points, and draw the curves of the intrados, extrados, and neutral axis.

6. Compute the dead loads at the panel points or at suitable intervals and lay out an equilibrium polygon passing through the crown and springing.

7. Alter the shape of the arch axis so that it will fit the equilibrium polygon as nearly as practicable. Lay out the arch thickness again and determine the radii of the intrados, extrados, and neutral axis.

8. Analyze the arch so determined by the elastic theory for maximum stresses in the steel and in the concrete. In most arches the maximum stresses occur either at the crown or at the springing, although where the ratio of live to dead load is large the maximum stresses may be found in the haunch. For aesthetic reasons the arch ring must gradually increase in thickness from crown to springing. Such a ring has a thickness much greater than required over the greater part of the distance between crown and springing. For this reason an investigation of the crown and springing sections is usually sufficient.

256. Design of a Reinforced Concrete Arch.

Type—Filled spandrel

Clear span—70 ft.-0 in.

Rise of intrados—10 ft.-0 in.

Live loading—Cooper's E-60.

Ballast—6 in. under ties.

Minimum fill—2 ft.-0 in.

Unit stresses—for dead load, live load, and arch shortening:

$f'_c = 2000.$

$f_s = 16{,}000.$

When temperature stresses are included an increase of 25 per cent in the above stresses is permissible.

Arch to be designed for a rise in temperature of 20 deg. Fahrenheit and for a fall in temperature of 30 deg. Fahrenheit.

Considering the weight of one locomotive distributed over 50 ft. of track and the load per foot distributed to the arch as

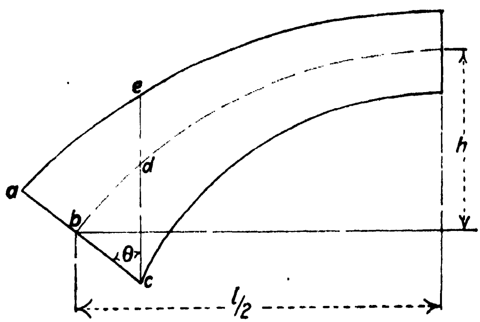


FIG. 169.

shown in Fig. 160, the live load per foot section is about 600 lb. per ft. The rails and fastenings are assumed to weigh 150 lb. per ft. of track, and ballast and fill 120 lb. per cu. ft.

Ties are 8 in. by 8 in. by 10 ft.-0 in. In making computations for dead load it is assumed that the top of the ballast is level with the base of rail and the weight of the ties neglected.

From Schwada's curves (as given in Fig. 163), the crown thickness is assumed as 22 in. and the vertical springing thickness as two times the crown thickness, or 44 in.

In Fig. 169, assuming ae as a straight line, and $dc = \frac{1}{2}ce = 22$ in.

$h = 120 \text{ in.} + 11 \text{ in.} - 22 \text{ in.} \cos^2 \theta$

$l = 840 \text{ in.} + 2(22 \text{ in.} \cos \theta \sin \theta)$

Assume $\theta = 38°$; $\sin 38° = 0.616$, $\cos 38° = 0.788$

$h = 117.3$ in.
$l = 861.4$ in.
$r = 0.136$

But $\tan \theta = \frac{4r}{1 + 3r} \cdot (1 + 7.5r) =$ (See page 437.)

$$\frac{4 \times 0.136}{1 + 3 \times 0.136} \cdot (1 + 7.5 \times 0.136) = 0.781$$

$\tan 38° = 0.781.$

Radial springing thickness $= 44 \times 0.788 = 34.7$

A springing thickness of 34 in. is assumed.

Reinforcement at crown ⅞-in. round rods 6 in. center to center, one row 2 in. from extrados and one row 2 in. from intrados. Therefore $p = \frac{2.41}{12 \times 22} = 0.0091$ and $\frac{d'}{a} = \frac{2}{22} = 0.09$. At a point 10 ft.-0 in. from the springing, these rods are lapped with 1-in. round rods, the latter being carried through the springing section. Therefore, at the springing, $p = \frac{3.14}{12 \times 34} = 0.0077$ and $\frac{d'}{a} = \frac{2}{34} = 0.06$. The dead load at the crown consists of

Fill	240 lb.
Ballast	140 lb.
Rails	10 lb.
Concrete	275 lb.
	665 lb

Approximate Method of Testing Trial Arch.

$l = 71.8$
$h = 9.8$
$r = 0.136$
$u_s = \frac{34}{22} = 1.55$
$w_c = 665$
$w = 600$
$\omega = 0.000006$
$E = 288,000,000$ lb. per sq. ft.

$$t = +20 \text{ deg. Fahrenheit or } -30 \text{ deg. Fahrenheit}$$

$$\omega tE = 34{,}560 \text{ or } 51{,}840$$

$$I_0 = \frac{1}{12} \times 1 \times \left(\frac{22}{12}\right)^3 + 14 \times \frac{2.41}{144} \times \left(\frac{9}{12}\right)^2 = 0.643$$

$$A_0 = \frac{22}{12} + \frac{2.41}{144} \times 14 = 2.07$$

$$wl = 600 \times 71.8 = 43{,}100 \text{ lb.}$$

$$wl^2 = 3{,}093{,}000 \text{ ft.-lb.}$$

Section at Crown.[1]

Dead Load:

$$T_c = -\frac{1 + 3 \times 0.136}{8 \times 0.136} \times 665 \times 71.8 = -61{,}800$$

$$M_c \textit{ assumed} = 0$$

Live Load for Maximum Positive Moment:

$$T_c = -\frac{57.6 + (189 - 8 \times 1.55) \times 0.136 - 220 \times 0.136^2}{1000 \times 0.136} \times 43{,}100$$

$$= -0.570 \times 43{,}100 = -24{,}600$$

$$M_c = +\frac{72 + 105 \times 0.136 + 220 \times 0.136^2 - (17 + 10 \times 0.136) \times 1.55 + 1.5 \times 1.55^2}{10{,}000}$$

$$\times 3{,}093{,}000 = 0.00656 \times 3.093{,}000 = +20{,}300$$

Live Load for Maximum Negative Moment:

$$T_c = -\frac{57.8 + 2 \times 1.55 + (10 + 30 \times 1.55) \times 0.136 - (380 + 30 \times 1.55) \times 0.136^2}{1000 \times 0.136}$$

$$\times 43{,}100 = -0.446 \times 43{,}100 = -19{,}230$$

Temperature (Fall of 30 deg.):

$$T_c = +[19.4 \times 1.55 - 7.5 + (17 \times 1.55 - 31) \times 0.136 - 140(1.55 - 1) \times 0.136^2]$$

$$\times \frac{51{,}840 \times 0.643}{9.8^2} = \frac{20.6 \times 51{,}840 \times 0.643}{9.8^2} = +7100$$

[1] Mr. Cochrane's analysis considers a compressive stress to be of positive sign. The authors prefer the opposite convention and in the example here shown compression is indicated by a minus sign.

$$M_c = +(38.5 - 12.8 \times 1.55 + 1.6 \times \overline{1.55^2}) \times \frac{9.8 \times 7100}{100}$$

$$= \frac{22.5 \times 9.8 \times 7100}{100} = +15{,}700$$

Section at Springing.

Dead Load:

$$V_s = -\frac{2 + 15 \times 0.136}{4} \times 665 \times 71.8 = -48{,}200$$

$$T_s = \sqrt{61{,}800^2 + 48{,}200^2} = -78{,}400$$

$$M_s \text{ assumed} = 0$$

Live Load for Maximum Negative Moment:

$$T_s = -\frac{27.6+(125+6\times1.55)\times0.136+320\times\overline{0.136}^2}{1000 \times 0.136} \times 43{,}100$$

$$= 0.381 \times 43{,}100 = -16{,}400$$

$$T_c = -\left[0.223 - \frac{0.0026(1.55-2)^2}{0.136}\right] \times 43{,}100 = 0.219 \times 43{,}100$$

$$= -9400$$

$$M_s =$$

$$-\frac{283-480\times0.136-9(4.22-2.8\times0.136-1.55)^2\times3{,}093{,}000}{10{,}000}$$

$$= -0.0171 \times 3{,}093{,}000 = -52{,}900$$

Temperature (Fall of 30 deg.):

$$T_s = (1.13 - 2.55 \times 0.136) \times 7100 = 0.78 \times 7100 = +5500$$

$$M_s = +15{,}700 - 9.8 \times 7100 = -53{,}900$$

Average Stresses.

For Dead Load:

$$f_a = -\left[1.030 + 2.5(.136 + 0.05)^2 - \frac{(20 \times 0.136 + 8) \times 1.55 - (1.55 - 1)^2}{100}\right]$$

$$\times \frac{61{,}800}{2.07} = \frac{0.954 \times 61{,}800}{2.07} = -28{,}500$$

For Live Load Producing Maximum Positive Moment at Crown:

$$f_a = -\left[0.920 + 2.6 \times \overline{0.136}^3 - 0.04 \times 1.55 + \frac{(6.7 + 33 \times 0.136)(4 - 1.55)^2}{1000}\right] \times \frac{24{,}600}{2.07}$$

$$= \frac{0.932 \times 24{,}600}{2.07} = -11{,}100$$

For Live Load Producing Maximum Negative Moment at Springing:

$$f_a = -\left[0.950 + 1.7 \times \overline{0.136}^2 - 0.05 \times 1.55 + \frac{(4 + 48 \times 0.136)(4 - 1.55)^2}{1000}\right]$$

$$\times \frac{9400}{2.07} = -\frac{0.967 \times 9400}{2.07} = -4400$$

For Fall of Temperature (30 deg.) *and Arch Shortening Stresses:*

$$f_a = [1.075 - 0.8 \times 0.136 - (0.081 - 0.11 \times 0.136) \times 1.55] \times \frac{7100}{2.07} = \frac{0.864 \times 7100}{2.07} = +3000$$

SUMMARY FOR MAXIMUM POSITIVE MOMENT AT CROWN

	Thrust	Moment	Average stress
Dead load	−61,800	0	−28,500
Live load	−24,600	+20,300	−11,100
Arch shortening[1]	+ 5,100	+11,300	+ 2,200
	−81,300	+31,600	−37,400
Dead load + live load	−86,400	+20,300	−39,600
Temperature	+ 7,100	+15,700	+ 3,000
Arch shortening	+ 4,700	+10,500	+ 2,000
	−74,600	+46,500	−34,600

[1] See typical computation on p. 460.

SUMMARY FOR MAXIMUM NEGATIVE MOMENT AT SPRINGING

Dead load	−78,400	0	−28,500
Live load	−16,400	−52,900	− 4,400
Arch shortening	+ 3,300	−32,400	+ 1,800
	−91,500	−85,300	−31,100
Dead load + live load	−94,800	−52,900	−32,900
Temperature	+ 5,500	−53,900	+ 3,000
Arch shortening	+ 3,000	−29,400	+ 1,600
	−86,300	−136,200	−28,300

UNIT STRESSES

Section	Load	M	N	$\frac{e}{a}$	np_g	K	f_c
Crown	$D + L + S$	31,600	81,300	0.212	0.14	1.84	580
	$D + L + S + T$	46,500	74,600	0.340	0.14	2.65	760
Springing	$D + L + S$	85,300	91,500	0.274	0.12	2.19	610
	$D + L + S + T$	136,200	86,300	0.557	0.12	3.67	920

D = dead load, L = live load, S = arch shortening, T = temperature.

Analysis by the Elastic Theory. The shape of the arch axis may be determined from equation (104). The values of $cl = x$ and the corresponding values of y are tabulated below:

c	cl	c^5	$24c^5r$	c^2	$c^2 + 24c^5r$	y
.25	17.95	.00098	.0032	.0625	.0657	1.73
.30	21.54	.00243	.0079	.0900	.0979	2.72
.35	25.13	.00525	.0171	.1225	.1396	3.87
.40	28.72	.01024	.0334	.1600	.1934	5.37
.45	32.31	.01845	.0602	.2025	.2627	7.29
.50	35.90	.03125	.1020	.2500	.3520	9.77

The curve of the half arch axis is plotted, and half the radial thickness of various points along the axis laid off on either side of the axis. These thicknesses may be taken from the first table of Art. 253. The arch of Fig. 170 is laid out in this manner. In order to determine the approximate line of thrust due to dead

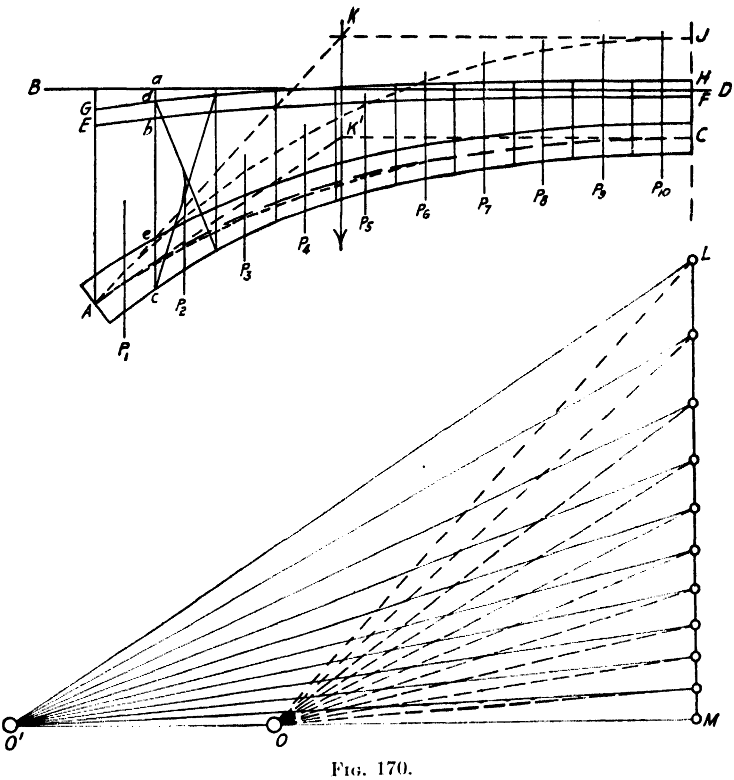


FIG. 170.

load, an equilibrium polygon for dead load may be drawn through the crown and springing. Such an equilibrium polygon closely represents the line of thrust produced by the dead load. The half span is divided into ten (or more) equal divisions. The horizontal line *BD* represents the top of the fill, and the line *EF* the reduce load line for fill. (The fill is assumed to weigh 120 lb. per cu. ft. and the concrete 150 lb. per cu. ft. Therefore,

be is $\frac{120}{150}$ times *ae*.) The weight of the ballast, rails, and joints per foot section of arch is reduced to equivalent weight of concrete and *GH* plotted parallel to *EF*. The dead load on the arch is then represented by the area between the line *GH* and the curve of the intrados. The load for each section is determined by measuring the ordinates *AG*, *cd*, etc., taking the average of each two adjacent ordinates, and multiplying this average by the width of each division times 150. The loads so determined are as follows:

$$P_1 = \frac{11.0 + 11.4}{2} \times 3.59 \times 150 = 6000$$

$$P_2 = \frac{9.1 + 11.0}{2} \times 3.59 \times 150 = 5400$$

$$P_3 = \frac{7.7 + 9.1}{2} \times 3.59 \times 150 = 4500$$

$$P_4 = \frac{6.6 + 7.7}{2} \times 3.59 \times 150 = 3800$$

$$P_5 = \frac{5.9 + 6.6}{2} \times 3.59 \times 150 = 3400$$

$$P_6 = \frac{5.4 + 5.9}{2} \times 3.59 \times 150 = 3000$$

$$P_7 = \frac{5.0 + 5.4}{2} \times 3.59 \times 150 = 2800$$

$$P_8 = \frac{4.7 + 5.0}{2} \times 3.59 \times 150 = 2600$$

$$P_9 = \frac{4.5 + 4.7}{2} \times 3.59 \times 150 = 2500$$

$$P_{10} = \frac{4.4 + 4.5}{2} \times 3.59 \times 150 = 2400$$

The center of gravity of each trapezoidal load is determined, and the verticals P_1, P_2, etc., drawn through these centers, which represent the points of application of the loads. The load line is now constructed, any convenient pole *O* assumed, and the rays of the force polygon drawn. After drawing the corresponding equilibrium polygon *AJ*, the first and last rays are prolonged to

their intersection at K. The vertical through K represents the resultant of all the loads on the half span. To construct an equilibrium polygon passing through both A and C, CK' is drawn horizontally, and LO' parallel to AK'. O' is the pole of the force polygon required for a corresponding equilibrium polygon passing through both A and C. If this equilibrium polygon fails to coincide with the neutral axis at all sections, the line of thrust for dead load will be eccentric and a bending moment will be produced at such sections. If it is desired to have no bending moments produced by dead load, the shape of the arch axis should be altered to coincide with the equilibrium polygon passing through the crown and the springing. If the difference between the assumed axis and the equilibrium polygon is great, the loads should be revised. In the present case, this is not necessary.

Radii of Neutral Axis. Three-centered curves are to be used for the intrados and the neutral axis, the larger radius in each case being used from crown to quarter point. From the equations of Art. 247, the radii of the neutral axis are computed as follows:

$$R_1 = \frac{17.95^2 + 1.73^2}{2 \times 1.73} = 93.99 \text{ ft}$$

$$\sin\theta = 0.191 \qquad \cos\theta = 0.982$$

$$R_2 = \frac{1}{2} \cdot \frac{\overline{17.95}^2 + \overline{8.04}^2}{8.04 \times 0.982 - 17.95 \times 0.191} = 43.30 \text{ ft.}$$

The length of the neutral axis may be computed, scaled, or estimated from the second table of Art. 253. The computations involved are as follows:

$$\theta = 11 \text{ degrees } 2 \text{ minutes}$$

Length crown to quarter point

$$\frac{11\tfrac{1}{30}}{360} \times 93.99 \times 2\pi = 18.10$$

$$\sin \frac{1}{2} \angle \text{ subtending chord of } R_2 = \frac{19.66}{2 \times 43.30} = 0.227$$

The $\angle$ = 26 degrees 15 minutes.

Length quarter point to springing

$$\frac{26\frac{1}{4}}{360} \times 43.30 \times 2\pi = 19.84 \text{ ft.}$$

Total length = 2(18.10 + 19.84) = 75.88 ft.

By Art. 253, the length = 0.529 × 2 × 71.8 = 75.96 ft.

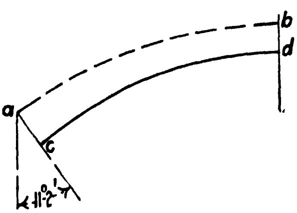


FIG. 171.

Radii of Intrados. The radial thickness of the arch at the quarter point is, from Art. 253,

$$1.065 \times 22 = 23.4 \text{ in.}$$

In Fig. 171

The horizontal distance from b to a is 17.95 ft. The horizontal distance from b to c is $17.95 - \frac{23.4}{24} \times 0.191 = 17.76$ ft. The vertical distance from b to a = 1.73 ft.

The vertical distance d to $c = 1.73 - \frac{11}{12} + \frac{23.4}{24} \times 0.982 =$ 1.77 ft. Then

$$R_1 = \frac{17.76^2 + 1.77^2}{2 \times 1.77} = 89.99 \text{ ft.}$$

$$\sin\theta = 0.197 \qquad \cos\theta = .980$$

and

$$R_2 = \frac{17.24^2 + 8.23^2}{2(8.23 \times 0.980 - 17.24 \times 0.197)} = 39.07 \text{ ft}$$

Location of Axis. Dividing the half of the neutral axis into 10 equal divisions, the length of each division is 3.79 ft. Laying off the centers of each one of the divisions on Fig. 172, the coordinates with the origin of coordinates at the crown are scaled and tabulated in the seventh and last columns on page 452. The radial thickness of the arch at the center of each division is also scaled and the moment of inertia computed and tabulated.

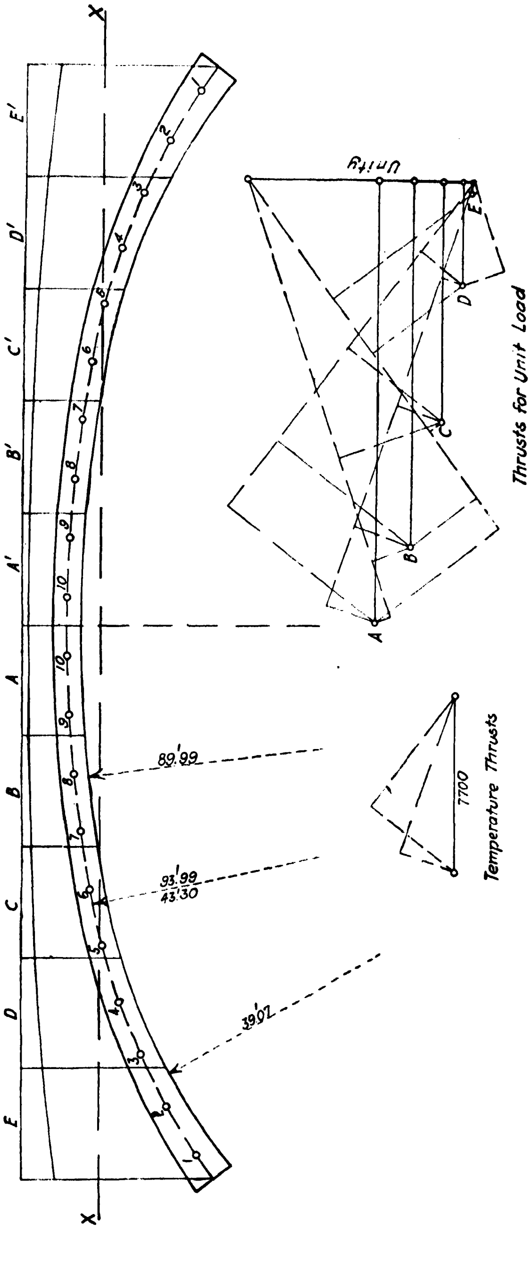


Fig. 172.

Point	a	I_c	$\left(\frac{a}{2} - d'\right)^2$	$14I_c$	I	y_c	$\frac{y_c}{I}$	$\frac{1}{I}$	y	x
10	1.85	.528	0.578	.135	.663	.02	.03	1.51	+2.15	1.9
9	1.87	.545	0.593	.139	.684	.19	.28	1.46	+1.98	5.7
8	1.90	.572	0.608	.142	.714	.50	.70	1.40	+1.67	9.5
7	1.92	.590	0.624	.146	.736	.96	1.30	1.36	+1.21	13.2
6	1.95	.618	0.640	.150	.768	1.57	2.04	1.30	+0.60	17.0
5	1.97	.637	0.672	.157	.794	2.37	2.99	1.26	−0.20	20.7
4	2.02	.687	0.706	.165	.852	3.49	4.10	1.17	−1.32	24.3
3	2.13	.805	0.810	.247	1.052	4.90	4.66	.95	−2.73	27.8
2	2.34	1.068	1.000	.305	1.373	6.61	4.81	.73	−4.44	31.1
1	2.66	1.568	1.346	.411	1.979	8.64	4.37	.51	−6.47	34.4
Crown	1.83	.511	0.563	.132	.643		$\sum\frac{y_c}{I}$	$\sum\frac{1}{I}$		
Springing	2.83	1.889	1.563	477	2.366		25.28	11.65		

From the above tabulation the axis is located so that

$$\sum\frac{y}{I} = 0, \text{ that is, } y' = \frac{25.28}{11.65} = 2.17 \text{ ft. below the crown.}$$

(See footnote on page 432.)

Determination of Loads. The load is assumed applied to the arch at the equidistant points as indicated on Fig. 172. The dead loads are computed in the same manner as on page 448 and are as follows:

A—4900 lb.
B—5400 lb.
C—6500 lb.
D—8500 lb.
E—11,100 lb.

The live load is 4300 lb. per section.

Moments and Thrusts for Unit Loads. The table on page 455 gives the value of H_0, V_0, and M_0 for unit load applied at each of the several load points. The tables on pages 456 to 458 give the moments and thrusts produced at the crown, sixth point, and springing sections respectively by unit load. The sixth point is chosen, not because it is necessarily the point of highest

stress, but to show the method of computation for a section other than the crown or springing (see Art. 255).

The value of the thrust N is determined as follows:

In Fig. 173, H_0 and V_0 represent the thrust and shear, due to a load unity P, applied as shown. These forces produce right and left reactions of R_R and R_L, respectively. At section a, the thrust produced by the load P is equal and opposite to the component of R_L which is parallel to the neutral axis at that

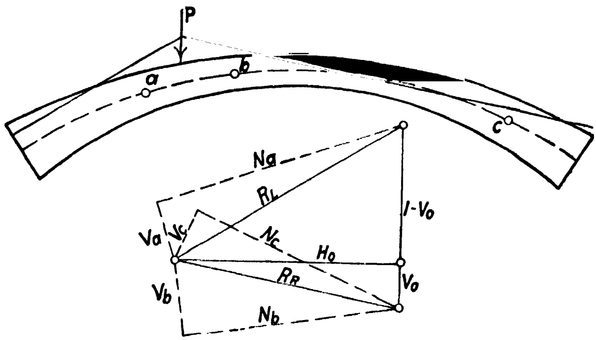


FIG. 173.

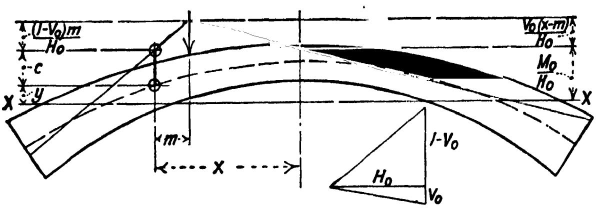


FIG. 174.

point. Similarly, the shear at section a is equal and opposite to the component of R_L perpendicular to the neutral axis. At section b, the components of R_R determine the thrust and shear. For any section on the right half of the arch, section c, the components of R_R determine the thrust and shear. In Fig. 172, the thrusts and shears are laid off as previously computed, and the values of the thrusts at the various sections determined as above, by scaling from the diagram.

The moment may be computed as indicated in the tables on pages 456 to 458 or it may be obtained graphically. From

Fig. 174 it may be seen that $M = H_0 c$, H_0 always being negative and c being positive when measured upward *to* the neutral axis, and negative when measured downward *to* the neutral axis.

Loading for Maximum Stresses. From the values of M and N for unit load as given in the tables it can readily be seen what portions of the arch should be considered loaded in order to obtain the maximum positive and negative moments for the sections investigated. Finally the values of thrusts and moments for the design dead and live loads are obtained by multiplying the values for unit load by the section loads as previously determined. The summations give the resultant maximum moments and thrusts.

s = a constant

$\sum \frac{y}{I} = 0$

$$H_o = \frac{\sum\left(\frac{my}{I}\right)}{2\sum\left(\frac{y^2}{I}\right)} \qquad V_o = \frac{\sum\left(\frac{mx}{I}\right)}{2\sum\left(\frac{x^2}{I}\right)} \qquad M_o = \frac{\sum\left(\frac{m}{I}\right)}{2\sum\left(\frac{1}{I}\right)}$$

	x	y	$\frac{1}{I}$	x^2	y^2	$\frac{x^2}{I}$	$\frac{y^2}{I}$	Unit load at A				B				C				D				E			
								m	$\frac{m}{I}$	$\frac{mx}{I}$	$\frac{my}{I}$	m	$\frac{m}{I}$	$\frac{mx}{I}$	$\frac{my}{I}$	m	$\frac{m}{I}$	$\frac{mx}{I}$	$\frac{my}{I}$	m	$\frac{m}{I}$	$\frac{mx}{I}$	$\frac{my}{I}$	m	$\frac{m}{I}$	$\frac{mx}{I}$	$\frac{my}{I}$
A	3.6																										
B	10.8																										
C	18.0																										
D	25.1																										
E	32.3																										
10	1.9	+2.15	1.51	3.6	4.62	5	7.0																				
9	5.7	+1.93	1.46	32.5	3.92	47	5.7	2.1	3.1	15	+ 6.1																
8	9.5	+1.67	1.40	90.3	2.79	126	3.9	5.9	8.3	79	+ 13.9																
7	13.2	+1.21	1.36	174.2	1.46	237	2.0	9.6	13.1	173	+ 15.8	2.4	3.3	44	+ 4.0												
6	17.0	+0.60	1.30	289.0	0.36	376	0.5	13.4	17.4	296	+ 10.4	6.2	8.1	138	+ 4.9												
5	20.7	−0.20	1.26	428.5	0.04	540	0.1	17.1	21.5	445	− 4.3	9.9	12.5	259	− 2.5	2.7	3.4	70	− 0.7								
4	24.3	−1.32	1.17	590.5	1.74	691	2.0	20.7	24.2	588	− 31.9	13.5	15.8	384	− 20.9	6.3	7.4	180	− 9.8								
3	27.8	−2.73	0.95	772.8	7.45	734	7.1	24.2	23.0	640	− 62.8	17.0	16.2	450	− 44.2	9.8	9.3	259	− 25.4	2.7	2.6	72	− 7.1				
2	31.1	−4.44	0.73	967.2	19.71	705	14.4	27.5	20.1	625	− 89.2	20.3	14.8	460	− 65.7	13.1	9.6	293	− 42.6	6.0	4.4	137	−19.5				
1	34.4	−6.47	0.51	1183.4	41.86	604	21.3	30.8	15.7	540	−101.6	23.6	12.0	413	− 77.6	16.4	8.4	289	− 54.4	9.3	4.7	162	−30.4	2.1	1.1	38	−7.1
Σ			11.65			4066	64.0		146.4	3404	−243.6		82.7	2148	−202.0		38.1	1096	−132.9		11.7	371	−57.0		1.1	38	−7.1

	Unit load at A	B	C	D	E
H_o	$\frac{-243.6}{2\times 64.0} = -1.903$	$\frac{-202.0}{2\times 64.0} = -1.578$	$\frac{-132.9}{2\times 64.0} = -1.038$	$\frac{-57.0}{2\times 64.0} = -0.445$	$\frac{-7.1}{2\times 64.0} = -0.055$
V_o	$\frac{3404}{2\times 4066} = 0.418$	$\frac{2148}{2\times 4066} = 0.264$	$\frac{1096}{2\times 4066} = 0.135$	$\frac{371}{2\times 4066} = 0.046$	$\frac{38}{2\times 4066} = 0.005$
M_o	$\frac{146.4}{2\times 11.65} = 6.28$	$\frac{82.7}{2\times 11.65} = 3.55$	$\frac{38.1}{2\times 11.65} = 1.64$	$\frac{11.7}{2\times 11.65} = 0.50$	$\frac{1.1}{2\times 11.65} = 0.05$

CROWN SECTION

$x = 0$	$\frac{d'}{a} = 0.09$	$I = 0.643$	Live load	Dead loads, M, and N,
		$a = 1.83$	4300 lb.	in thousand pound units
$y = +2.17$	$p_o = 0.0091$	$A = 2.07$	per section	$M = M_o + H_o y + V_o x - m$

Unit Load at										Dead load		Live loading for max. comp.			
												Upper Fiber		Lower Fiber	
Section	H_o	$H_o y$	V_o	$V_o x$	$-m$	M_o	M	N	Dead load	M	N	M	N	M*	N*
E	−0.055	−0.12				0.05	−0.07	−0.06	11.1	− 0.8	− 0.7			•	•
D	−0.445	−0.97				0.50	−0.47	−0.45	8.5	− 4.0	− 3.8			•	•
C	−1.038	−2.25				1.64	−0.61	−1.04	6.5	− 4.0	− 6.8			•	•
B	−1.578	−3.42				3.55	+0.13	−1.58	5.4	+ 0.7	− 8.5	•	•	•	•
A	−1.903	−4.13				6.28	+2.15	−1.90	4.9	+10.5	− 9.3	•	•		
A'												•	•		
B'												•	•		
C'															
D'															
E'															
Σ										+ 4.8	−58.2	+19.6	−29.9	− 4.4	−13.5

* These loadings assume that broken loads are not considered. In a highway arch, values for loads E', D', C', and B' should also be included in these summations.

SIXTH POINT

$x = 23.9$ $\quad \frac{d'}{a} = 0.08$ $\quad I = 0.845$ $\quad$ Live load 4300 lb. per section $\quad$ Dead loads, M, and N, in thousand pound units
$y = -1.18$ $\quad p_o = 0.0083$ $\quad a = 2.01$ $\quad A = 2.24$ $\quad M = M_o + H_oy + V_ox - m$

	Section	H_o	H_oy	V_o	V_ox	$-m$	M_o	M	N	Dead load	Dead load		Live loading for max. comp.			
											M	N	Upper Fiber		Lower Fiber	
													M	N	M	N
Unit Load at	E	−0.055	+0.06	+0.005	+ 0.12		0.05	+0.23	−0.05	11.1	+ 2.6	− 0.6	•	•		
	D	−0.445	+0.52	+0.046	+ 1.10		0.50	+2.12	−0.41	8.5	+18.0	− 3.5	•	•		
	C	−1.038	+1.22	+0.135	+ 3.23	− 5.90	1.64	+0.19	−1.26	6.5	+ 1.2	− 8.2	•	•	•	•
	B	−1.578	+1.86	+0.264	+ 6.31	−13.10	3.55	−1.38	−1.73	5.4	− 7.5	− 9.3			•	•
	A	−1.903	+2.24	+0.418	+10.00	−20.30	6.28	−1.78	−1.99	4.9	− 8.7	− 9.8			•	•
	A'	−1.903	+2.24	−0.418	−10.00		6.28	−1.48	−1.94	4.9	− 7.3	− 9.5			•	•
	B'	−1.578	+1.86	−0.264	− 6.31		3.55	−0.90	−1.57	5.4	− 4.9	− 8.5			•	•
	C'	−1.038	+1.22	−0.135	− 3.23		1.64	−0.37	−1.03	6.5	− 2.4	− 6.7			•	•
	D'	−0.445	+0.52	−0.046	− 1.10		0.50	−0.08	−0.44	8.5	− 0.7	− 3.7			•	•
	E'	−0.055	+0.06	−0.005	− 0.12		0.05	−0.01	−0.05	11.1	− 0.1	− 0.6			•	•
	Σ										− 9.8	−60.4	+10.9	−7.4	−25.0	−43.0

SPRINGING

$x = 35.9$ $\quad$ $y = -7.60$

$\frac{d'}{a} = 0.06$ $\quad$ $p_o = 0.0077$

$I = 2.366$ $\quad$ $a = 2.83$ $\quad$ $A = 3.14$

Live load 4300 lb. per section

Dead loads, M, and N, in thousand pound units $M = M_o + H_o y + V_o x - m$

Unit Load at Section	H_o	$H_o y$	V_o	$V_o x$	$-m$	M_o	M	N	Dead load	Dead load M	Dead load N	Live loading for max. comp. Upper Fiber M	Upper Fiber N	Lower Fiber M	Lower Fiber N
E	−0.055	+ 0.42	+0.005	+ 0.18	− 3.60	0.05	−2.95	−0.64	11.1	−32.7	− 7.1			•	•
D	−0.445	+ 3.38	+0.046	+ 1.65	−10.80	0.50	−5.27	−0.93	8.5	−44.8	− 7.9			•	•
C	−1.038	+ 7.89	+0.135	+ 4.84	−17.90	1.64	−3.53	−1.35	6.5	−22.9	− 8.8			•	•
B	−1.578	+11.99	+0.264	+ 9.48	−25.10	3.55	−0.08	−1.70	5.4	− 4.3	− 9.2	•	•	•	•
A	−1.903	+14.46	+0.418	+15.02	−32.30	6.28	+3.46	−1.87	4.9	+17.0	− 9.2	•	•		
A'	−1.903	+14.46	−0.418	−15.02		6.28	+5.72	−1.77	4.9	+28.0	− 8.7	•	•		
B'	−1.578	+11.99	−0.264	− 9.48		3.55	+6.06	−1.41	5.4	+32.7	− 7.6	•	•		
C'	−1.038	+ 7.89	−0.135	− 4.84		1.64	+4.69	−0.91	6.5	+30.5	− 5.9	•	•		
D'	−0.445	+ 3.38	−0.046	− 1.65		0.50	+2.23	−0.38	8.5	+19.0	− 3.2	•	•		
E'	−0.055	+ 0.42	−0.005	− 0.18		0.05	+0.29	−0.05	11.1	+ 3.2	− 0.6	•	•		
Σ										+25.7	−68.2	+96.2	−34.8	−50.8	−19.9

Temperature Stresses. The value of H_0 for a fall in temperature of 30 deg. is from equation (99)

$$H_0 = \frac{288{,}000{,}000 \times 30 \times 0.000006}{2 \times 64.0} \cdot \frac{71.8}{3.79} = +7700$$

For a rise of 20 deg.

$$H_0 = -5100$$

STRESSES DUE TO THRUST

		Crown	Springing	Average
	Dead load	−28,100	−21,700	−24,900
For live loading producing the maximum compression in the upper fiber at the:—	Crown	−14,400	− 9,200	−11,800
	Sixth point	− 3,200	− 4,000 − 1,800	− 3,100
	Springing	−17,700	−11,100 −13,200	−14,900
For live loading producing the maximum compression in the lower fiber at the:—	Crown	− 6,500	− 6,300 − 3,800	− 5,800
	Sixth point	−19,800	−12,900 −14,500	−16,800
	Springing	− 6,500	− 6,300 − 3,800	− 5,800
Temperature	Fall of 30°F.	+ 3,700	+ 1,900	+ 2,800
	Rise of 20°F.	− 2,500	− 1,300	− 1,900

Since the arch shortening stresses are proportional to those for a fall in temperature, these are more easily obtained in the final summation for maximum moments and thrusts.

Maximum Moments and Thrusts

Crown Section

Maximum Compression in Upper Fiber

	Thrust	Moment	c_a
Dead load	−58,200	+ 4,800	−24,900
Live load	−29,900	+19,600	−11,800
Arch shortening*	+ 5,200	+11,200	+ 1,900
	−82,900	+35,600	−34,800
Dead load + live load	−88,100	+24,400	−36,700
Temperature (30° fall)	+ 7,700	+16,700	+ 2,800
Arch shortening	+ 4,800	+10,400	+ 1,700
	−75,600	+51,500	−32,200

Maximum Compression in Lower Fiber

	Thrust	Moment	c_a
Dead load	−58,200	+ 4,800	−24,900
Live load	−13,500	− 4,400	− 5,800
Arch shortening	+ 4,300	+ 9,400	+ 1,600
	−67,400	+ 9,800	−29,100
Dead load + live load	−71,700	+ 400	−30,700
Temperature (20° rise)	− 5,100	−11,100	− 1,900
Arch shortening	+ 4,600	+ 9,900	+ 1,700
	−72,200	− 800	−30,900

* For a 30-deg. fall in temperature $\omega tE = -51{,}840$. (For a 20-deg. rise +34,560.) c_a for dead and live load $= -36{,}700$. The thrust, moment, and c_a for arch shortening are equal to $\dfrac{-36{,}700}{-51{,}840}$ times the similar quantities for a 30-deg. fall in temperature, or +5500, +11,800, and +2000, respectively. The last value, when added algebraically to the value of c_a for dead and live load, results in a smaller numerical value for the summation, and consequently the ratio between the arch shortening quantities and the temperature quantities becomes smaller. Hence, the thrust, moment, and c_a as computed above are slightly too large. Assume c_a due to arch shortening = +1900. Then the total $c_a = -34{,}800$ and the thrust, moment, and c_a due to arch shortening are +5200, +11,200, and +1900, respectively. If the last value does not check the value assumed, another computation must be made.

SIXTH POINT

Maximum Compression in Upper Fiber

	Thrust	Moment	c_e
Dead load	−60,400	− 9,800	−24,900
Live load	− 7,400	+10,900	− 3,100
Arch shortening	+ 3,800	− 4,600	+ 1,500
	−64,400	− 3,500	−26,500
Dead load + live load	−67,800	+ 1,100	−28,000
Temperature (20° rise)	− 4,900	+ 6,000	− 1,900
Arch shortening	+ 4,000	− 4,900	+ 1,600
	−68,700	+ 2,200	−28,300

Maximum Compression in Lower Fiber

	Thrust	Moment	c_e
Dead load	− 60,400	− 9,800	−24,900
Live load	− 43,000	−25,000	−16,800
Arch shortening	+ 5,600	− 7,000	+ 2,100
	− 97,800	−41,800	−39,600
Dead load + live load	−103,400	−34,800	−41,700
Temperature (30° fall)	+ 7,300	− 9,100	+ 2,800
Arch shortening	+ 5,200	− 6,500	+ 2,000
	− 90,900	−50,400	−36,900

SPRINGING SECTION

Maximum Compression in Upper Fiber

	Thrust	Moment	c_2
Dead load	− 68,200	+ 25,700	−24,900
Live load	− 34,800	+ 96,200	−14,900
Arch shortening	+ 4,400	− 42,500	+ 2,100
	− 98,600	+ 79,400	−37,700
Dead load + live load	−103,000	+121,900	−39,800
Temperature (20° rise)	− 4,000	+ 39,000	− 1,900
Arch shortening	+ 4,600	− 44,600	+ 2,200
	−102,400	+116,300	−39,500

Maximum Compression in Lower Fiber

	Thrust	Moment	c_2
Dead load	−68,200	+ 25,700	−24,900
Live load	−19,900	− 50,800	− 5,800
Arch shortening	+ 3,400	− 32,900	+ 1,600
	−84,700	− 58,000	−29,100
Dead load + live load	−88,100	− 25,100	−30,700
Temperature (30° fall)	+ 6,000	− 58,500	+ 2,800
Arch shortening	+ 3,100	− 30,400	+ 1,500
	−79,000	−114,000	−26,400

Final Maximum Unit Stresses

Section	Fiber	Load	M	N	$\frac{e}{a}$	ba	p_g	np_g	$\frac{d'}{a}$	K	f_c
Crown	Upper	D + L + S	+ 35,600	− 82,900	0.235	264	0.0091	0.14	0.09	2.00	620
		D + L + S + T	+ 51,500	− 75,600	0.372					2 89	830
	Lower	D + L + S	+ 9,800	− 67,400							
		D + L + S + T	− 800	− 72,200	0.005					0.91	250
Sixth point	Upper	D + L + S	− 3,500	− 64,000		290	0.0083	0.12	0.08		
		D + L + S + T	+ 2,200	− 68,700	0.016					0.96	230
	Lower	D + L + S	− 41,800	− 97,800	0.213					1.93	650
		D + L + S + T	− 50,400	− 90,900	0.276					2 31	720
Springing	Upper	D + L + S	+ 79,400	− 98,600	0.284	408	0.0077	0.12	0.06	2.34	570
		D + L + S + T	+116,300	−102,400	0.402					3 15	790
	Lower	D + L + S	− 58,000	− 84,700	0.242					2.06	440
		D + L + S + T	−114,000	− 79,000	0.510					4.01	780

257. Design of Abutments. Since a slight settling of its supports will produce large stresses in an arch, it is important that the abutments be so designed that no such settlement occurs. On soft ground it is difficult in the extreme to obtain an abutment large enough to insure stability, without the use of piles. As the size of the abutment increases, its weight and the weight of the filling above it increase so rapidly that in some types of arches an abutment without a pile foundation becomes nearly as large as the arch itself. It is a question whether some other type of structure is not preferable where hardpan or rock is not accessible as a foundation bed, or where a good pile foundation cannot easily be made.

The abutments of a reinforced concrete arch are often designed for full live load and also for live load over one-half the arch. While the first condition of loading may give the maximum total pressure on the abutment, the two most extreme conditions are those loadings which cause maximum compression in the

upper and lower fibers of the arch at the springing section. The moments and thrusts for this section are given on page 462. In a similar manner the shears for unit load are obtained from Fig. 172, and the total dead- and live-load shears computed and tabulated below. The shear at the springing due to a fall in temperature of 30 deg. Fahrenheit scaled from Fig. 172 is −4700 lb. The shears due to rise of temperature and to arch shortening are proportional.

Load at section	Dead load	Shears			
		For unit load	For dead load	For live loading producing maximum compression in upper fiber	For live loading producing maximum compression in lower fiber
E	11.1	−0.76	−8.4		*
D	8.5	−0.50	−4.3		*
C	6.5	−0.07	−0.5		*
B	5.4	+0.36	+1.9	*	*
A	4.9	+0.68	+3.3	*	
A'	4.9	+0.82	+4.0	*	
B'	5.4	+0.75	+4.1	*	
C'	6.5	+0.52	+3.4	*	
D'	8.5	+0.24	+2.0	*	
E'	11.1	+0.03	+0.3	*	
Σ			+5.8	+14.6	−4.2

Summary of Moments, Thrusts, and Shears
For Maximum Compression in Upper Fiber at Springing

	M	N	V
Dead load	+ 25.7	− 68.2	+ 5.8
Live load	+ 96.2	− 34.8	+14.6
Arch shortening	− 42.5	+ 4.4	− 3.4
	+ 79.4	− 98.6	+17.0 $e = +0.81$
Dead load + live load	+121.9	−103.0	+20.4
Temperature (20° rise)	+ 39.0	− 4.0	+ 3.1
Arch shortening	− 44.6	+ 4.6	− 3.5
	+116.3	−102.4	+ 20.0 $e = +1.14$

For Maximum Compression in Lower Fiber at Springing

	M	N	V
Dead load	+ 25.7	−68.2	+5.8
Live load	− 50.8	−19.9	−4.2
Arch shortening	− 32.9	+ 3.4	−2.6
	− 58.0	−84.7	−1.0 $e = -0.69$
Dead load + live load	− 25.1	−88.1	+1.6
Temperature (30° fall)	− 58.5	+ 6.0	−4.7
Arch shortening	− 30.4	+ 3.1	−2.4
	−114.0	−79.0	−5.5 $e = -1.44$

An abutment section $ABCDE$ (Fig. 175) is assumed. From the center of BC, the eccentric distance e (+1.14 ft.) is laid off upward, the value of N (−102,400 lb.) drawn perpendicular to BC, and the resultant of N and V obtained. This resultant must be combined with the forces due to the weight of the earth, filling, and abutment itself.

The forces due to the weight of the filling and the abutment are: (1) the weight of the filling; (2) the weight of the abutment; and

(3) the horizontal pressure due to the weight of the filling. When there is live load over the abutment a fourth force must be considered.

A more simple method giving results nearly the same as the more detailed analysis can be used for all but very large or important structures.

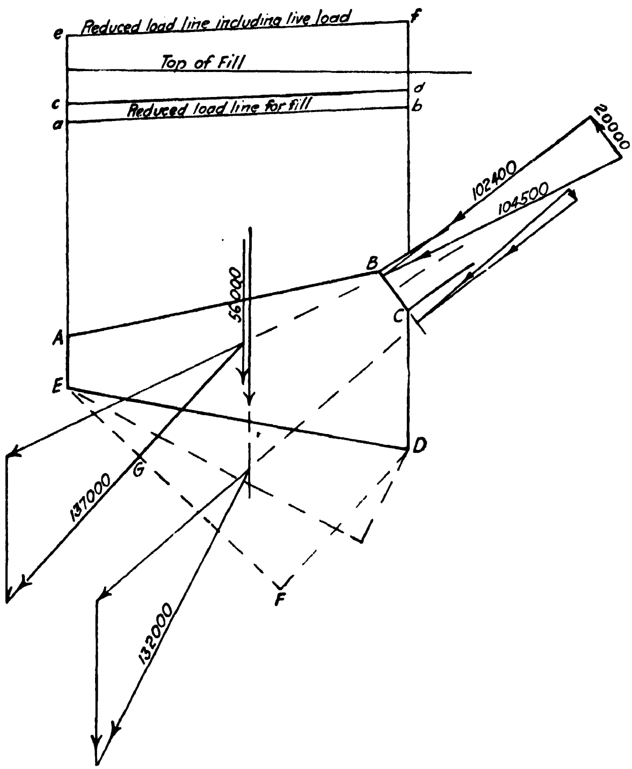


Fig. 175.

The reduced load line for fill at 120 lb. per cu. ft. (*ab*) is constructed as in the design of the arch. The line *cd* is drawn representing the top of the ballast as before. The trapezoid *cdDE* may now be considered as material of the same weight (150 lb. per cu. ft.) and its amount and point of application determined. The horizontal pressure due to the filling (in this case less than 2900 lb.) is neglected. The vertical force and the

resultant from the arch are combined graphically, and the total resultant pressure with its line of action determined. EF is constructed as the projection of the base of the footing on a plane perpendicular to this resultant, and the distances EG and EF scaled.

Then the maximum unit pressure is, from equation (78),

$$\frac{4EF - 6EG}{EF^2} \times 137{,}000 = \frac{4 \times 17.0 - 6 \times 5.8}{17.0^2} \times 137{,}000 =$$
$$15{,}700 \text{ lb. per sq. ft.}$$

In a similar manner the other condition of loading, that is, that producing maximum compression in the lower fiber at the springing, is analyzed and the maximum unit pressure found to be

$$\frac{4 \times 19.4 - 6 \times 7.8}{19.4^2} \times 132{,}000 = 10{,}500 \text{ lb. per sq. ft.}$$

In determining the resultant pressure for this condition the live load is considered completely to cover the abutment, and the total vertical force is represented by the trapezoid $efDE$.

Details of typical arches are shown in Figs. 176 to 178.

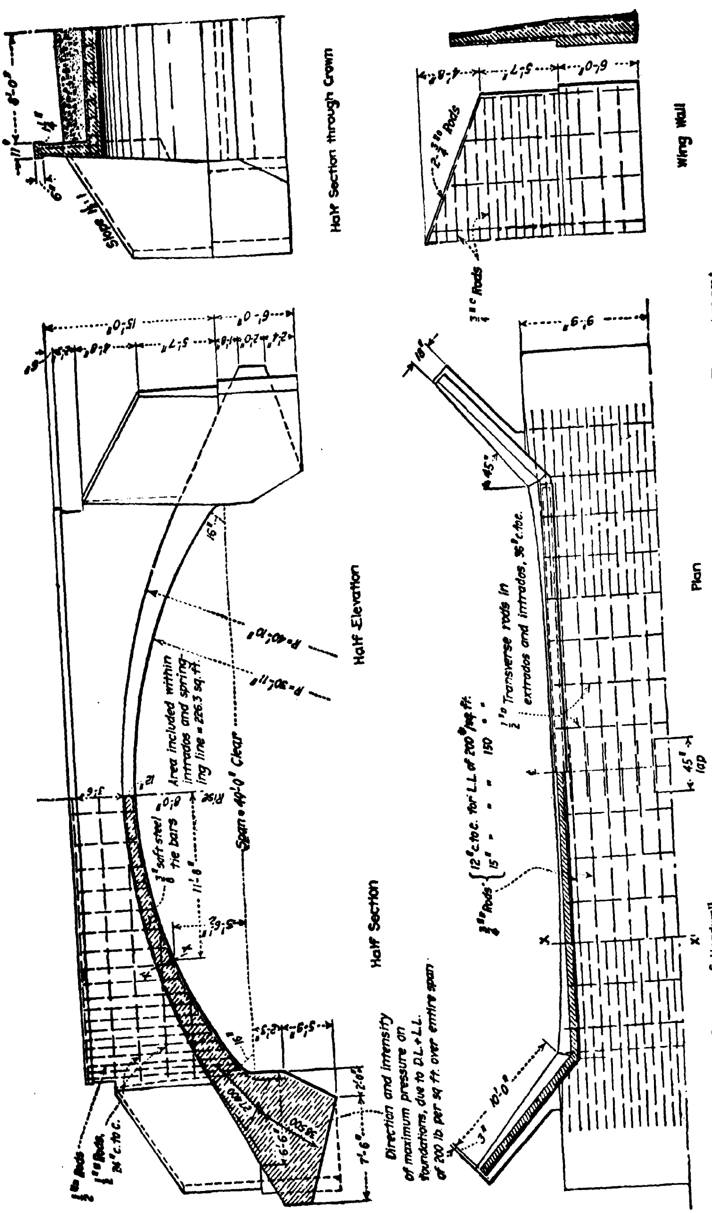


FIG. 176.—Standard 40-ft. arch, State of Missouri Highway Department.

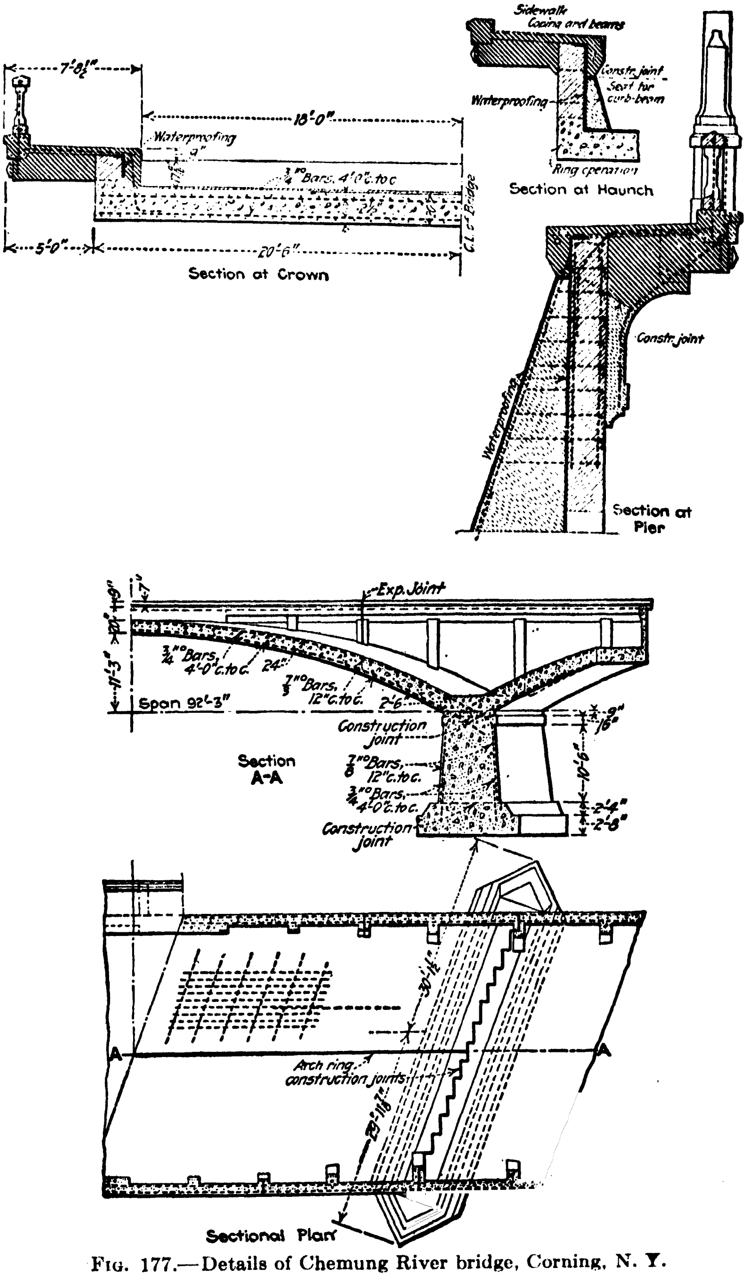


FIG. 177.—Details of Chemung River bridge, Corning, N. Y.

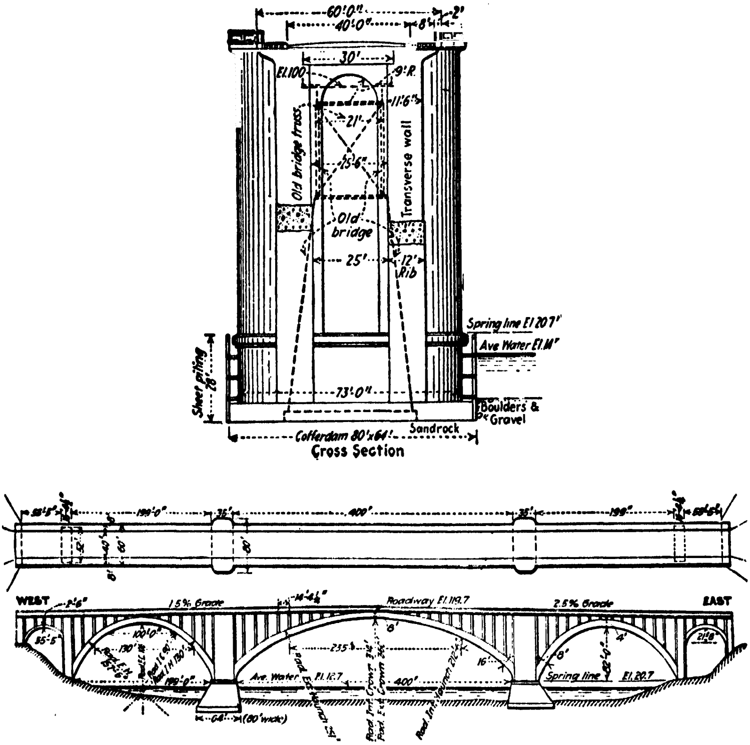


FIG. 178.—Franklin Avenue concrete arch bridge, Minneapolis, Minnesota.

CHAPTER XI

SLAB, BEAM, AND GIRDER BRIDGES

258. Types of Bridges. Reinforced concrete is particularly adapted to use in short-span highway bridges, because of its durability, rigidity, and economy, as well as the comparative ease with which a pleasing architectural appearance can be secured. The most widely used general types of concrete bridges for short spans are the slab bridge, Fig. 179, the T-beam or deck-

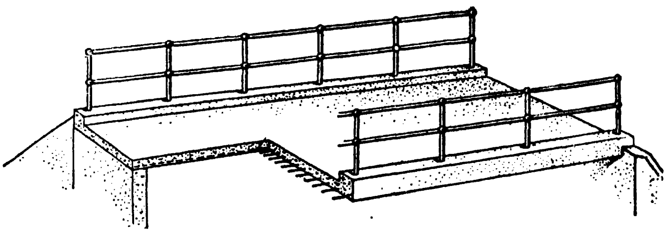


Fig. 179.—Slab bridge.

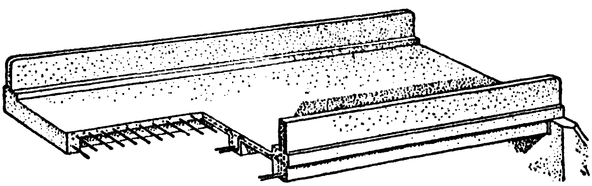


Fig. 180.—Deck-girder bridge.

girder bridge, Fig. 180, and the through-girder bridge, Fig. 181. Concrete girder bridges are normally economical only for spans up to about 65 ft. They have been used however, for spans of 100 ft. or more, the longest in the United States being 142 ft., but in most of these bridges economy has been sacrificed for other considerations which were of more importance in each specific case.

Structural steel is used more generally than reinforced concrete for bridges which are intended primarily for railroad traffic,

except the shortest spans, for which reinforced concrete slab bridges are frequently constructed. This chapter will therefore be devoted to the discussion of the details and design of highway bridges only.

259. Classification of Highway Bridges. Highway bridges of all types are classified according to the kind of traffic which is anticipated during the probable life of the structure. The classes which are given by the American Association of State Highway Officials, and for which the recommended loadings are given in a subsequent article, are as follows:

Class AA. Bridges for exceptionally heavy traffic units in locations where the passage of such units is frequent.

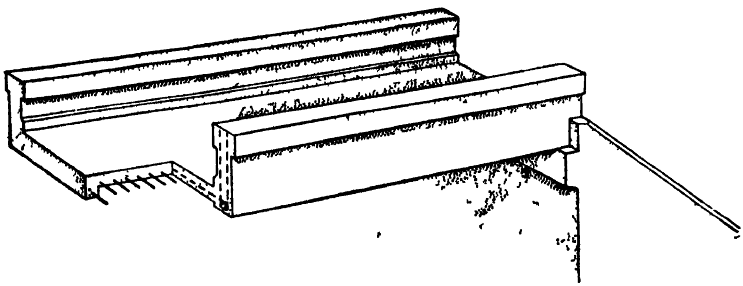


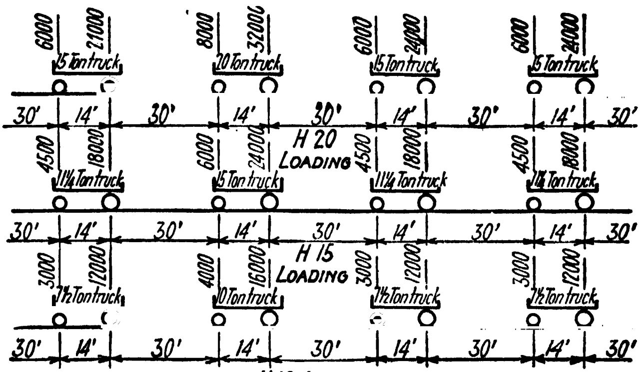
FIG. 181.—Through-girder bridge.

Class A. Bridges for normally heavy traffic units and the occasional passage of exceptionally heavy loads.

Class B. Bridges for light traffic units and the occasional passage of normally heavy loads. Class *B* bridges shall be considered as temporary or semitemporary structures.

Class C. Bridges for electric railway traffic in addition to highway traffic. The latter may correspond to any one of the classes described above.

260. Live Loads. *Truck Loadings.* For loaded lengths up to 60 ft. the live load specified by the American Association of State Highway Officials consists of a series of trucks as shown in Fig. 182. The wheel spacing, weight distribution, and amount of clearance required for the individual trucks are shown in Fig. 183. A width of 9 ft. is required for each line of trucks; this width is called the lane width.



H 10 LOADING

FIG. 182.

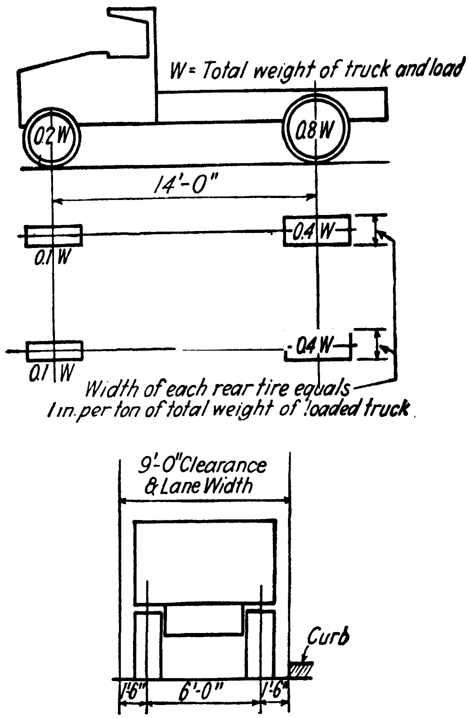


FIG. 183.—Standard truck-train loadings.

The highway live loads of the above specifications are divided into three classes, $H20$, $H15$ and $H10$. The number

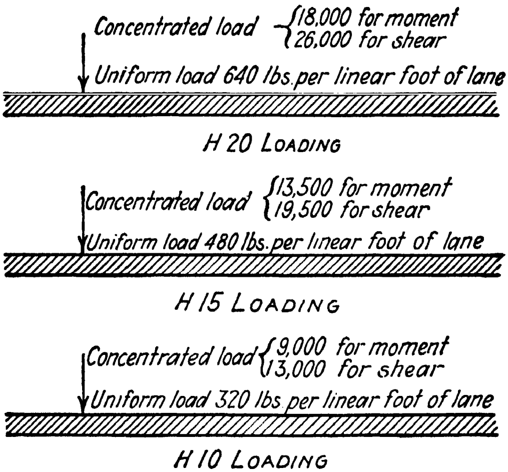


Fig. 184.—Equivalent highway loadings.

of the loading indicates the gross weight in tons of the heaviest truck in the series, while the other trucks of the series have a

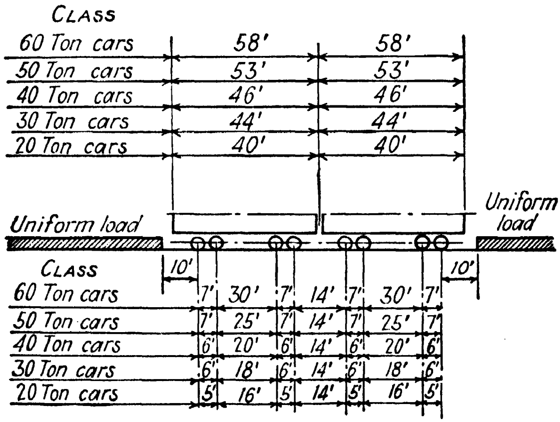
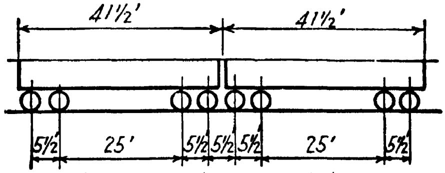


Fig. 185.—Electric railway loadings.

gross weight of three quarters of that amount. The gross weight of each truck is divided between the front and rear axles in the proportions shown in Fig. 183.

For loaded lengths of 60 ft. or more a uniform live load plus a concentrated load as shown in Fig. 184 is used.

Electric Railway Loadings. For bridges carrying electric railway traffic the loading is determined on the basis of the class of traffic which the bridge may be expected to carry. Such a loading, for design purposes, consists of a train of two cars, followed by and/or preceded by, a uniform load as shown in Fig. 185. Where freight cars are to be carried one of the classes shown in



Total loaded weight per car including 10 percent overload
40-Ton capacity-132,000 lbs.
70-Ton capacity-212,000 lbs.

Fig. 186.—Freight car loadings.

Fig. 186 may be assumed. These loadings are assumed to occupy 10 ft. of the roadway width.

Sidewalk Loadings. Sidewalk floors, stringers, and their immediate supports are usually designed for a live load of 100 lb. per sq. ft.

Selection of Loadings. Bridges of the different classes are designed for loadings as follows:

Class of bridge	Loading
AA	*H*20
A	*H*15
B	*H*10

261. Application of Loadings. The selected loading is applied by whichever of the following methods produces the greatest stress in the member under consideration.

1. Each traffic-lane loading shall be considered as a unit and the number and position of the loaded lanes shall be such as will produce maximum stress.

2. The roadway shall be considered as loaded over its entire width with a load per foot of width equal to one-ninth of the load of one traffic lane.

Reduction in Load Intensity. If the loaded width of the roadway exceeds 18 ft., the specified loads may be reduced 1 per cent

for each foot of loaded roadway in excess of 18 ft. with a maximum reduction of 25 per cent, corresponding to a loaded roadway width of 43 ft. If the loads are lane loads, the loaded width of roadway is the aggregate width of the lanes considered; if the loads are distributed over the entire width of the roadway, the loaded width of the roadway is the full width of the roadway between curbs.

262. Impact. Live-load stresses due to truck loading and electric railway and freight car loading are increased to make allowance for vibration and the sudden application of the load. This increase is computed by the formula.

$$I = \frac{50}{l + 125}$$

in which I = impact fraction of the live-load stress.
l = loaded length in feet.

263. Distribution of Loads. When a concentrated load is placed on a reinforced concrete slab, the load is distributed over a larger area than the actual contact area. For example, if a concentrated load with a bearing area of 1 sq. ft. were placed on the slab of a through-girder bridge such as is shown in plan in Fig. 188*a*, it is reasonable to assume that, on account of the stiffness of the slab, the strips of slab at right angles to the girder and adjacent to the 1-ft. strip in direct contact with the load would assist in carrying the load. Similarly, with the beams of a T-beam bridge, such as is shown in plan in Fig. 188*b*, spaced fairly close, a concentrated load placed directly over one of the beams would not be carried entirely by that beam, for the concrete slab is sufficiently rigid to transfer part of the load to adjacent beams.

No distribution is assumed in the direction of the span of the member. The effect of any such distribution would be comparatively small, increasing from zero at the contact surface to a maximum at the bottom surface of the member.

The following recommendations for the distribution of loads are taken from the specifications of the American Association of State Highway Officials.

264. Distribution of Wheel Loads on Concrete Slabs. In the direction perpendicular to the span of the slab, the wheel load shall be considered as distributed uniformly over a width of slab

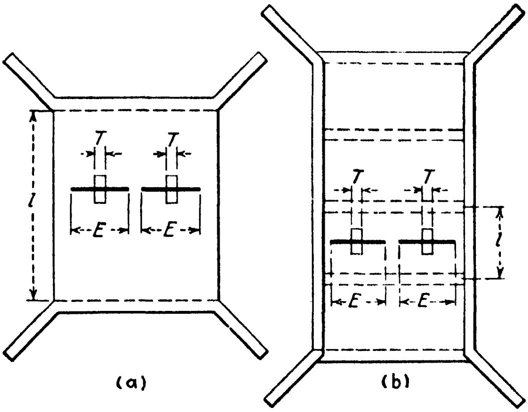


FIG. 187.

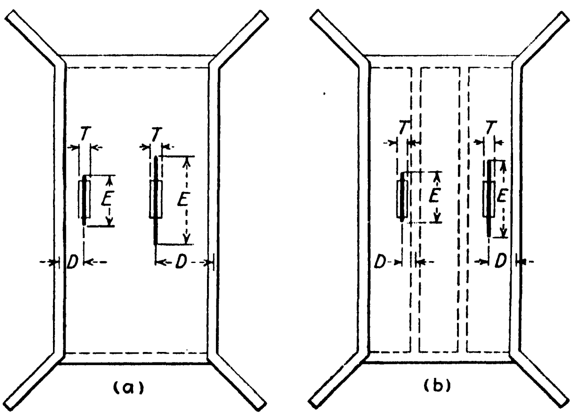


FIG. 188.

which is termed the "effective width." The following notation is used:

l = span of slab in feet.

T = width of wheel or tire in feet.

D = distance in feet from the center of the near support to the center of the wheel.

E = effective width in feet for one wheel.

Case I. *Main Reinforcement Parallel to Direction of Traffic.* This condition occurs in a slab bridge (Fig. 187*a*) and in a through-girder bridge with floor beams (Fig. 187*b*). For these,

$$E = 0.7l + T$$

in which E shall have a maximum value of 7.0 ft.

Where two wheels are so located on a slab that their effective widths overlap, the effective width for each wheel is $\frac{1}{2}(E + C)$, in which E is the value determined above for one wheel and C is the distance between centers of wheels. Where two adjacent lines of trucks must be considered E cannot be greater than $\frac{1}{2}(C + C')$, where C is the distance between wheels of the same truck and C' the distance between wheels of adjacent trucks.

Case II. *Main Reinforcement Perpendicular to Direction of Traffic.* This condition occurs in a through-girder bridge without floor beams (Fig. 188*a*) and in a T-beam or deck-girder bridge (Fig. 188*b*). For these,

$$E = 0.7(2D + T)$$

For this case the bending moment on a strip of slab 1 ft. in width is determined by placing the wheel loads in the position to produce the maximum moment, assuming no distribution, determining the effective width for each wheel, and assuming the load of each wheel on the 1-ft. strip to be the wheel load divided by its respective effective width.

265. Distribution of Wheel Loads to Longitudinal Beams. In the calculation of shears and end reactions, no lateral or longitudinal distribution of the wheel load is assumed. In calculating moments for bridges designed for one traffic lane with beams spaced not farther apart than 6 ft., the portion of one wheel load sustained by each interior beam is $\frac{S}{6}$, where S is the spacing of the beams in feet. If S is greater than 6 ft., one wheel load is placed directly over a beam and the additional load from the other wheel on the axle is obtained by considering the slab between beams as a simple beam and computing its reactions accordingly. For bridges designed for more than one traffic

lane each interior beam sustains $\frac{S}{4.5}$ wheel loads, provided S is less than 10 ft. If S is greater than 10 ft., the total beam load is obtained by considering the slab between beams as a simple beam.

The live load supported by the outside beams is the reaction of the truck wheels, placed as close to the curb as the clearance diagram (Fig. 183) will permit, assuming the slab between beams to act as a simple beam.

266. Distribution of Wheel Loads to Floor Beams. In the calculation of shears and end reactions no lateral or longitudinal distribution of the wheel load is assumed. In calculating moments, where the spacing of the beams is 6 ft. or less, each beam sustains $\frac{S}{6}$ of each wheel load, where S is the spacing of the beams in feet. Where the spacing of the beams is greater than 6 ft. the beam load is determined by treating the slab between beams as a simple beam and computing its reactions accordingly.

267. Abutments. Bridge abutments serve to transmit the load from the superstructure to the foundation and they also act as retaining walls to hold back the earth fill behind them.

Bridge Seats. The bridge seats of abutments which support the fixed ends of concrete highway bridges are quite frequently built as horizontal surfaces without parapets or backwalls, as shown in Fig. 189. The slab or the beams of the deck rest directly on the bridge seat; in deck-girder bridges, transverse diaphragm walls are constructed between the beams to hold back the earth above the bridge seat. These diaphragms are thin walls, 6 or 8 in. in thickness, with a nominal amount of reinforcement.

Figure 190 shows a modified form of the construction described above, in which a very low backwall is used, primarily for the purpose of counteracting the tendency of the abutment to move inward under the deck. The dowels b and c which are shown in the abutment and in the deck construction serve to tie the deck, the abutment, and the approach slab together.

Another type of bridge seat is shown in Fig. 191. This type is particularly adapted to the fixed ends of deck-girder bridges.

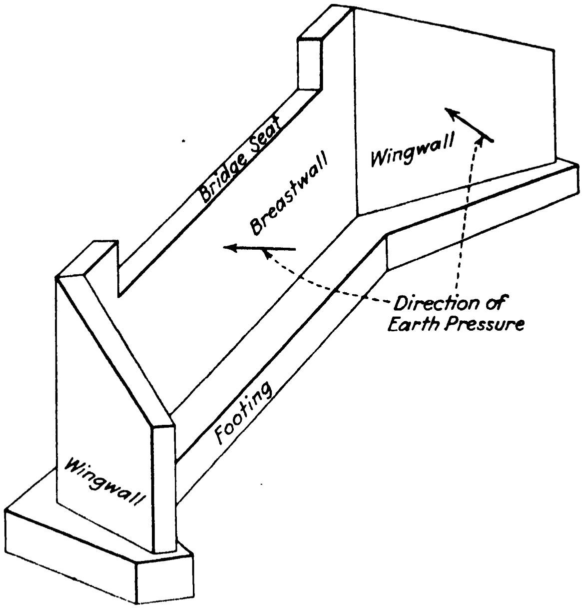


FIG. 189.—Details of abutment without backwall.

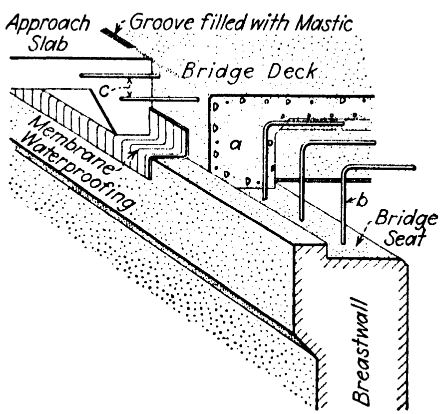


FIG. 190.—Details of bridge seat with low backwall.

The bridge seat is constructed with notches into which the bridge beams are built. Fig. 191 is a diagrammatic sketch, and does not show the final relative position of the deck and abutment;

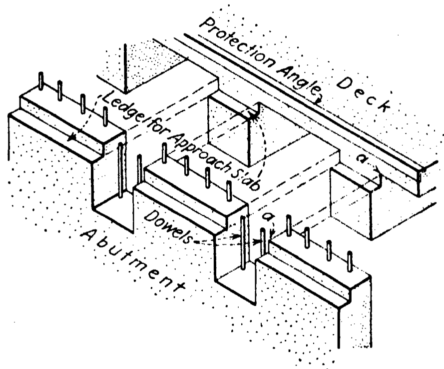


FIG. 191.—Details of bridge seat with pockets for beams.

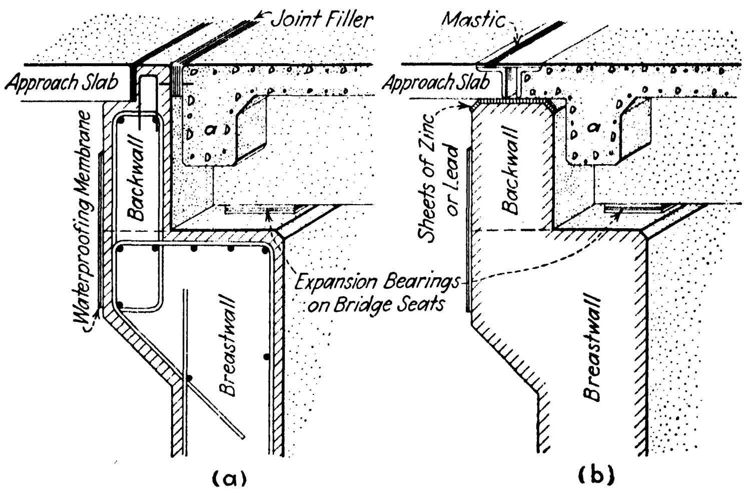


FIG. 192.—Details of bridge seats with backwalls.

the two parts of the figure actually fit together as shown by the dotted lines. Ledges are provided in the abutment and at the top of the deck beams to support the approach slab, as shown in the figure. Dowels are placed in the abutment in such a manner as to project up into the beams and deck slab.

A third type of bridge seat construction is shown in Fig. 192. This is the usual type with backwall, and is used at the expansion end of the bridge. Two arrangements of joints are shown in the figure. The backwall is extended upward to the roadway surface in *a*, while it stops at the bottom of the roadway slabs in *b*. Type *b* is usually preferred, in that only one joint is required in the roadway surface, as compared with two joints in type *a*.

Breastwalls. The two most common types of breastwalls are, the cantilever wall of reinforced concrete, and the type that acts as a vertical slab supported horizontally by the deck and by the foundation. Buttressed breastwalls are rarely used in short-span concrete bridges. Gravity walls of plain concrete are sometimes used, especially where the height of the abutment is not great.

The cantilever breastwall is generally used at the expansion end of the bridge. The type that is supported horizontally by the deck and the footing can be used only at the fixed end; in such cases it is always advisable to have at least a low parapet or backwall, as shown in Fig. 190, in order to obtain the direct horizontal resistance which is assumed to be furnished by the deck.

Wingwalls. Gravity or cantilever-type wingwalls may be used. These are topped with a simple coping, which is built parallel to and slightly above the earth fill.

It is advisable to construct vertical expansion joints at the junction of the wingwalls and breastwall, in order to avoid unsightly cracks which are apt to form along these planes. Sometimes, instead of an expansion joint, merely a vertical groove is constructed at the joint; any crack that may be developed will then be inconspicuous. The expansion joint or the groove should preferably be placed at the end of the bridge seat, as shown in Fig. 193.

268. Miscellaneous Construction Details. *Diaphragms.* In deck-girder bridges, thin transverse walls called diaphragms are constructed between the beams at the ends of the bridge, as shown at *a* in Figs. 190 and 192. These diaphragms serve two purposes: first, they furnish lateral support for the beams, and second, when no parapet or backwall is built above the bridge

seat they prevent the earth fill from spilling out on the bridge seat. At the fixed ends, the diaphragms may rest directly on the bridge seat, as shown in Fig. 190; at the expansion ends, the diaphragms should be kept clear of the bridge seat, as shown in Fig. 192, so as not to develop friction which would interfere with the freedom of longitudinal movement. The diaphragms are usually from 6 to 8 in. thick, with a nominal amount of reinforcement.

Intermediate diaphragms should be constructed between the beams of deck-girder bridges with spans greater than 40 ft.;

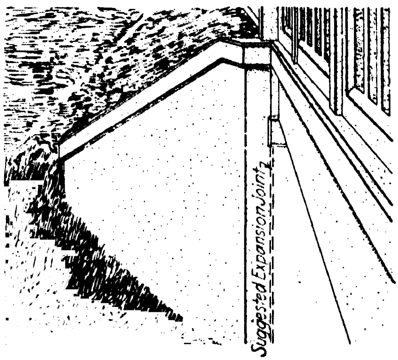


FIG. 193.—Vertical expansion joint in breastwall.

the intermediate diaphragms are placed at the center of the span, or at the third-points of the span.

Fixed Bearings. Fixed bearings can safely be used at both ends of spans up to 40 ft. Two such bearings are illustrated in Figs. 190 and 191. The dowels which serve to tie together the deck construction and the abutment are not designed to resist any appreciable amount of bending moment, and hence no joint rigidity can be assumed in the design of the deck or abutment.

Expansion Bearings. For spans greater than 40 ft., an expansion bearing should be provided at one end of the bridge. A suitable form of expansion bearing for spans up to 50 ft. is shown in Fig. 192. A pair of steel plates is placed under each girder; the lower plate is anchored to the concrete in the abutment, and the upper plate is fastened to the under side of the girder.

Frequently, a thin layer of graphited asbestos, zinc, or copper, is placed between the plates to reduce the friction.

For the shorter spans and light loadings, frequently the only expansion device is a layer of tar paper inserted between the bridge seat and the deck. Obviously this detail is not desirable where provision for expansion is of any importance, because of the questionable efficiency of the construction.

For spans greater than 50 ft., the deflection of the deck may rotate the ends of the girders so much that the bearing pressure may become concentrated on a narrow strip along the front edge of the bridge seat, and cause local damage if flat bearing plates

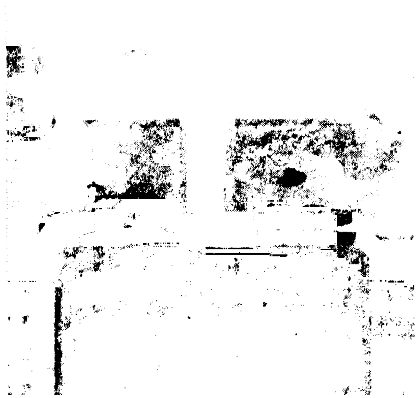


FIG. 194.—Expansion bearings.

are used. In order to counteract this tendency, one plane and one curved plate should be used, as shown in Fig. 194.

Deck Joints. The number of expansion joints in a bridge deck should be kept as low as possible. In single-span bridges of the type under discussion, one such joint at most is usually necessary; that joint is at one end of the bridge. A suitable detail for the joint is shown in Fig. 192*b*; in this detail an angle is anchored to the end of the deck slab, and a similar angle is anchored to the end of the approach slab. A tread plate is fastened, by means of rivets or tap screws, to the top of the angle at the end of the deck, and this plate bears on the angle

at the end of the approach slab. The tread plate does not cover the latter angle completely, sufficient provision for expansion of the deck slab being made. The groove between the end of the tread plate and the edge of the approach slab is filled with mastic, as shown in Fig. 192*b*, in order to make the joint water-tight. The mastic is soft enough to permit movement of the tread plate, but firm enough to resist displacement.

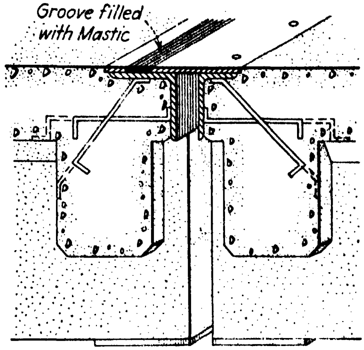


Fig. 195.—Details of expansion joint.

An enlarged detail of a similar joint is shown in Fig. 195; this detail applies specifically to the joint between the ends of two adjacent spans in a multi-span bridge.

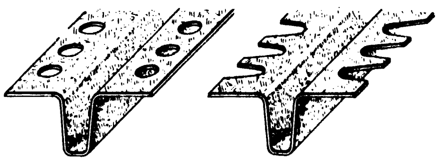


Fig. 196.—Metal strips for expansion joints.

Thin copper or zinc strips, bent in the form of the letter T as shown in Fig. 196, are sometimes placed across a joint, and embedded in the concrete on either side of the joint, in order to add to the water-tightness. The joint is filled with a mastic compound. A detail of this type is shown in Fig. 192*a*.

Joints at the fixed end of a span are made water-tight by simply filling a groove at the top of the joint with mastic, as shown in Fig. 190. The membrane water-proofing which is shown on top of the low parapet in this figure is furnished to insure water-tightness in the horizontal joint.

Drainage. Surface water on bridge decks should be disposed of as quickly and as directly as possible. This is accomplished

by crowning the roadway about 1/8 in. per ft., and pitching the gutters to drain into inlets. In single-span bridges which are built on a grade, no special provision for longitudinal drainage is ordinarily necessary. The water is carried by the transverse crown to the curbs or gutters and thence to the low end of the bridge, where it can easily be deflected away from the roadway.

If the bridge is horizontal, longitudinally, a longitudinal camber can be built into the bridge deck by elevating the middle

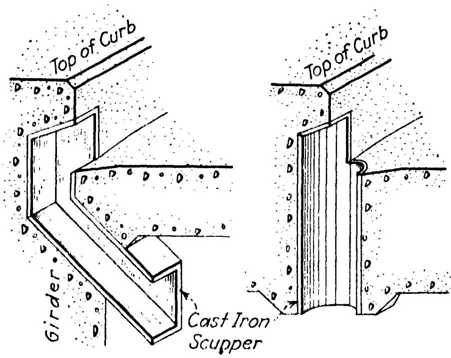


FIG. 197.—Typical scupper details.

portion of the span when the forms are erected. This camber will serve to carry the water in the gutters to either end of the span. A camber of about 1/10 in. per ft. is satisfactory for concrete decks.

When necessary, special drain inlets can be constructed by building cast-iron scuppers in the gutters. Two types of scuppers are shown in Fig. 197. The scuppers should be so designed as to prevent the drain water from touching the concrete. Drain inlets are generally necessary in multi-span bridges, because of the impracticability of carrying the surface water to the ends of the bridge.

269. Design of a Slab Bridge. A slab bridge similar to that shown in Fig. 179, but with a concrete railing, is to be designed according to the following data:

Clear span	15 ft.-0 in.
Clear width	20 ft.-0 in.
Live loading	*H*20
Wearing surface	30 lb. per sq. ft.

$f'_c = 2500$; $f_s = 18{,}000$; $n = 12$.

The effective span of the slab is assumed as 15 ft. + 1 ft. = 16 ft. and a total thickness of slab of 16 in. is selected for trial. The total dead load per square foot is then 230 lb. and the dead-load moment $\frac{1}{8}wl^2 = \frac{1}{8} \times 230 \times 16^2 \times 12 = 88{,}300$ in.-lb.

The load on each rear wheel is 16,000 lb. and the width of tire is 20 in., or 1.67 ft. Then

$$E = 0.7 \times 16 + 1.67 = 12.87 \text{ ft.}$$

but the maximum allowable value of E is 7 ft. Since, however, this bridge carries two traffic lanes E cannot be greater than the sum of one-half the distance between wheels on one axle and one-half the distance between adjacent wheels of two adjacent trucks[1] or

$$\tfrac{1}{2}(6 + 3) = 4.5 \text{ ft.}$$

The load on a unit width of slab is therefore $\dfrac{16{,}000}{4.5} = 3550$ lb. and the live-load moment $\dfrac{3550}{2} \times \dfrac{16}{2} \times 12 = 170{,}400$ in.-lb. The impact moment is $\dfrac{50}{16 + 125} \times 170{,}400 = 60{,}500$ in.-lb. and the total moment 319,200 in.-lb.

For the unit stresses specified, $K = 173.3$ (Table 6), and $d = \sqrt{\dfrac{319{,}200}{173.3 \times 12}} = 12.4$ in. Taking $d = 12.5$ in. and $t = 14.5$ in., the revised dead load is $181 + 30 = 211$ lb. per sq. ft., the dead-load moment 81,000 in.-lb., and the total moment 311,900 in.-lb. which requires a d of 12.29 in. so that the 12.5 in. which was taken above is adopted. Then

[1] A rather improbable loading but required by the provisions of Art. 264.

$$A_s = \frac{311{,}900}{18{,}000 \times 0.867 \times 12.5} = 1.60 \text{ sq. in.}$$

which is furnished by 7/8-in. round bars 4½ in. center to center.

While there is no negative moment at the ends of the span, in order to provide for a possible development of such a moment on account of the ends becoming somewhat fixed, one-third of the steel is bent upward 2 ft.-0 in. from the edge of the support

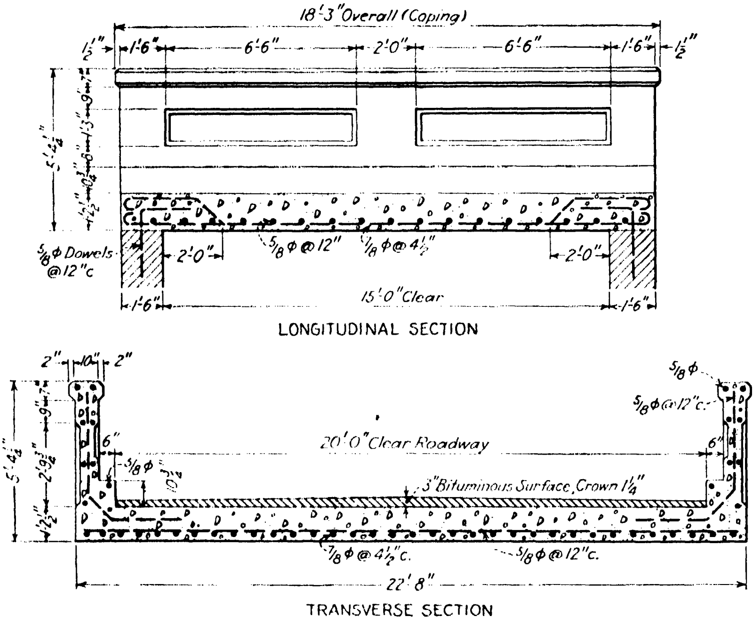


FIG. 198.—Details of slab bridge.

and carried along the top of the slab. All bars are anchored by means of hooks. In order to insure proper distribution of the concentrated loads and to provide for transverse shrinkage, transverse 5/8-in. round bars 12 in. center to center are placed directly on top of the longitudinal reinforcement. The railing is poured monolithically with the slab and tied to it with 5/8-in. round bars, 12 in. center to center. These bars extend horizontally into the slab and vertically into the railing. The crown of the roadway is obtained by varying the thickness of the bituminous wearing surface.

No expansion bearings are necessary for a span of this length and a fixed bearing is constructed at each end by placing vertical dowels in the abutment so that they will project up into the concrete deck slab. The bridge seat is to be coated with a bituminous mixture before the slab is poured. Details are shown in Fig. 198.

Design of a T-beam or Deck-girder Bridge

270. Data and Specifications. It is required to design a T-beam or deck-girder bridge for the following conditions:

Clear span........................	48 ft.-0 in.
Clear width.......................	20 ft.-0 in.
Live loading......................	$H20$

$f'_c = 2500$; $f_s = 18{,}000$; $f_v = 20{,}000$; $n = 12$. The bridge is to consist of three intermediate beams and two outside beams supporting a floor slab. The outside beams are to project at least 9 in. above the floor slab to act as a curb.

271. Slab. Since the outside beams will be rather heavy in comparison with the floor slab and since all beams and slabs will be poured monolithically and reinforced by diaphragms,

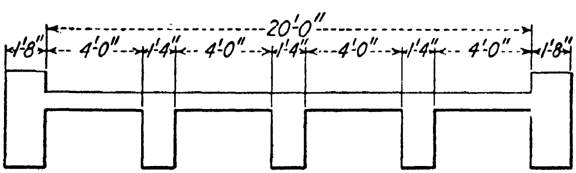


Fig. 199.

the slab may be considered as fully restrained by the outside beams and the span taken as the clear distance between supports. Assuming that the intermediate beams will be 16 in. in width, the clear span of the slab will be 4 ft.-0 in. (see Fig. 199). The outside beams will be assumed to require a width of 20 in. According to the specifications of the American Association of State Highway Officials, concrete floor slabs built continuously over supporting beams are designed for 80 per cent of the maximum bending moment of a simply supported slab of the same span.

Assuming a total thickness of slab (including a $\frac{3}{4}$-in. wearing surface) of $6\frac{1}{2}$ in. and allowing 15 lb. per sq. ft. for possible future protective covering, the total dead load per sq. ft. is 93 lb. and the dead-load moment $0.8\ (\frac{1}{8} \times 93 \times 4^2 \times 12) =$ 1790 in.-lb. For the center spans $D = \dfrac{5.33}{2} = 2.67$ ft., $T =$ 1.67 ft. (see Art. 263), and $E = 0.7(2 \times 2.67 + 1.67) = 4.9$ ft. For the outside spans $D = \dfrac{5.50}{2} = 2.75$ ft., $T = 1.67$ ft., and $E = 0.7(2 \times 2.75 + 1.67) = 5.02$ ft. Using the smaller of these two values, the concentrated load at the center of a 1-ft. strip of slab is $\dfrac{16{,}000}{4.9} = 3270$ lb. and the live-load moment is $0.8\left(\dfrac{3270}{2} \times \dfrac{4}{2} \times 12\right) = 31{,}400$ in.-lb. Since the loaded length is small the impact coefficient is practically 0.4; the impact moment is 12,500 in.-lb. and the total moment 45,700 in.-lb. Then

$$d = \sqrt{\frac{45{,}700}{12 \times 173.3}} = 4.68 \text{ in.}$$

Taking $d = 4\frac{3}{4}$ in. with 1 in. of insulation below the center of the bars, the total thickness is $6\frac{1}{2}$ in. as assumed.

$$A_s = \frac{45{,}700}{18{,}000 \times 0.867 \times 4.75} = 0.62 \text{ sq. in.}$$

which is furnished by $\frac{5}{8}$-in. round bars $5\frac{1}{2}$ in. center to center. Each alternate bar is bent up over the supports, and additional straight bars 11 in. center to center are placed in the top of the slab continuous from outside beam to outside beam to complete the negative-moment reinforcement. All bars are anchored to the exterior beams by a 90-degree bend 6 in. in length at each end of the bars. Temperature and distribution stresses in the direction of the span are provided for by placing four $\frac{1}{2}$-in. round bars in the top and bottom of each slab panel, parallel to the beams, at about 12-in. centers. Complete details of the slab are shown in Fig. 204.

272. Intermediate Beams. The intermediate beams are T-beams with a flange width equal to the distance center to center of beams. The required dimensions are governed by either the maximum moment or maximum shear. The bridge seats will be assumed 2 ft.-0 in. in width and the effective span length center to center of bearings taken as 50 ft.-0 in.

Dead-load Moments. The weight from the slab per foot of beam is $93 \times 5.33 = 496$ lb. The cross-section of the beam below the slab is assumed as 16 in. $\times$ 30 in., which adds an additional weight of 500 lb. per ft. making the total weight 996 lb. per ft. Then the dead-load moment at the center of the span is

$$M_D = \tfrac{1}{8} \times 996 \times \overline{50}^2 \times 12 = 3{,}735{,}000 \text{ in.-lb.}$$

In order to determine the points at which some of the horizontal steel may be bent up, it is necessary to compute the moment at some sections between the point of maximum moment and the

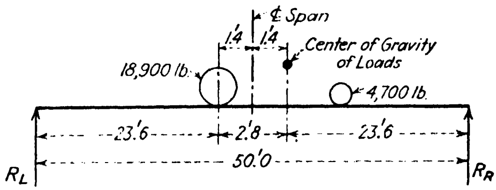


FIG. 200.

support. At 10 ft. from the support, the dead-load moment is 2,390,000 in.-lb. and at 20 ft. 3,586,000 in.-lb.

Live-load Moments. The absolute maximum live-load moment will occur with the 20-ton truck on the bridge in the position[1] shown in Fig. 200. With the distribution of loads as specified in Art. 265 each intermediate beam must support $\dfrac{5.33}{4.5} = 1.184$ wheel loads per wheel. Therefore, the load from the rear wheel is $1.184 \times 16{,}000 = 18{,}900$ lb. and from the front wheel $1.184 \times 4000 = 4700$ lb.

$$R_L = [(18{,}900 + 4700)23.6] \div 50 = 11{,}140 \text{ lb.}$$

[1] For a discussion of the position of moving loads for absolute maximum moment see the authors' "Stresses in Simple Structures," p. 174.

and

$$M_L = 11{,}140 \times 23.6 \times 12 = 3{,}155{,}000 \text{ in.-lb.}$$

At 10 ft. from the left support, the maximum live-load moment occurs with the rear wheel at that point and the front wheel 24 ft. from the left support. With this position of the loads:

$$R_L = (18{,}900 \times 40 + 4700 \times 26) \div 50 = 17{,}560 \text{ lb.}$$

and

$$M_{10} = 17{,}560 \times 10 \times 12 = 2{,}107{,}000 \text{ in.-lb.}$$

Similarly,

$$M_{20} = 3{,}082{,}000 \text{ in.-lb.}$$

Impact Moments. In computing impact moments it is usual to consider the whole span as the loaded length. The impact coefficient is therefore $\dfrac{50}{50 + 125} = 0.285$, and the impact moments are as follows:

$$\begin{aligned} M_{\text{max.}} &= 899{,}000 \text{ in.-lb.} \\ M_{20} &= 879{,}000 \text{ in.-lb.} \\ M_{10} &= 601{,}000 \text{ in.-lb.} \end{aligned}$$

Maximum Total Moments. The sum of the maximum dead-load, live-load, and impact moments is 7,749,000 in.-lb. The total moments at the 10-ft. and 20-ft. points are 5,098,000 in.-lb. and 7,547,000 in.-lb., respectively.

Dead-load Shears. The maximum dead-load shear at the end of the beam is $996 \times 25 = 24{,}900$ lb. At 10 ft. from the support the shear is 14,900 lb. and at 20 ft. from the support 5000 lb.

Live-load Shears. The maximum shear in the center beam occurs with the truck train on the span in the position shown in Fig. 201*a*. Owing to the fact that two trucks proceeding in the same direction may cross the bridge simultaneously, the center beam must be able to sustain considerably more than one wheel load. The design arrangement of the truck axles is shown in Fig. 202. With the loads so placed, the center beam sustains

$\dfrac{2.83 + 4.83}{5.33} = 1.44$ times the value of each wheel load as shown in Fig. 201*b* and the maximum live-load shear is 29,300 lb. The shears at the 10-ft., 20-ft., and 25-ft. points are 21,400 lb., 15,700 lb., and 12,900 lb., respectively.

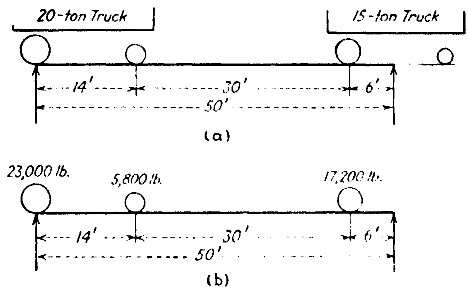


FIG. 201.

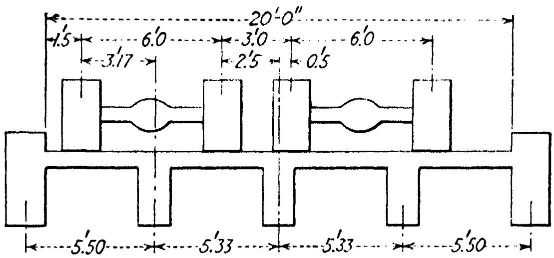


FIG. 202.

Impact Shears. Using loaded lengths of 50 ft., 40 ft., 30 ft. and 25 ft., respectively, the impact shears are as follows:

End shear	8400 lb.
10-ft. section	6500 lb.
20-ft. section	5100 lb.
Mid-span	4300 lb.

Total Shears. The total shears are as follows:

End shear	62,600 lb.
10-ft. section	42,800 lb.
20-ft. section	25,800 lb.
Mid-span	17,200 lb.

Determination of Cross-section and Steel Area. The area $b'd$ required to sustain the maximum shear is $\dfrac{62{,}600}{\frac{7}{8} \times 150} = 477$ sq. in. With $b' = 16$ in., the required $d = 29.8$ in. Practical considerations indicate that a slightly greater depth should be used. With d taken as 32 in. and the main reinforcement placed in two rows $2\frac{1}{2}$ in. center to center, with $2\frac{1}{2}$-in. insulation for the lower row, the depth of beam below the slab is $32 + 3\frac{3}{4} - 5\frac{3}{4} = 30$ in., as assumed.

$$A_s = \frac{7{,}749{,}000}{18{,}000\left(32 - \dfrac{5.75}{2}\right)} = 14.8 \text{ sq. in.}$$

which is furnished by ten $1\frac{1}{4}$-in. square bars.

$$\Sigma_0 = \frac{62{,}600}{125 \times \frac{7}{8} \times 32} = 17.9 \text{ in.}$$

This is furnished by 4 of the $1\frac{1}{4}$-in. bars allowing the remaining 6 to be bent up to assist in resisting the diagonal tension stresses.

The actual maximum stress in the concrete is computed as follows:

$$p = \frac{15.62}{63 \times 32} = 0.0078 \qquad np = 0.094 \qquad \frac{t}{d} = \frac{5.75}{32} = 0.18$$

From Diagram 2, $k = 0.40$ and $j = 0.92$.
From equation (24),

$$f_c = \frac{7{,}749{,}000}{\left(1 - \dfrac{5\frac{3}{4}}{2 \times 0.40 \times 32}\right)63 \times 5\frac{3}{4} \times 0.92 \times 32} = 940 \text{ p.s.i.}$$

Web Reinforcement. The portion of the beam over which web reinforcement is required is determined by computing the unit shears at various points on the beam. This is shown graphically in Fig. 203. The concrete resists a unit shear of 50 p.s.i. and the remainder must be cared for by bent up bars or stirrups. The upper row of bars will be bent up at an angle of

45 degrees; a single bar will be bent first, then two successive pairs. In the moment diagram of Fig. 203c the points *c*, *d*, and *e* represent the points at which one, two additional, and two more additional bars may be bent up.

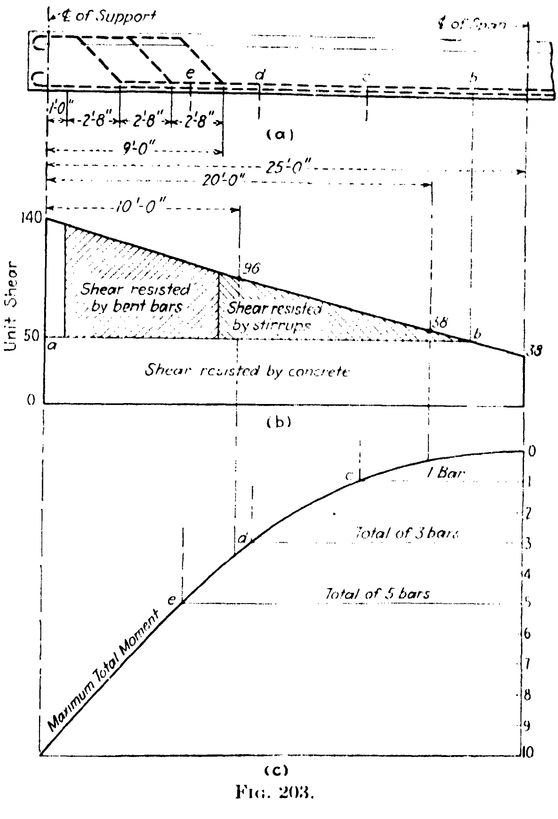


FIG. 203.

The maximum distance over which inclined bars may be considered effective in resisting diagonal tension is equal to 32 in. (see Art. 79). Therefore the pair nearest the support will be bent up 2 ft.-8 in. from the support and the others as shown in Fig. 203*a*.

At a distance of 9 ft.-0 in. from the center of the support the unit shear is 100 p.s.i. and the required spacing of ⅝-in. round

U-stirrups is

$$s = \frac{20,000 \times 2 \times 0.307}{(100 - 50)16} = 15.35 \text{ in.}$$

The maximum allowable spacing of stirrups is $\frac{1}{2} \times 32 = 16$ in. (see Art. 79).

The final arrangement of stirrups will be as follows: With the first stirrup 4 in. from the edge of the support, 6 at 16 in., 4 at 15 in. and 8 at 16 in. Theoretically, stirrups are not needed over the first 9 ft.-0 in. from the support where the inclined bars resist the diagonal tension nor from the point b to the center of the span, but the arrangement here given assures the proper tying of the flange to the web and gives an additional safeguard against diagonal tension cracks.

273. Exterior Beams. Since the exterior beams are to project a minimum of 9 in. above the slab to form curbs for the protection of the railings, they must be designed as rectangular beams.

Dead-load Moments. The weight from the slab per ft. of beam is $93 \times 2 = 186$ lb. The weight of the railing, details of which are shown in Fig. 204, is assumed as 300 lb. per ft. The cross-section of the beam is assumed as 20×51 in. which weighs 1062 lb. per ft. The total dead load is 1548 lb. per ft. and the maximum dead-load moment at the center of the span is

$$M_D = \frac{1}{8} \times 1548 \times 50^2 \times 12 = 5{,}805{,}000 \text{ in.-lb.}$$

Live-load and Impact Moments. A portion of each wheel load which rests on the exterior slab panel is supported by the exterior beam. That portion is obtained by placing the wheels as close to the curb as the clearance diagram will permit and treating the exterior slab panel as a simple beam. The position is shown in Fig. 202 and the proportion of the load is $\frac{3.17}{5.50} = 0.576$. The longitudinal position of the load which will produce the absolute maximum bending moment is the same as for the intermediate beams, as shown in Fig. 200. The moments are directly propor-

tional, and the absolute maximum moment in the exterior beam is

$$M_L = \frac{0.576}{1.184} \times 3{,}155{,}000 = 1{,}535{,}000 \text{ in.-lb.}$$

The impact moment is $0.285 \times 1{,}535{,}000 = 437{,}000$ in.-lb.

Total Maximum Moment. The total maximum moment is 7,777,000 in.-lb.

Shears. The maximum dead-load shear at the end of the beam is

$$V_D = 1548 \times {}^{50}\!/_{2} = 38{,}700 \text{ lb.}$$

and the shear at the 10-ft. section is 23,200 lb.

The maximum live-load shear is proportional to the maximum live-load shear in the intermediate beams and is

$$V_L = \frac{0.576}{1.44} \times 29{,}300 = 13{,}600 \text{ lb.}$$

Similarly the shear at the 10-ft. section is 8600 lb.

The impact shears are 3900 lb. and 2600 lb., respectively, and the total shears are:

End shear	56,200 lb.
10-ft. section	34,400 lb.

Determination of Cross-section and Steel Area. For the assumed width of 20 in. the required depth is

$$d = \sqrt{\frac{7{,}777{,}000}{173.3 \times 20}} = 47.3 \text{ in.}$$

Taking $d = 47\frac{1}{2}$ in. and assuming the same arrangement of steel and insulation as in the intermediate beam, the total height of the cross-section is $47\frac{1}{2} + 2\frac{1}{4} + 1\frac{1}{4} = 51$ in., as assumed.

$$A_s = \frac{7{,}777{,}000}{18{,}000 \times 0.867 \times 47.5} = 10.5 \text{ sq. in.}$$

which is furnished by seven $1\frac{1}{4}$-in. square bars. Four bars are placed in the lower row and three in the upper row. Three bars are sufficient to develop the bond stress, so that the remain-

ing four bars can be bent up to assist in resisting diagonal tension if desired.

Web Reinforcement. At the support the unit shear is $\dfrac{56{,}200}{20 \times \frac{7}{8} \times 47.5} = 68$ p.s.i. and at the 10 ft. section 42 p.s.i., so that web reinforcement is required for a distance of only $\dfrac{68 - 50}{68 - 42} \times 10 = 6.9$ ft. from the support. Bending up two bars 36 in. from the edge of the support and one bar 36 in. farther toward the center, the diagonal tension is theoretically provided for (1 ft.-0 in. + 3 ft.-0 in. + 3 ft.-0 in. = 7 ft.-0 in.), but in order to reinforce the beam against shrinkage and to tie it together, ½-in. round U-stirrups will be placed about 2 ft.-0 in. center to center throughout its length. A diagram similar to Fig. 203 could be constructed for this beam, but it is not necessary, for it is evident from a study of that figure that ample steel is provided in the bottom of the beam to care for the positive moment.

274. Miscellaneous Details. *Diaphragms.* A transverse diaphragm *a*, Figs. 190 and 192, will be built between the beams at either end of the bridge. The chief function of these diaphragms is to furnish lateral support to the beams; with some abutment details, the diaphragms also serve to prevent the backfill from spilling out on the bridge seats. A similar diaphragm will be built between the beams at mid-span. Such intermediate diaphragms are recommended for all spans in excess of 40 ft.-0 in.

Fixed Bearing. A fixed bearing similar to Fig. 190 is provided at one end. Vertical dowels *b* are placed in the breastwall of the abutment and are bent so as to project into the longitudinal beams or into the deck slab; horizontal dowels *c* are embedded in the deck and approach slabs. The diaphragm *a* may rest directly on the bridge seat.

Expansion Bearing. An expansion bearing is provided at one end of the span. Its details are similar to Fig. 192*b*. The diaphragm *a* is flush with the bottoms of the intermediate beams and not in contact with the bridge seat.

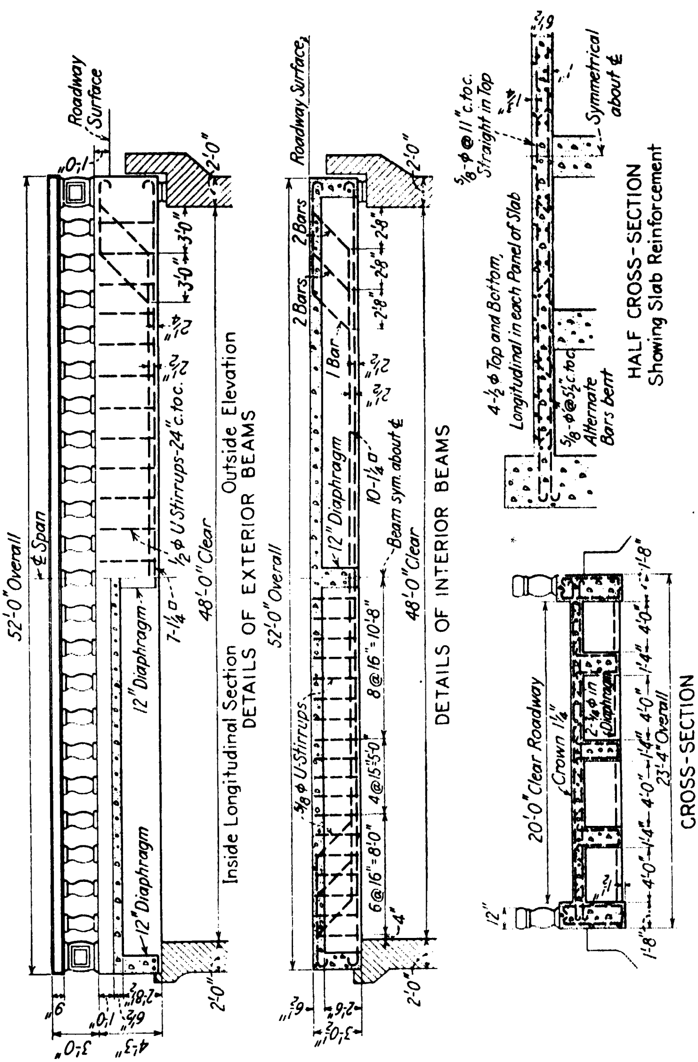


FIG. 204.—Details of T-beam bridge.

Water-proofing and Drainage. All joints are to be filled with a mastic compound to prevent water from seeping through the joints.

Removal of surface water will be accomplished by crowning the roadway 1¼ in. and pitching the gutters toward the ends by providing a camber of 2½ in. at the center. This latter is accomplished by raising the forms at the center. Besides facilitating drainage, this camber serves to prevent the appearance of sag that would be evident if the girders were at the same level throughout the span. Full details are shown in Fig. 204.

Design of a Through-girder Bridge

275. Data and Specifications. It is required to design a through-girder bridge for the following conditions:

Clear span	48 ft.-0 in.
Clear width	20 ft.-0 in.
Live loading	*H*20

$f'_c = 2500$; $f_s = 18,000$; $f_v = 20,000$; $n = 12$.

The bridge is to consist of a slab supported between two girders.[1]

276. Slab. Although the slab is poured so as to act monolithically with the girders, the girders themselves are unrestrained, and since the slab is in junction with the girders near the bottom of the latter, a torsional moment of considerable amount is brought to the girders, so that they may not be considered to fully restrain the ends of the slab. Tests have shown a point of inflection in the slab about 1 ft. from the edge of the girders, but many designers prefer to consider the slab as simply supported and that practice will be followed in the accompanying design. Slabs may be designed for tensile steel only or for both tensile and compressive steel. The latter design allows a thinner slab

[1] With this width of roadway it might be more economical to use cross-beams, in order materially to reduce the thickness of the floor slab. For greater widths there is no doubt of the economy of the use of such beams.

but requires more steel. Both types are used. The present design will consider tensile steel only.

Assuming an effective depth of slab of 14 in. with 1½ in. insulation, ¾ in. for monolithic wearing surface, and 15 lb. for future protective covering, the dead load per sq. ft. is 219 lb. The span of the slab is taken as the clear span plus the structural thickness or $20 + \frac{15.5}{12} = 21.3$ ft. The maximum dead-load moment per foot of slab is

$$M_D = \tfrac{1}{8} \times 219 \times (21.3)^2 \times 12 = 149{,}200 \text{ in.-lb.}$$

Since the bridge has sufficient width to allow for two lines of trucks traveling abreast, according to Art. 264 the rear wheels of two trucks should be placed in the position for absolute maximum moment, and the effective width E, for each wheel, determined. This will give four unequal loads and a new position of these loads for absolute maximum moment must be determined. Since the values of D will now be somewhat different, E for each load has a different value from that computed above and the whole cycle must be repeated.

Such an extremely theoretical analysis is unnecessary and it is sufficiently accurate to center the two trucks on the bridge, obtain values of D and E and compute the moment under the inner wheel of either truck.

Assuming the width of the main girder as 26 in. or 2.17 ft., for the inner wheels (see Fig. 205), $D = 9.58$ ft. and

$$E = 0.7(2 \times 9.58 + 1.67) = 14.58 \text{ ft.}$$

Similarly, for the outer wheels,

$$E = 0.7(2 \times 3.58 + 1.67) = 6.18 \text{ ft.}$$

The actual loads for which the unit slab is to be designed are 1100 and 2590 lb., respectively, and

$$M_L = [(1100 + 2590)8.5 - 2590 \times 6]12 = 189{,}900 \text{ in.-lb.}$$

Since the loaded length is zero, the impact coefficient is 0.40 and

$$M_I = 0.40 \times 189{,}900 = 76{,}000 \text{ in.-lb.}$$

The total maximum moment is

$$M = 149{,}200 + 189{,}900 + 76{,}000 = 415{,}100 \text{ in.-lb.}$$

$$d = \sqrt{\frac{415{,}100}{12 \times 173.3}} = 14.12 \text{ in.}$$

Adopting an effective depth of $14\frac{1}{4}$ in. the total slab thickness is $16\frac{1}{2}$ in., which is sufficiently close to the assumed thickness to

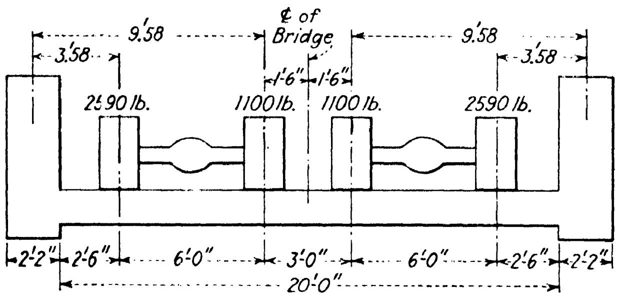


FIG. 205.

require no revision. The required steel area is

$$A_s = \frac{415{,}100}{18{,}000 \times 0.867 \times 14.25} = 1.87 \text{ sq. in.}$$

which is furnished by 1-in. round bars at 5 in. on centers.

Reinforcement for temperature and shrinkage in the direction of the span will be provided by placing $\frac{5}{8}$-in. round bars at 12-in. centers, directly on top of the main reinforcement.

Since the slab and girders are to be poured monolithically some negative moment will be developed at their junction. Therefore one-third of the main reinforcing bars will be bent up near the supports and continued near the top of the slab to within about 2 in. of the outside face of each girder, where they will be anchored by means of hooks. In addition, 1-in. round bars at 15-in. centers will be placed in the top of the slab near the girders and bent up into the girders to within about 3 in. of the top of the girders as shown in Fig. 206.

277. Main Girders. The weight from the slab per foot of girder is $221 \times 10 = 2210$ lb. The cross-section of the girder is assumed as 26×76 in. overall, which would give a weight of 2060 lb. per ft., but since the sides are panelled in the architectural treatment, the weight will be assumed as 1900 lb. per ft. The dead-load moment is therefore

$$M_D = \tfrac{1}{8} \times 4110 \times 50^2 \times 12 = 15{,}410{,}000 \text{ in.-lb.}$$

In order to obtain the maximum live-load moment, one line of trucks is placed as close to the face of one girder as the clearance diagram will permit. Considering the left-hand girder of Fig. 205, all of the wheels are placed 1 ft. farther to the left. Then treating the slab as a simple beam and taking moments about the right-hand girder, the proportionate part of one wheel load P, carried by the left-hand girder is

$$P(4.58 + 10.58 + 13.58 + 19.58) \div 22.17 = 2.180P$$

The position of the loads longitudinally is the same as for the deck-girder bridge previously designed (see Fig. 200) and the moments are proportional. Therefore,

$$M_L = \frac{2.180}{1.184} \times 3{,}155{,}000 = 5{,}828{,}000 \text{ in.-lb.}$$

The impact moment is

$$M_I = 0.285 \times 5{,}828{,}000 = 1{,}667{,}000 \text{ in.-lb.}$$

and the total maximum moment is

$$M = 15{,}410{,}000 + 5{,}828{,}000 + 1{,}667{,}000 = 22{,}905{,}000 \text{ in.-lb.}$$

$$d = \sqrt{\frac{22{,}905{,}000}{26 \times 173.3}} = 71.2 \text{ in.}$$

Adopting an effective depth of $71\tfrac{1}{2}$ in., assuming two rows of bars 3 in. center to center with the center of the lower row 3 in. from the bottom of girder, the total depth is $71\tfrac{1}{2} + 1\tfrac{1}{2} + 3 = 76$ in. as assumed.

$$A_s = \frac{22{,}905{,}000}{18{,}000 \times 0.867 \times 71.5} = 20.4 \text{ in.}$$

which is furnished by thirteen 1¼-in. square bars, seven in the lower row and six in the upper row. All of the bars will be continued at the bottom from one end of the girder to the other.

The shears are computed in a manner similar to that used for the intermediate beams of the deck-girder bridge (see Art. 272). The values are as follows:

	Total shear	Unit shear
Center of support	159,700 lb.	121 p.s.i.
5-ft. section	130,500 lb.	100 p.s.i.
10-ft. section	103,800 lb.	79 p.s.i.
15-ft. section	78,000 lb.	60 p.s.i.

In computing the above unit shears the minimum width, which is $26 - (2 \times 2\frac{1}{2}) = 21$ in., was used (see Fig. 206).

Using ⅝-in. round U-stirrups, $\dfrac{f_vA_v}{b} = \dfrac{20{,}000 \times 2 \times 0.307}{21} =$ 585, and the required spacings are as follows:

Center of support	$585 \div (121 - 50)$	$= 8.2$ in.
5-ft. section	$585 \div (100 - 50)$	$= 11.7$ in.
10-ft. section	$585 \div (79 - 50)$	$= 20.2$ in.
15-ft. section	$585 \div (60 - 50)$	$= 58.5$ in.

The maximum allowable spacing is $\frac{1}{2} \times 71.5 = 36$ in.

The first stirrup will be placed at the center of the support and the spacings thereafter will be as follows: 7 at 8 in., 5 at 12 in., 3 at 20 in., and 4 at 31 in.

278. Miscellaneous Details. The details of the architectural treatment of the girders are shown in Fig. 206. The floor slab is framed into the girders 5½ in. above the bottom of the girders in order to avoid the necessity of bending the slab reinforcement to clear the bars in the girders.

A fixed bearing will be constructed at one end by placing ¾-in. dowels in the breastwall of the abutment, at 12-in. centers, and bending these dowels into the slab and girders as shown in Fig. 206. Horizontal dowels will also be used. The slab will be

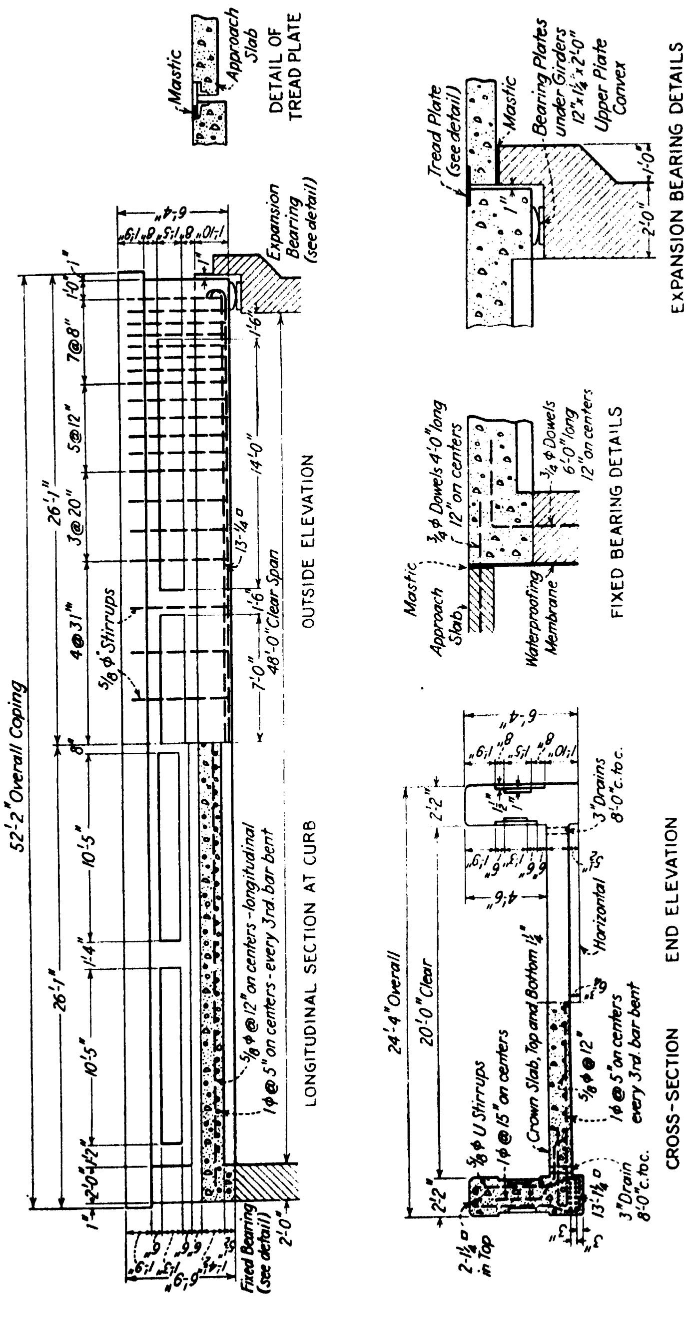


FIG. 206.—Details of through-girder bridge.

deepened so that the bottom of the slab will rest on the bridge seat and serve as a backwall for the abutment.

An expansion bearing at the other end of the bridge is similar to the type described in Art. 268 and illustrated in Fig. 192. The details are shown in Fig. 206. The required area of the bearing plates is $159{,}700 \div 600 = 266$ sq. in.

Transverse drainage is provided by a 1¼-in. crown and longitudinal drainage by sloping the gutters toward 3-in. drains which are placed close to each girder at 8-ft. centers.

Abutments are designed according to the principles outlined in Chap. IX. Some abutment and bridge details are given in Figs. 207 to 212.

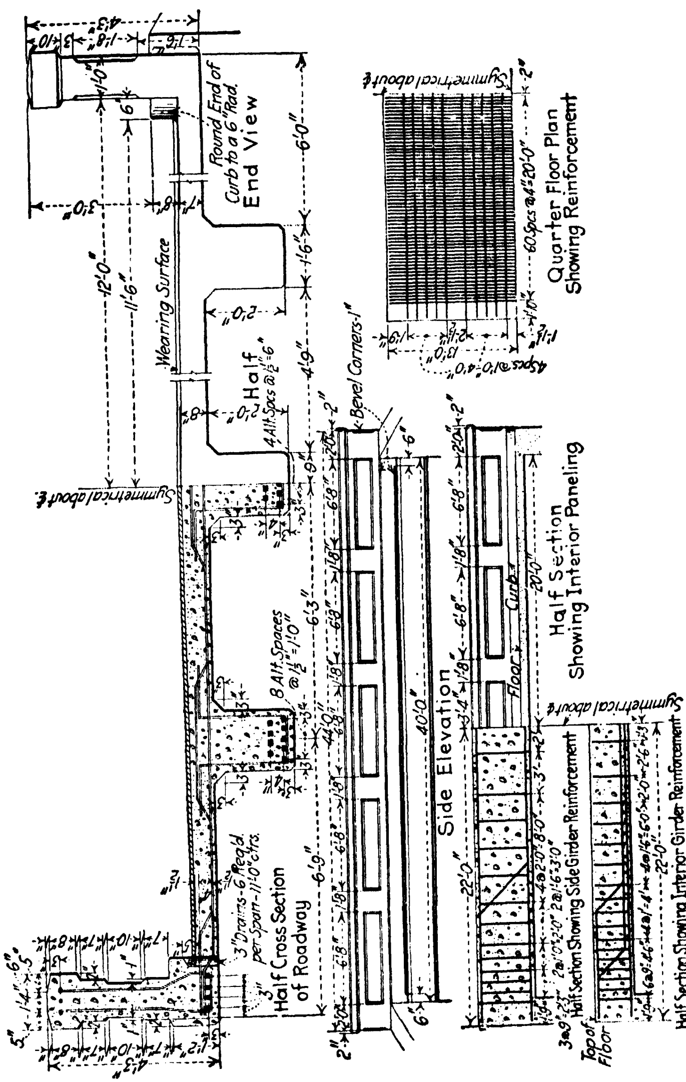


FIG. 207.—Deck reinforced concrete girder bridge, Wisconsin Highway Commission.

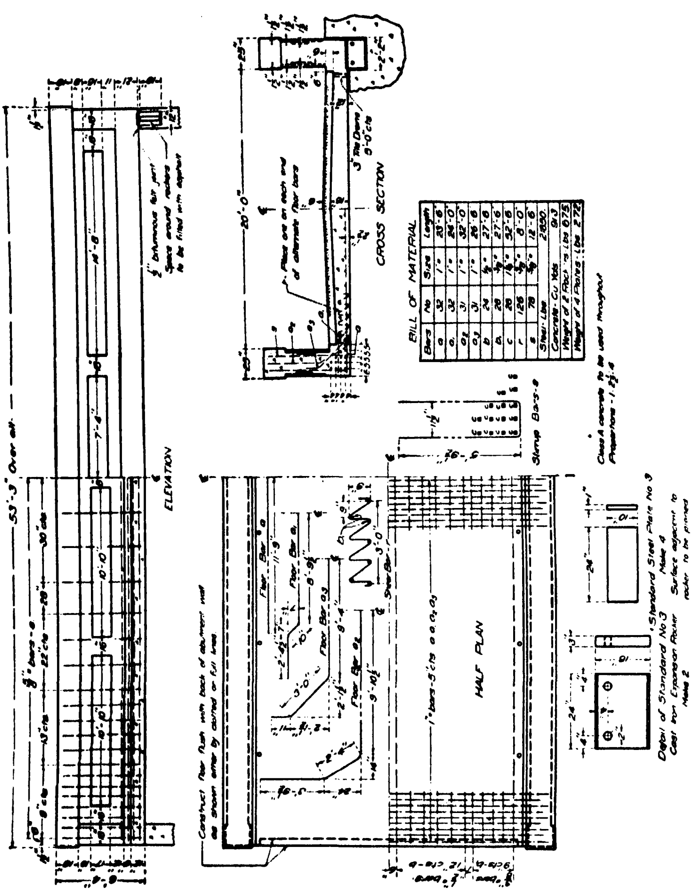


FIG. 208.—Through reinforced concrete girder bridge, Illinois State Highway Department.

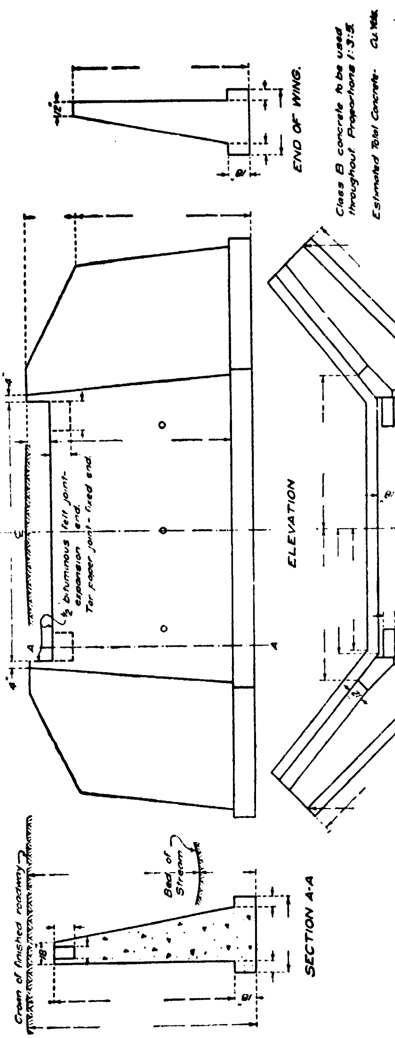


FIG. 209.—Plain concrete abutment for girder bridge, Illinois State Highway Department.

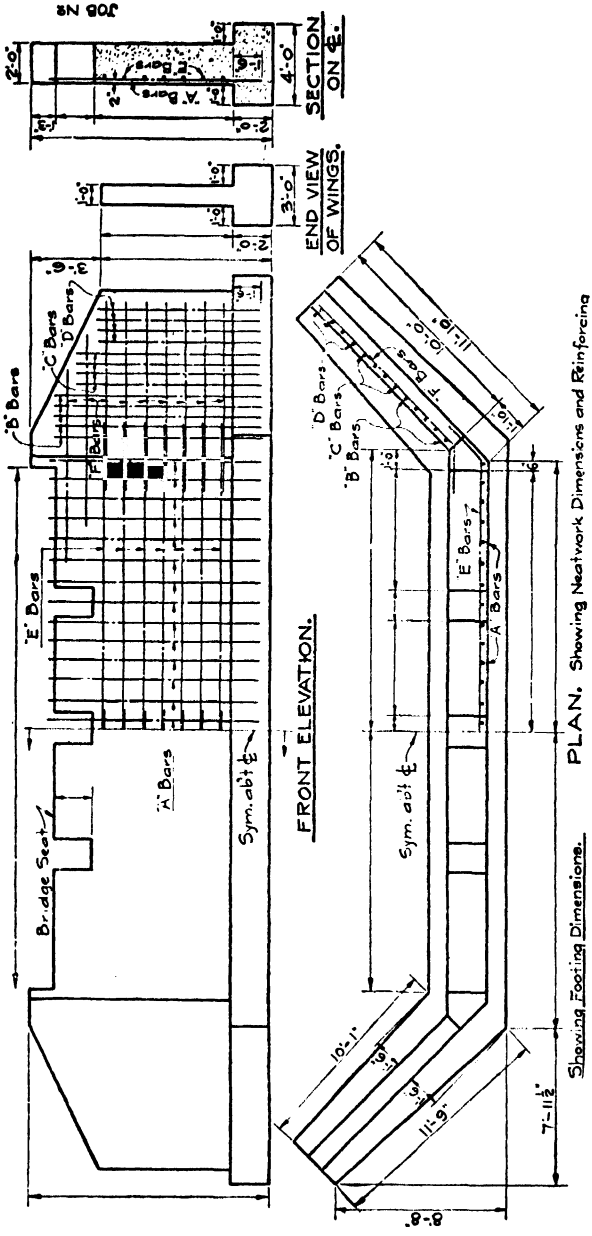


FIG. 210.—Reinforced concrete abutment, Wisconsin Highway Commission.

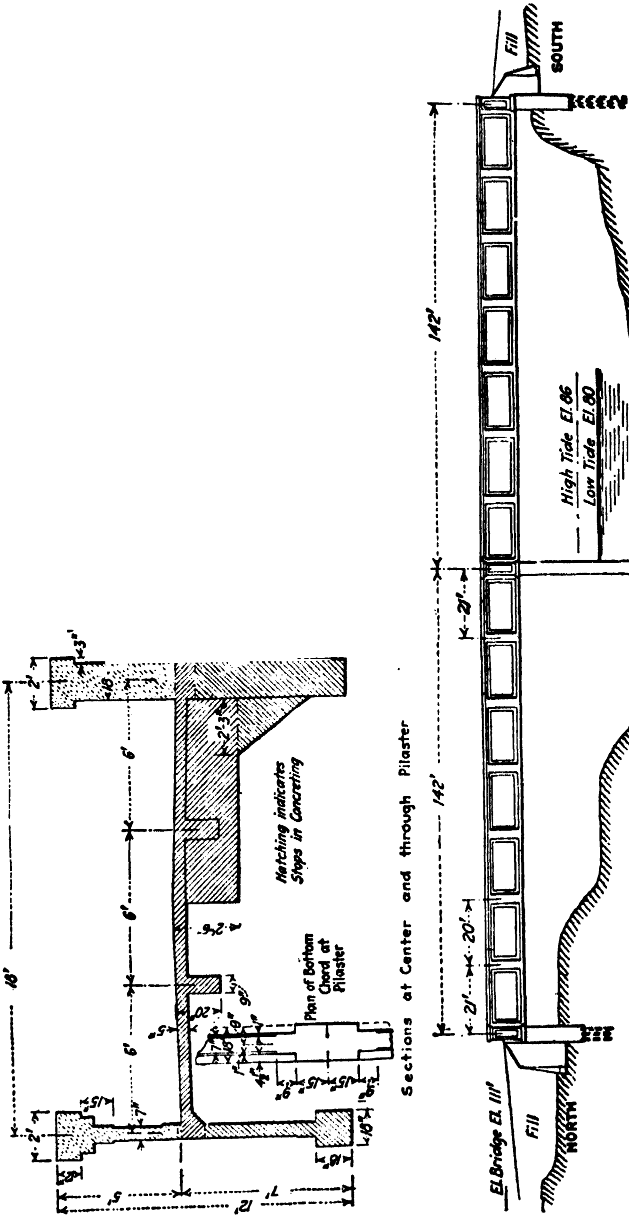


FIG. 211.—Salt River concrete bridge, Humboldt County, California.

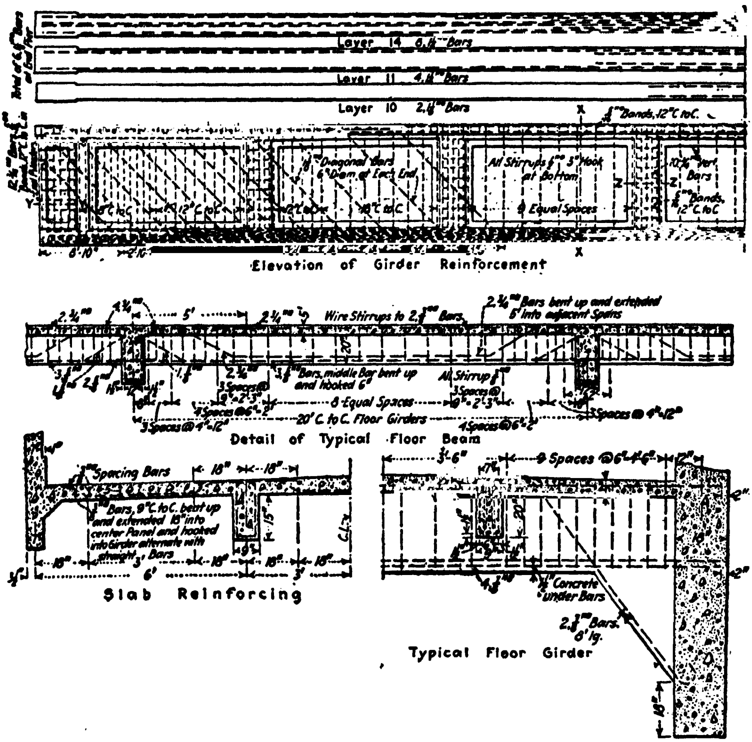


FIG. 212.—Details of Salt River bridge.

APPENDIX A

STANDARD NOTATION

The principal symbols used in the previous discussions have been collected here for the convenience of reference.

a = side of column parallel to principal beam, or overall depth of any member.

α = angle between inclined web bars and longitudinal bars.

A = area of effective cross-section of column = $A_c + A_s$.

A_c = net area of concrete in this section.

A_g = overall or gross area of concrete section.

A_s = effective cross-sectional area of metal reinforcement in tension in beams, and of longitudinal bars in columns.

A'_s = effective cross-sectional area of metal reinforcement in compression in beams.

A_v = total area of web reinforcement in tension within a distance of $s(s_1, s_2, s_3$, etc.) or the total area of all bars bent up in any one plane.

b = width of rectangular beam or width of flange of T-beam.

b' = width of stem of T-beam.

c = projection of footing from face of column; distance from gravity axis to extreme fiber in compression.

C = total compressive stress in concrete.

C' = total compressive stress in reinforcement.

d = depth from compression surface of beam or slab to center of longitudinal tension reinforcement; diameter of circular column.

d' = depth from compression surface of beam or slab to center of compression reinforcement.

E_c = modulus of elasticity of concrete in compression.

E_s = modulus of elasticity of steel in tension = 30,000,000 p.s.i.

f_c = compressive unit working stress in extreme fiber of concrete.

f'_c = ultimate compressive strength of concrete at age of 28 days, based on tests of 6- $\times$ 12-in. or 8- $\times$ 16-in. cylinders made and tested in accordance with the Standard Methods of Making and Storing Specimens of Concrete in the Field and Standard Methods of Making Compression Tests of Concrete of the American Society for Testing Materials.

f_s = tensile unit working stress in longitudinal reinforcement in beams, and compressive unit working stress in longitudinal bars in columns.

f'_s = compressive unit stress in longitudinal reinforcement in beams.
f_v = tensile unit stress in web reinforcement.
h = unsupported length of column.
I = moment of inertia of a section about the neutral axis for bending.
i = diameter or side of bar.
j = ratio of lever arm of resisting couple to depth d.
jd = $d - z$ = arm of resisting couple.
k = ratio of depth of neutral axis to depth d.
K = $\frac{1}{2}f_c kj$ or $pf_s j$ in rectangular beams; the ratio $\frac{I}{l}$ or $\frac{I}{h}$ for beams and columns, respectively.
l = span length of beam or slab (generally distance from center to center of supports).
M = bending moment or moment of resistance in general.
n = $\frac{E_s}{E_c}$ = ratio of modulus of elasticity of steel to that of concrete.
Σ_0 = sum of perimeters of bars in one set.
p = ratio of effective area of tension reinforcement to effective area of concrete in beams = $\frac{A_s}{bd}$.
p' = ratio of effective area of compression reinforcement to effective area of concrete in beams.
p_g = ratio of total effective reinforcement in member subject to compression to gross concrete section.
P = total safe axial load on column whose $\frac{h}{a \text{ (or } d)}$ is less than 10.
P' = total safe axial load on long column.
R = least radius of gyration of a section; ratio of gross area of column to core area.
s = spacing of web members, measured at the plane of the lower reinforcement and in the direction of the longitudinal axis of the beam; diameter of circle on which the longitudinal bars in a circular column are placed.
t = thickness of flange of T-beam.
T = total tensile stress in longitudinal reinforcement.
u = bond stress per unit of area of surface of bar.
v = shearing unit stress.
V = total shear.
V_c = total shear that can be resisted by the concrete.
V' = total shear on any section after deducting that carried by the concrete, *i.e.*, $V' = V - V_c$.
w = uniformly distributed load per unit of length of beam or slab.
z = depth from compression surface of beam or slab to resultant of compressive stresses.

APPENDIX B

SUMMARY OF WORKING STRESSES RECOMMENDED BY THE JOINT CODE

Allowable Unit Stresses in Concrete

Description		Allowable unit stresses: For any strength of concrete as fixed by test $n = \frac{30{,}000}{f'_c}$	f'_c = 2000 p.s.i. $n = 15$	f'_c = 2500 p.s.i. $n = 12$	f'_c = 3000 p.s.i. $n = 10$	f'_c = 3750 p.s.i. $n = 8$
Flexure:						
Extreme fiber stress in compression†	f_c	$0.40f'_c$	800	1000	1200	1500
Extreme fiber stress in compression adjacent to supports of continuous or fixed beams or of rigid frames	f_c	$0.45f'_c$	900	1125	1350	1688
Shear:						
Beams with no web reinforcement and without special anchorage of longitudinal steel	v_c	$0.02f'_c$	40	50	60	75
Beams with no web reinforcement but with special anchorage of longitudinal steel	v_c	$0.03f'_c$	60	75	90	113
Beams with properly designed web reinforcement but without special anchorage of longitudinal steel	v	$0.06f'_c$	120	150	180	225
Beams with properly designed web reinforcement and with special anchorage of longitudinal steel	v	$0.12f'_c$	240	300	360	450
Flat slabs at distance d from edge of column capital or drop panel	v_c	$0.03f'_c$	60	75	90	113
Footings	v_c		60	75	75	75
Bond:						
In beams and slabs and one-way footings:*						
Plain bars	u	$0.04f'_c$	80	100	120	150
Deformed bars	u	$0.05f'_c$	100	125	150	188
In two-way footings:						
Plain bars	u	$0.045f'_c$	90	113	135	169
Deformed bars	u	$0.056f'_c$	112	140	168	210
Bearing:						
On full area	f_c	$0.25f'_c$	500	625	750	938
Pedestals	r_a	See Art. 153				

* Where special anchorage is provided, 1½ times these values in bond may be used in beams, slabs and one-way footings. The values given for two-way footings include an allowance for special anchorage.

† The Joint Code (1941) and the Joint Committee (1940) propose that the extreme fiber stress in compression (flexure) be increased to $0.45f'_c$.

Allowable Unit Stresses in Reinforcement

(*a*) *Tension:*

(f_s = tensile unit stress in longitudinal reinforcement)
(f_v = tensile unit stress in web reinforcement)
20,000 p.s.i. for rail-steel concrete reinforcement bars, billet-steel concrete reinforcement bars (of intermediate and hard grades), axle-steel concrete reinforcement bars (of intermediate and hard grades), and cold drawn steel wire for concrete reinforcement.
18,000 p.s.i. for billet-steel concrete reinforcement bars (of structural grade), and axle-steel concrete reinforcement bars (of structural grade).

(*b*) *Compression, vertical column reinforcement:*

(f_s = nominal working stress in vertical column reinforcement)
20,000 p.s.i. for rail or hard grade steel.
16,000 p.s.i. for intermediate grade steel.

APPENDIX C

FLAT-SLAB REGULATIONS OF THE JOINT CODE

The rules given in this appendix form a part of the report of the American Concrete Institute Committee 501 on Reinforced Concrete Building Design and Specifications, adopted by the American Concrete Institute in 1936. Only those portions of the report which deal specifically with flat slab design and construction are included. The symbols and notations used in the code are defined as follows:

b_1 = width of dropped panel in feet.

c = diameter in feet of the column capital at the under side of the slab or drop panel.

f'_c = ultimate compression strength of the concrete.

h = unsupported length of column.

I = moment of inertia of a column section about the neutral axis for bending.

i = diameter of bar.

l = span length of the panel (usually expressed in feet) center to center of column, in the direction in which moments are considered.

l_1 = span length of the panel, center to center of columns, perpendicular to the rectangular direction in which moments are considered.

M = bending moment or moment of resistance in general.

M_0 = sum of the positive and negative bending moments in one direction at the principal design sections of a flat slab panel.

p = ratio of effective area of tension reinforcement to effective area of concrete in beams and slabs.

p_g = ratio of effective area of longitudinal reinforcement to the area of the concrete core in columns.

t_1 = thickness of the slab, in panels without drops; or the total thickness of the slab and the drop panel, where drops are used.

t_2 = thickness of the slab outside of the perimeter of the drop, in panels where drops are used.

w' = uniformly distributed dead and live load on the slab in lb. per sq. ft.

W = total dead and live load on one panel, in pounds, including the weight of the drops.

1. Shearing Stresses in Flat Slabs:

(*a*) In flat slabs, the shearing unit stress on a vertical section which lies at a distance $t_1 - 1\frac{1}{2}$ in. from the edge of the column capital and parallel with it, shall not exceed the following values when computed by the formula $v = \frac{V}{\frac{7}{8}bd}$ (in which d shall be taken as $t_1 - 1\frac{1}{2}$ in.).

(1) $0.03f'_c$ when at least 50 per cent of the total negative reinforcement passes directly over the column capital.

(2) $0.025f'_c$ when 25 per cent of the total negative reinforcement passes directly over the column capital (which is the least that shall be permitted).

(3) For intermediate percentages, intermediate values of the shearing unit stress shall be used.

(*b*) In flat slabs, the shearing unit stress on a vertical section which lies at a distance of $t_2 - 1\frac{1}{2}$ in. from the edge of the dropped panel and parallel with it shall not exceed $0.03f'_c$ when computed by the formula $v = \frac{V}{\frac{7}{8}bd}$ (in which d shall be taken as $t_2 - 1\frac{1}{2}$ in.). At least 50 per cent of the cross-sectional area of the negative reinforcement in the column strip must be within the width of the strip directly over the dropped panel.

2. Limitations:

(*a*) The term flat slabs as used in these regulations refers to concrete slabs, having reinforcing bars extending in two or four directions, without beams or girders to carry the load to supporting members.

(*b*) The moment coefficients, moment distribution, and slab thicknesses specified herein are for a series of slabs of approximately uniform size arranged in three or more rows of panels in each direction, and in which the ratio of length to width of panel does not exceed 1.33.

(*c*) For structures having a width of less than three rows of panels, or in which irregular panels are used, an analysis shall be made of the moments developed in both slabs and columns. When so required, computations shall be submitted to the commissioner of buildings for approval.

3. Panel Strips and Principal Design Sections:

(*a*) For convenience of reference, a flat slab shall be considered as consisting of strips as follows:

A middle strip one-half panel in width, symmetrical with respect to the panel center line and extending through the panel in the direction in which moments are being considered;

A column strip one-half panel in width, occupying the two quarter-panel areas outside of the middle strip.

(*b*) The critical sections for moment calculations are referred to as principal design sections and are located as follows:

Sections for negative moment shall be taken along the edges of the panel, on lines joining the column centers, and following the circumference of the column capital.

Sections for positive moment shall be taken on the center lines of the panel.

(*c*) In the two-way system it shall be assumed that the various moments in the strips are resisted by the bands located within the strips, each band being $0.5l_1$ in width.

(*d*) In the four-way system, it shall be assumed that the column strip positive moment is resisted by the direct band; that the column strip negative moment is resisted by the direct band plus the two diagonal bands multiplied by the cosine of the angle between the direct band and the diagonal bands; that the middle strip positive moment is resisted by the two diagonal bands multiplied by the cosine of the angle between the axis of the middle strip and the diagonal bands; and that the middle strip negative moment is resisted by an independent top band across the middle of the direct band. The width of direct and middle strip negative bands shall be approximately $0.4l_1$; the width of the diagonal bands shall be approximately 0.4 of the average span length or $\frac{l + l_1}{5}$.

(*e*) The width of the column head section for compression shall be taken as the width of the dropped panel (b_1), or half the width of the panel ($0.5l_1$) where no dropped panel is used.

4. Flat Slab Design:

(*a*) Tables A, B, C, D contain values of thicknesses, dimensions and moments which are to govern flat slab design when 2000 lb. per sq. in. ultimate strength concrete is used. The general formulas are given under the heading "General Case"; the formulas for the case where the diameter of the column capital $c = 0.225l$ are given under the heading "Special Case for $c = 0.225l$." In these tables l, l_1, b_1, and c are always expressed in feet, while the units to which the formulas develop are shown in the column headed "units."

(*b*) Where concrete of higher ultimate strength than 2000 lb. per sq. in. is used, the required and minimum slab thickness given in the following table may be reduced by multiplying by the factor $\sqrt[3]{\frac{2000}{f'_c}}$, in which f'_c is the ultimate strength of the concrete to be used.

(*c*) The average c for the columns at the four corners of a panel shall be used in obtaining the slab thickness, the numerical sum of the total positive and negative moments (M_0) in either direction, and the middle strip positive and negative moments in either direction.

The average c for two adjacent columns shall be used in obtaining the positive and negative moments in the column strip between these adjacent columns.

(*d*) Any of the moments in Tables B, C, and D may be varied by not more than 6 per cent, provided that the total numerical sum of the positive and negative moments on the principal sections is not reduced.

(*e*) The ratio of reinforcement considered in any strip shall not exceed the value of p calculated for balanced reinforcement. The ratio of reinforcement in any strip shall not be less than 0.0025. Bars shall not be spaced farther apart than one and one-half times the slab thickness for the full width of the bands.

(*f*) Moments for the four-way system are shown in Table B by strips, and, for convenience, also by bands.

(*g*) Slabs supported by marginal beams on opposite edges shall be designed as solid one or two-way slabs to carry the entire panel load.

5. Arrangement of Reinforcement:

(*a*) The slab reinforcement shall be accurately placed so as to resist not only the moments at the critical sections, but also the moments at intermediate sections, and shall be secured and supported by concrete or metal chairs or spacers. Required lengths of bars and location of points of bending are shown in Table E.

(*b*) The positive moment reinforcement perpendicular to a discontinuous edge shall extend to this edge and have an embedment of at least 6 in. in spandrel beams or columns. All negative-moment reinforcement shall be bent or hooked at spandrel beams or columns to provide adequate bond resistance.

6. Brackets:

Brackets extending the full width of the column may be substituted for column capitals at exterior columns, provided the sloping face of the bracket makes an angle not less than 45 degrees with the face of the column, projected upward.

The value of c where brackets are used is twice the distance from the center of the column to a point where the bracket is 1½ in. thick.

7. Columns without Capitals or Brackets:

Brackets and column capitals may be omitted altogether, provided the slab thickness is sufficient to fully resist the moments and shears at the column head section.

The value of c where brackets and column capitals are omitted is the width of the column in the direction in which moments are considered, except that, when a beam of greater depth than the thickness of the slab or dropped panel extends into the column in the direction in which moments are considered, the value of c may be taken as the width of the column plus twice the projection of the beam below the slab or dropped panel.

8. Openings in Flat Slabs:

(*a*) Openings of any size may be cut through the floor in the area common to two intersecting middle strips, provided the total positive and negative

resisting moments be maintained as required in Art. 4 and that these total positive and total negative moments be redistributed between the remaining principal design sections to meet the new conditions.

(b) In any area common to two column strips, not more than one opening shall be allowed and the greatest dimension of such an opening shall not exceed $\frac{1}{20}l$.

(c) In any area common to one column strip and one middle strip, openings shall not interrupt more than one-quarter of the bars in either strip and the equivalent of the bars so interrupted shall be provided by extra steel on both sides of the opening.

(d) Any opening larger than described above shall be completely framed on all sides with beams to carry the loads to the columns.

TABLE A.—SLAB THICKNESS AND SIZE OF DROP

	Symbol	Unit	General case*	Special case for* $c = 0.225l$
Minimum floor-slab thickness	t_1 or t_2	Inches	0.375 (long l)	0.375 (long l)
Minimum roof-slab thickness	t_1 or t_2	Inches	0.300 (long l)	0.300 (long l)
Slab thickness without dropped panel	t_1	Inches	$0.038\left(1 - 1.44\frac{c}{l}\right)l\sqrt{w'} + 1\frac{1}{2}$	$0.025l\sqrt{w'} + 1\frac{1}{2}$
Slab thickness beyond dropped panel	t_2	Inches	$0.02l\sqrt{w'} + 1$	$0.02l\sqrt{w'} + 1$
Slab thickness through dropped panel	t_1	Inches	Minimum $= 1.25t_2$ Maximum $= 1.50t_2$	Minimum $= 1.25t_2$ Maximum $= 1.50t_2$
(Maximum l to be used in thickness formulas)				
Minimum side or diameter of dropped panel	b_1	Feet	$0.35l_1$	$0.35l_1$

* In substituting in these equations l and c are taken in feet and w' in pounds per square foot. The resulting slab thicknesses are in inches.

TABLE B.—MOMENTS FOR DESIGN OF AN INTERIOR PANEL

Numerical sum of positive and negative moments in direction of either side of interior rectangular panel	Symbol	Unit	General case $0.09\,Wl\left(1-\frac{2c}{3l}\right)^2$	Special case* $0.065\,Wl$
Two-way system with dropped panel:				
Column strip, negative moment	$-M_c$	Ft.-lb.	$0.50M_0$	$0.0325\,Wl$
Column strip, positive moment	$+M_c$	Ft.-lb.	$0.20M_0$	$0.0130\,Wl$
Middle strip, negative moment	$-M_m$	Ft.-lb.	$0.15M_0$	$0.00975\,Wl$
Middle strip, positive moment	$+M_m$	Ft.-lb.	$0.15M_0$	$0.00975\,Wl$
Two-way system without dropped panel:				
Column strip, negative moment	$-M_c$	Ft.-lb.	$0.46M_0$	$0.030\,Wl$
Column strip, positive moment	$+M_c$	Ft.-lb.	$0.22M_0$	$0.0142\,Wl$
Middle strip, negative moment	$-M_m$	Ft.-lb.	$0.16M_0$	$0.0104\,Wl$
Middle strip, positive moment	$+M_m$	Ft.-lb.	$0.16M_0$	$0.0104\,Wl$
Four-way system with dropped panels				
(Moments by strips):				
Column strip, negative moment	$-M_c$	Ft.-lb.	$0.54M_0$	$0.0351\,Wl$
Column strip, positive moment	$+M_c$	Ft.-lb.	$0.19M_0$	$0.0124\,Wl$
Middle strip, negative moment	$-M_m$	Ft.-lb.	$0.08M_0$	$0.0052\,Wl$
Middle strip, positive moment	$+M_m$	Ft.-lb.	$0.19M_0$	$0.0124\,Wl$
(Moments by bands):				
Direct band, negative moment	$-M$	Ft.-lb.	$0.307M_0$	$0.0200\,Wl$
Direct band, positive moment	$+M$	Ft.-lb.	$0.19M_0$	$0.0124\,Wl$
Diagonal band, negative moment	$-M$	Ft.-lb.	$0.168M_0$	$0.0109\,Wl$
Diagonal band, positive moment	$+M$	Ft.-lb.	$0.134M_0$	$0.0087\,Wl$
Cross band, negative moment	$-M$	Ft.-lb.	$0.08M_0$	$0.0052\,Wl$
Four-way system without dropped panels				
(Moments by strips):				
Column strip, negative moment	$-M$	Ft.-lb.	$0.50M_0$	$0.0325\,Wl$
Column strip, positive moment	$+M$	Ft.-lb.	$0.20M_0$	$0.0130\,Wl$
Middle strip, negative moment	$-M$	Ft.-lb.	$0.10M_0$	$0.0065\,Wl$
Middle strip, positive moment	$+M$	Ft.-lb.	$0.20M_0$	$0.0130\,Wl$
(Moments by bands):				
Direct band, negative moment	$-M$	Ft.-lb.	$0.30M_0$	$0.0195\,Wl$
Direct band, positive moment	$+M$	Ft.-lb.	$0.20M_0$	$0.0130\,Wl$
Diagonal band, negative moment	$-M$	Ft.-lb.	$0.141M_0$	$0.0092\,Wl$
Diagonal band, positive moment	$+M$	Ft.-lb.	$0.141M_0$	$0.0092\,Wl$
Cross band, negative moment	$-M$	Ft.-lb.	$0.10M_0$	$0.0065\,Wl$

* Special case for $c = 0.225l_{av}$.

Table C.—Moments for Design of an Exterior Panel

Moments in the strips perpendicular to the discontinuous edge, where they differ from an interior panel, are given in the following table. Negative moments in the column strip and middle strip on the line of the first interior columns are the same as for an interior panel. Moments in the strips parallel to the discontinuous edge are the same as for an interior panel.

	Symbol	Unit	General case	Special case for $c = 0.225l$
Two-way system with dropped panel:				
Column strip, negative moment at discontinuous edge	$-M_c$	Ft.-lb.	$0.45M_0$	$0.029Wl$
Column strip, positive moment	$+M_c$	Ft.-lb.	$0.25M_0$	$0.016Wl$
Middle strip, negative moment at discontinuous edge	$-M_m$	Ft.-lb.	$0.10M_0$	$0.0065Wl$
Middle strip, positive moment	$+M_m$	Ft.-lb.	$0.19M_0$	$0.012Wl$
Two-way system without dropped panel:				
Column strip, negative moment at discontinuous edge	$-M_c$	Ft.-lb.	$0.41M_0$	$0.027Wl$
Column strip, positive moment	$+M_c$	Ft.-lb.	$0.28M_0$	$0.018Wl$
Middle strip, negative moment at discontinuous edge	$-M_m$	Ft.-lb.	$0.10M_0$	$0.007Wl$
Middle strip, positive moment	$+M_m$	Ft.-lb.	$0.20M_0$	$0.013Wl$
Four-way system with dropped panels (Moments by strips):				
Column strip, negative moment at discontinuous edge	$-M$	Ft.-lb.	$0.485M_0$	$0.0315Wl$
Column strip, positive moment	$+M$	Ft.-lb.	$0.24M_0$	$0.0156Wl$
Middle strip, negative moment at discontinuous edge	$-M$	Ft.-lb.	$0.05M_0$	$0.0032Wl$
Middle strip, positive moment	$+M$	Ft.-lb.	$0.24M_0$	$0.0156Wl$
(Moments by bands) (for square panel):				
Direct band at column head at discontinuous edge	$-M$	Ft.-lb.	$0.28M_0$	$0.018Wl$
Direct band at center	$+M$	Ft.-lb.	$0.24M_0$	$0.0156Wl$
Diagonal bands at column head at discontinuous edge	$-M$	Ft.-lb.	$0.15M_0$	$0.010Wl$
Diagonal bands at center	$+M$	Ft.-lb.	$0.17M_0$	$0.011Wl$
Top band (across middle of direct) at discontinuous edge	$-M$	Ft.-lb.	$0.05M_0$	$0.003Wl$
Four-way system without dropped panels (Moments by strips):				
Column strip, negative moment at discontinuous edge	$-M$	Ft.-lb.	$0.475M_0$	$0.029Wl$
Column strip, positive moment	$+M$	Ft.-lb.	$0.25M_0$	$0.0163Wl$
Middle strip, negative moment at discontinuous edge	$-M$	Ft.-lb.	$0.062M_0$	$0.004Wl$
Middle strip, positive moment	$+M$	Ft.-lb.	$0.25M_0$	$0.016Wl$
(Moments by bands) (for square panel):				
Direct band at column head at discontinuous edge	$-M$	Ft.-lb.	$0.27M_0$	$0.017Wl$
Direct band at center	$+M$	Ft.-lb.	$0.25M_0$	$0.016Wl$
Diagonal bands at column head at discontinuous edge	$-M$	Ft.-lb.	$0.13M_0$	$0.0084Wl$
Diagonal bands at center	$+M$	Ft.-lb.	$0.18M_0$	$0.0117Wl$
Top band (across middle of direct) at discontinuous edge	$-M$	Ft.-lb.	$0.06M_0$	$0.004Wl$

TABLE D.—MOMENTS IN PANELS WITH MARGINAL BEAMS OR REINFORCED BEARING WALLS

	Marginal beams with depth greater than 1½ the slab thickness; or reinforced bearing wall					Marginal beam with depth 1½ the slab thickness or less				
(*a*) Load to be carried by marginal beam or wall	Loads directly superimposed upon it plus a uniform load equal to one-quarter of the total live and dead panel load					Loads directly superimposed upon it exclusive of any panel load				
		Two-way system		Four-way system			Two-way system		Four-way system	
		With drop	Without drop	With drop	Without drop		With drop	Without drop	With drop	Without drop
(*b*) Moment to be used in the design of half column strip adjacent and parallel to marginal beam or wall	Neg.	$0.125M_0$	$0.115M_0$	$0.135M_0$	$0.125M_0$	Neg.	$0.25M_0$	$0.23M_0$	$0.27M_0$	$0.25M_0$
	Pos.	$0.05M_0$	$0.055M_0$	$0.0475M_0$	$0.05M_0$	Pos.	$0.10M_0$	$0.11M_0$	$0.095M_0$	$0.10M_0$
(*c*) Negative moment to be used in design of middle strip continuous over beam or wall	Neg.	$0.195M_0$	$0.208M_0$	$0.104M_0$	$0.13M_0$	Neg.	$0.15M_0$	$0.16M_0$	$0.08M_0$	$0.10M_0$

TABLE E.—LENGTHS OF BARS AND POINTS OF BEND

	With drop		Without drop	
	General case	$c = 0.225l$	General case	$c = 0.225l$
Two-way slab				
(Column strip):				
Length of straight bars (not less than 0.4 of total band steel).	$l - b +$ (2′ or $40i$)	$0.65l +$ (2′ or $40i$)	$0.75l$	$0.75l$
Length of truss bars (not less than 0.4 total band steel)................	$1.5l + 0.6c$*	$1.635l$*	$1.44l + 0.66c$*	$1.59l$*
Length of additional straight bars over column head (if required)..	$0.5l + 0.6c$	$0.635l$	$0.44l + 0.66c$	$0.59l$
Point of top bend in truss bars (from column center)................	$0.25l$	$0.25l$	$0.25l$	$0.25l$
(Middle strip):				
Length of straight bars (not more than 0.5 total band steel)............	$0.65l$	$0.65l$	$0.7l$	$0.7l$
Length of truss bars (not less than 0.5 total band steel)................	$1.5l$*	$1.5l$*	$1.5l$*	$1.5l$*
Point of top bend in truss bars (from column centers)..............	$0.175l$	$0.175l$	$0.15l$	$0.15l$
Four-way slab:				
(Column strip):				
Length of straight bars (not less than 0.4 total band steel)	$l - b +$ (2′ or $40i$)	$0.65l +$ (2′ or $40i$)	$0.75l$	$0.75l$
Length of truss bars (not less than 0.4 total band steel)................	$1.5l + 0.6c$*	$1.635l$*	$1.44l + 0.66c$*	$1.59l$*
Length of additional straight bars over column head (if required).	$0.5l + 0.6c$	$0.635l$	$0.44l + 0.66c$	$0.59l$
Point of bend for truss bars (from column centers)	$0.2l$	$0.2l$	$0.2l$	$0.2l$
(Diagonal band):				
Length of straight bars (not more than 0.6 total band steel)	$l - b +$ (2′ or $40i$)	$0.65l +$ (2′ or $40i$)	$0.75l$	$0.75l$
Length of truss bars (not less than 0.4 total band steel)................	$2.21l$*	$2.21l$*	$2.21l$*	$2.21l$*
Point of bend for truss bars (from column centers)..	$0.33l$	$0.33l$	$0.33l$	$0.33l$
Length of additional straight bars over column head (if required)..	$0.8l$	$0.8l$	$0.8l$	$0.8l$
Top band across middle of direct band (length of straight bars)..........	$0.5l$	$0.5l$	$0.5l$	$0.5l$

* NOTE: To these lengths proper allowance to be added for bends.

APPENDIX D

TABLES AND DIAGRAMS

TABLE 1.—AREAS, PERIMETERS, AND WEIGHTS OF STANDARD BARS

○ Round bars				□ Square bars			
Size, in.	Area, sq. in.	Perimeter, in.	Weight, lb. per ft.	Size, in.	Area, sq. in.	Perimeter, in.	Weight, lb. per ft.
1/4	0.0491	0.785	0.17	1/2	0.2500	2.00	0.85
3/8	0.1104	1.178	0.38	1	1.0000	4.00	3.40
1/2	0.1963	1.571	0.67	1 1/8	1.2656	4.50	4.30
5/8	0.3068	1.964	1.04	1 1/4	1.5625	5.00	5.31
3/4	0.4418	2.356	1.50				
7/8	0.6013	2.749	2.04				
1	0.7854	3.142	2.67				

TABLE 2.—AREAS, PERIMETERS, AND WEIGHTS OF NON-STANDARD BARS

○ Round bars				□ Square bars			
Size, in.	Area, sq. in.	Perimeter, in.	Weight, lb. per ft.	Size, in.	Area, sq. in.	Perimeter, in.	Weight, lb. per ft.
5/16	0.0767	0.982	0.26	1/4	0.0625	1.00	0.21
7/16	0.1503	1.374	0.51	5/16	0.0977	1.25	0.33
9/16	0.2485	1.767	0.85	3/8	0.1406	1.50	0.48
11/16	0.3712	2.160	1.26	7/16	0.1914	1.75	0.65
13/16	0.5185	2.553	1.76	9/16	0.3164	2.25	1.08
15/16	0.6903	2.945	2.35	5/8	0.3906	2.50	1.33
1 1/8	0.9940	3.534	3.38	11/16	0.4727	2.75	1.61
1 1/4	1.2272	3.927	4.17	3/4	0.5625	3.00	1.91
				13/16	0.6602	3.25	2.25
				7/8	0.7656	3.50	2.60
				15/16	0.8789	3.75	2.99

TABLE 3.—AREAS OF GROUPS OF STANDARD BARS, SQUARE INCHES

Size of bar, in.	Number of bars												
	2	3	4	5	6	7	8	9	10	11	12	13	14
1/2 round...	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96	2.16	2.36	2.55	2.75
1/2 square..	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50
5/8 round ..	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68	3.99	4.30
3/4 round...	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.86	5.30	5.74	6.19
7/8 round...	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22	7.82	8.42
1 round. ..	1.57	2.35	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.43	10.21	11.00
1 square...	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
1⅛ square...	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66	13.92	15.19	16.45	16.72
1¼ square...	3.12	4.68	6.25	7.81	9.37	10.94	12 50	14.06	15.62	17.19	18.75	20.31	21.87

TABLE 4.—PERIMETERS OF GROUPS OF STANDARD BARS, INCHES

Size of bar, in.	Number of bars												
	2	3	4	5	6	7	8	9	10	11	12	13	14
1/2 round	3 1	4.7	6 2	7.8	9.4	11 0	12 6	14.1	15.7	17 3	18 8	20.4	22.0
1/2 square...........	4 0	6 0	8 0	10.0	12 0	14 0	16 0	18.0	20 0	22.0	24.0	26 0	28.0
5/8 round...........	3.9	5.9	7.8	9 8	11.8	13.7	15.7	17.7	19 5	21.6	23 6	25 5	27.5
3/4 round...........	4.7	7.1	9.4	11.8	14.1	16 5	18.8	21.2	23.6	25.9	28.3	30.6	33.0
7/8 round...........	5.5	8.2	11.0	12.7	16.5	19.2	22.0	24.7	27 5	30.2	33.0	35.7	38.5
1 round	6.3	9 4	12.6	15.7	18 9	22 0	25.1	28.3	31.4	34 6	37.7	40.9	44.0
1 square	8.0	12.0	16.0	20 0	24.0	28.0	32.0	36 0	40 0	44.0	48 0	52.0	56.0
1⅛ square	9 0	13.5	18.0	22.5	27 0	31.5	36 0	40 5	45 0	49.5	54.0	58.5	63.0
1¼ square	10.0	15.0	20 0	25.0	30.0	35.0	40.0	45 0	50.0	55.0	60 0	65.0	70.0

TABLE 5.—AREAS OF BARS IN SLABS, SQUARE INCHES PER FOOT

Spacing (inches)	○ Size of round bar, in.							□ Size of square bar, in.			
	¼	⅜	½	⅝	¾	⅞	1	½	1	1⅛	1¼
3	0.20	0.44	0 78	1.23	1.77	2.40	3.14	1.00	4.00	5.06	6 25
3½	0.17	0.38	0.67	1 05	1.51	2 06	2.69	0 86	3 43	4 34	5 36
4	0.15	0 33	0.59	0 92	1.32	1 80	2.36	0.75	3 00	3.80	4.69
4½	0.13	0.29	0 52	0 82	1.18	1.60	2 09	0.67	2 67	3.37	4.17
5	0 12	0 26	0.47	0.74	1.06	1 44	1 88	0 60	2.40	3 04	3 75
5½	0.11	0.24	0.43	0 67	0.96	1.31	1.71	0.55	2.18	2 76	3 41
6	0.10	0.22	0.39	0 61	0.88	1.20	1.57	0.50	2.00	2 53	3.12
6½		0.20	0.36	0.57	0.82	1.11	1.45	0.46	1.85	2.34	2.89
7		0.19	0.34	0.53	0.76	1.03	1.35	0.43	1.71	2.17	2.68
7½		0.18	0.31	0.49	0.71	0.96	1.26	0.40	1.60	2 02	2.50
8		0.17	0.29	0.46	0 66	0.90	1.18	0.37	1.50	1.89	2.34
9		0.15	0.26	0 41	0.59	0.80	1.05	0.33	1.33	1.69	2.08
10		0.13	0.24	0 37	0 53	0.72	0 94	0.30	1.20	1.52	1.87
12		0.11	0.20	0.31	0.44	0 60	0.78	0.25	1.00	1.27	1.56

TABLE 6.—DESIGN OF RECTANGULAR BEAMS AND SLABS

$$k = \frac{n}{n + r} \qquad j = 1 - \frac{k}{3} \qquad p = \frac{n}{2r(n + r)} \qquad K = \tfrac{1}{2}f_c kj \text{ or } pf_s j$$

n and f'_c	f_s	f_c	k	j	p	K
8 (3750)	18,000	1200	0.348	0.884	0.0116	185
		1400	0.384	0.872	0.0149	234
		1500	**0.400**	**0.867**	**0.0167**	**260**
		1688	*0.429*	*0.857*	*0.0201*	*310*
	20,000	1200	0.324	0.892	0.0097	173
		1400	0.359	0.880	0.0126	221
		1500	**0.375**	**0.875**	**0.0141**	**246**
		1688	*0.403*	*0.866*	*0.0170*	*294*
10 (3000)	18,000	1000	0.357	0.881	0.0099	157
		1100	0.379	0.874	0.0116	183
		1200	**0.400**	**0.867**	**0.0133**	**208**
		1350	*0.428*	*0.857*	*0.0161*	*248*
	20,000	1000	0.333	0.889	0.0083	148
		1100	0.355	0.882	0.0098	172
		1200	**0.375**	**0.875**	**0.0113**	**197**
		1350	*0.403*	*0.866*	*0.0136*	*235*
12 (2500)	16,000	750	0.360	0.880	0.0084	119
		800	0.375	0.875	0.0094	131
		900	0.403	0.866	0.0113	156
		1000	**0.429**	**0.857**	**0.0134**	**184**
		1125	*0.457*	*0.848*	*0.0161*	*218*
	18,000	750	0.333	0.889	0.0069	111
		800	0.348	0.884	0.0077	123
		900	0.375	0.875	0.0094	148
		1000	**0.400**	**0.867**	**0.0111**	**173**
		1125	*0.428*	*0.857*	*0.0134*	*207*
	20,000	750	0.310	0.897	0.0058	104
		800	0.324	0.892	0.0065	116
		900	0.351	0.883	0.0079	140
		1000	**0.375**	**0.875**	**0.0094**	**164**
		1125	*0.403*	*0.866*	*0.0113*	*196*
15 (2000)	16,000	650	0.379	0.874	0.0077	108
		700	0.397	0.868	0.0087	121
		750	0.414	0.862	0.0097	134
		800	**0.429**	**0.857**	**0.0107**	**147**
		900	*0.457*	*0.848*	*0.0129*	*176*
	18,000	650	0.351	0.883	0.0063	101
		700	0.368	0.877	0.0072	113
		750	0.385	0.872	0.0080	126
		800	**0.400**	**0.867**	**0.0089**	**139**
		900	*0.429*	*0.857*	*0.0107*	*165*
	20,000	650	0.328	0.891	0.0053	94
		700	0.344	0.885	0.0060	106
		750	0.359	0.880	0.0067	118
		800	**0.374**	**0.875**	**0.0075**	**131**
		900	*0.403*	*0.866*	*0.0091*	*157*

Boldface for $0.4f'_c$; italics for $0.45f'_c$. See Art. 50 and footnote on pg. 61.

TABLE 7.—REVIEW OF RECTANGULAR BEAMS AND SLABS

$k = \sqrt{2pn + (pn)^2} - pn$ $j = 1 - \frac{1}{3}k$

p	n = 8		n = 10		n = 12		n = 15	
	k	j	k	j	k	j	k	j
0.0010	0.119	0.960	0.132	0.956	0.145	0.952	0.158	0.947
0.0020	0.164	0.945	0.181	0.940	0.196	0.935	0.217	0.928
0.0030	0.196	0.935	0.217	0.928	0.235	0.922	0.258	0.914
0.0040	0.223	0.926	0.246	0.918	0.266	0.911	0.292	0.903
0.0050	0.246	0.918	0.270	0.910	0.291	0.903	0.320	0.893
0.0054	0.254	0.915	0.279	0.907	0.300	0.900	0.329	0.891
0.0058	0.262	0.913	0.287	0.904	0.309	0.897	0.337	0.888
0.0062	0.269	0.910	0.296	0.901	0.317	0.894	0.348	0.884
0.0066	0.276	0.908	0.304	0.899	0.325	0.892	0.356	0.881
0.0070	0.283	0.906	0.311	0.896	0.334	0.889	0.365	0.878
0.0072	0.286	0.905	0.314	0.895	0.338	0.887	0.369	0.877
0.0074	0.290	0.903	0.318	0.894	0.342	0.886	0.372	0.876
0.0076	0.293	0.902	0.321	0.893	0.345	0.885	0.376	0.875
0.0078	0.297	0.901	0.325	0.892	0.349	0.884	0.380	0.873
0.0080	0.300	0.900	0.328	0.891	0.353	0.882	0.384	0.872
0.0082	0.303	0.899	0.332	0.889	0.356	0.881	0.387	0.871
0.0084	0.306	0.898	0.336	0.888	0.360	0.880	0.390	0.870
0.0086	0.309	0.897	0.338	0.887	0.363	0.879	0.394	0.869
0.0088	0.312	0.896	0.341	0.886	0.366	0.878	0.398	0.867
0.0090	0.314	0.895	0.344	0.885	0.370	0.877	0.402	0.866
0.0092	0.317	0.894	0.347	0.884	0.373	0.876	0.405	0.865
0.0094	0.320	0.893	0.350	0.883	0.376	0.875	0.407	0.864
0.0096	0.322	0.893	0.353	0.882	0.379	0.874	0.411	0.863
0.0098	0.325	0.892	0.356	0.881	0.381	0.873	0.414	0.862
0.0100	0.328	0.891	0.358	0.881	0.385	0.872	0.418	0.861
0.0104	0.333	0.889	0.363	0.879	0.391	0.870	0.423	0.859
0.0108	0.339	0.887	0.369	0.877	0.396	0.868	0.429	0.857
0.0112	0.343	0.886	0.375	0.875	0.402	0.866	0.434	0.855
0.0116	0.348	0.884	0.380	0.873	0.407	0.864	0.440	0.853
0.0120	0.353	0.882	0.384	0.872	0.412	0.863	0.446	0.851
0.0124	0.357	0.881	0.389	0.870	0.417	0.861	0.451	0.850
0.0128	0.362	0.879	0.394	0.869	0.422	0.859	0.457	0.848
0.0132	0.366	0.878	0.398	0.867	0.427	0.858	0.461	0.846
0.0136	0.370	0.877	0.403	0.866	0.432	0.856	0.466	0.845
0.0140	0.374	0.875	0.407	0.864	0.436	0.855	0.471	0.843
0.0144	0.379	0.874	0.412	0.863	0.440	0.853	0.475	0.842
0.0148	0.383	0.872	0.416	0.861	0.444	0.852	0.479	0.840
0.0152	0.386	0.871	0.420	0.860	0.449	0.850	0.483	0.839
0.0156	0.390	0.870	0.424	0.859	0.453	0.849	0.487	0.838
0.0160	0.394	0.869	0.428	0.857	0.457	0.848	0.493	0.836
0.0170	0.403	0.866	0.437	0.854	0.467	0.845	0.502	0.833
0.0180	0.412	0.863	0.446	0.851	0.476	0.841	0.513	0.829
0.0190	0.420	0.860	0.455	0.848	0.485	0.838	0.522	0.826
0.0200	0.428	0.857	0.463	0.846	0.493	0.836	0.531	0.823

TABLE 8.—COLUMNS WITH LONGITUDINAL BARS AND SPIRALS
Loads in thousand pounds
$P = 0.225f'_c A_g + f_s A_s$

Diameter of column, in.	Gross area A_g, sq. in.	Load on bars, $f_s A_s$ for $p_g = 0.01$		Load on concrete, $0.225f'_c A_g$			
		f_s		f'_c			
		Intermediate grade 16,000	Hard grade 20,000	2000	2500	3000	3750
12*	113	14	18	41	51	61	76
13*	133	17	22	48	60	72	90
14	154	25	31	69	87	104	130
15	177	28	35	80	99	119	149
16	201	32	40	91	113	136	170
17	227	36	45	102	128	153	192
18	255	41	51	114	143	172	215
19	284	45	57	128	159	191	239
20	314	50	63	141	177	212	265
21	346	55	69	156	195	234	292
22	380	61	76	171	214	257	321
23	416	66	83	187	234	280	350
24	452	72	90	204	254	305	382
25	491	79	98	221	276	331	414
26	531	85	106	239	299	358	448
27	573	92	115	258	322	387	483
28	616	98	123	277	346	416	519
29	661	106	132	297	372	446	557
30	707	113	141	318	398	477	596

* Spirals for these are excess id seldom available. The loads given in the table are r circular columns with lateral ties.

TABLE 9.—COLUMNS WITH LONGITUDINAL BARS AND LATERAL TIES
Loads in thousand pounds
$P = 0.18f'_cA_g + 0.8f_sA_s$

Dimensions of column, in.		Gross area A_g, sq. in.	Load on bars, $0.8f_sA_s$ for $p_g = 0.01$		Load on concrete $0.18f'_cA_g$			
			Intermediate grade 16,000	Hard grade 20,000	f'_c 2000	2500	3000	3750
10	12	120	15	19	43	54	65	81
	14	140	18	22	50	63	76	95
	16	160	20	26	58	72	86	108
12	12	144	18	23	52	65	78	97
	14	168	22	27	60	76	91	113
	16	192	25	31	69	86	104	130
14	14	196	25	31	71	88	106	132
	16	224	29	36	81	101	121	151
	18	252	32	40	91	113	136	170
16	16	256	33	41	92	115	138	173
	18	288	37	46	104	130	156	194
	20	320	41	51	115	144	173	216
	22	352	45	56	127	158	190	238
18	18	324	41	52	117	146	175	219
	20	360	46	58	130	162	194	243
	22	396	51	63	143	178	214	267
	24	432	55	69	156	194	233	292
20	20	400	51	64	144	180	216	270
	22	440	56	70	158	198	238	297
	24	480	61	77	173	216	259	324
	26	520	67	83	187	234	281	351
22	22	484	62	77	174	218	261	327
	24	528	68	84	190	238	285	356
	26	572	73	92	206	257	309	386
	28	616	79	99	222	277	333	416
24	24	576	74	92	207	259	311	389
	26	624	80	100	225	281	337	421
	28	672	86	108	242	302	363	454
26	26	676	87	108	243	304	365	456
	28	728	93	116	262	328	393	491
28	28	784	100	125	282	353	423	529
30	30	900	115	144	324	405	486	608

TABLE 10.—AREAS, WEIGHTS, AND MOMENTS OF INERTIA
Moments of inertia about the axis A-A

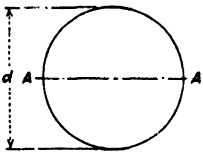
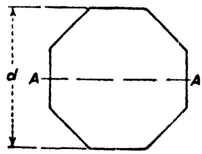
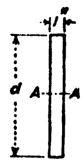
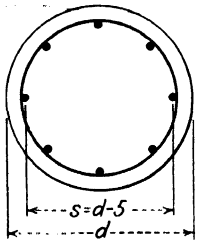
d									
	Area, sq. in.	Weight per ft., lb.	I, in.^{4}	Area, sq. in.	Weight per ft., lb.	I, in.^{4}	Area, sq. in.	Weight per ft., lb.	I, in.^{4}
12	113	118	1,018	119	124	1,136	12.0	12.5	144
13	133	138	1,402	140	146	1,565	13.0	13.5	183
14	154	160	1,886	162	169	2,105	14.0	14.6	229
15	177	184	2,485	186	194	2,775	15.0	15.6	281
16	201	210	3,217	212	221	3,591	16.0	16.7	341
17	227	237	4,100	239	249	4,577	17.0	17.7	409
18	255	265	5,153	268	280	5,753	18.0	18.8	486
19	284	295	6,397	299	312	7,142	19.0	19.8	572
20	314	327	7,854	331	345	8,768	20.0	20.8	667
21	346	361	9,547	365	381	10,658	21.0	21.9	772
22	380	396	11,499	401	418	12,837	22.0	22.9	887
23	416	433	13,737	438	457	15,335	23.0	24.0	1,014
24	452	471	16,286	477	497	18,181	24.0	25.0	1,152
25	491	511	19,175	518	539	21,406	25.0	26.1	1,302
26	531	553	22,432	560	583	25,042	26.0	27.1	1,465
27	573	597	26,087	604	629	29,123	27.0	28.1	1,640
28	616	642	30,172	650	677	33,683	28.0	29.2	1,829
29	661	688	34,719	697	726	38,759	29.0	30.2	2,032
30	707	736	39,761	746	777	44,388	30.0	31.2	2,250

TABLE 11.—MOMENT OF INERTIA OF COLUMN VERTICALS ARRANGED IN A CIRCLE OF DIAMETER 5 IN. LESS THAN DIAMETER OF COLUMN

Values of $(n - 1)I_s$ in inches[4] for $p_g = 0.01$

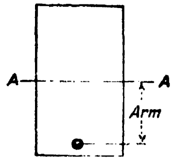


Diameter of column d, in.	Diameter of steel circle s, in.	A_s where $p_g = 0.01$	f'_c 2000	2500	3000	3750
12	7	1.13	97	76	62	49
13	8	1.33	149	117	96	74
14	9	1.54	218	172	140	109
15	10	1.77	310	243	199	155
16	11	2.01	426	335	274	213
17	12	2.27	572	449	368	286
18	13	2.55	754	593	485	377
19	14	2.84	974	765	626	487
20	15	3.14	1166	916	750	583
21	16	3.46	1550	1218	996	775
22	17	3.80	1922	1510	1236	961
23	18	4.16	2359	1853	1516	1179
24	19	4.52	2856	2244	1836	1428
25	20	4.91	3437	2701	2210	1719
26	21	5.31	4098	3220	2635	2049
27	22	5.73	4853	3813	3120	2427
28	23	6.16	5703	4481	3666	2852
29	24	6.61	6663	5235	4283	3331
30	25	7.07	7733	6076	4971	3867

The bars are assumed transformed into a thin-walled cylinder having the same sectional area as the bars. Then $I_s = A_s s^2 \div 8$.

For other values of p_g multiply the value from the table by $100p_g$.

TABLE 12.—MOMENTS OF INERTIA OF BARS IN INCHES[4]
For various distances from an axis A-A



Arm, in.	Round bars, inches $\frac{1}{2}$	Round bars, inches $\frac{5}{8}$	Round bars, inches $\frac{3}{4}$	Round bars, inches $\frac{7}{8}$	Round bars, inches 1	Square bars, inches $\frac{1}{2}$	Square bars, inches 1	Square bars, inches $1\frac{1}{8}$	Square bars, inches $1\frac{1}{4}$
2	1	1	2	2	3	1	4	5	6
$2\frac{1}{2}$	1	2	3	4	5	2	6	8	10
3	2	3	4	5	7	2	9	12	14
$3\frac{1}{2}$	2	4	5	7	10	3	12	16	19
4	3	5	7	10	13	4	16	20	25
$4\frac{1}{2}$	4	6	9	12	16	5	20	26	32
5	5	8	11	15	20	6	25	32	39
$5\frac{1}{2}$	6	9	13	18	24	8	30	38	47
6	7	11	16	22	28	9	36	46	56
$6\frac{1}{2}$	8	13	19	25	33	11	42	54	66
7	10	15	22	29	39	12	49	62	77
$7\frac{1}{2}$	11	17	25	34	44	14	56	71	88
8	13	20	28	39	50	16	64	81	100
$8\frac{1}{2}$	14	22	32	43	57	18	72	92	113
9	16	25	36	49	64	20	81	103	127
$9\frac{1}{2}$	18	28	40	54	71	23	90	114	141
10	20	31	44	60	79	25	100	127	156
$10\frac{1}{2}$	22	34	49	66	87	28	110	149	172
11	24	37	53	73	95	30	121	153	189
$11\frac{1}{2}$	26	41	58	80	104	33	132	168	207
12	28	44	64	87	113	36	144	182	225
13	33	52	75	102	133	42	169	214	264

Table 13.—Size and Pitch of Spirals, Joint Code (1940)

Hot Rolled, Intermediate Grade

Diameter of column, in.	Out to out spiral, in.	f'_c 2000	2500	3000	3750
14	11	⅜–1¾	⅜–1¾	⅜–1¾	½–2
15	12	⅜–2	⅜–2	⅜–1¾	½–2
16	13	⅜–2	⅜–2	⅜–1¾	½–2
17	14	⅜–2¼	⅜–2¼	⅜–1¾	½–2¼
18	15	⅜–2½	⅜–2¼	⅜–1¾	½–2½
19	16	⅜–2¼	⅜–2¼	⅜–1¾	½–2½
20	17	⅜–2¼	⅜–2¼	⅜–2	½–2¾
21	18	⅜–2	⅜–2	⅜–2	½–2¾
22	19	⅜–2	⅜–2	⅜–2	½–2¾
23	20	⅜–1¾	⅜–1¾	⅜–1¾	½–2¾
24	21	½–3¼	½–3¼	½–3¼	½–2¾
25	22	½–3¼	½–3¼	½–3¼	½–2¾
26	23	½–3	½–3	½–3	½–2¾
27	24	½–2¾	½–2¾	½–2¾	½–2¾
28	25	½–2¾	½–2¾	½–2¾	½–2¾
29	26	½–2¾	½–2¾	½–2¾	½–2¾
30	27	½–2½	½–2½	½–2½	½–2½

Values below dotted line are based on the minimum value for p' of 0.0112.

Above the dotted line the formula $p' = 0.45(R - 1)\frac{f'_c}{f'_s}$ governs.

Cold Drawn

Diameter of column, in.	Out to out spiral, in.	2000	2500	3000	3750
14	11	¼–1¾	⅜–1¾	⅜–1¾	⅜–1¾
15	12	¼–1¾	⅜–2	⅜–2	⅜–2
16	13	¼–1¾	⅜–2	⅜–2	⅜–2
17	14	¼–1¾	⅜–2¼	⅜–2¼	⅜–2¼
18	15	¼–1¾	⅜–2½	⅜–2½	⅜–2¼
19	16	⅜–2½	⅜–2½	⅜–2½	⅜–2¼
20	17	⅜–2¾	⅜–2¾	⅜–2¾	⅜–2¼
21	18	⅜–3	⅜–3	⅜–3	⅜–2¼
22	19	⅜–3	⅜–3	⅜–3	⅜–2¼
23	20	⅜–2¾	⅜–2¾	⅜–2¾	⅜–2¼
24	21	⅜–2¾	⅜–2¾	⅜–2¾	⅜–2¼
25	22	⅜–2½	⅜–2½	⅜–2½	⅜–2¼
26	23	⅜–2½	⅜–2½	⅜–2½	⅜–2¼
27	24	⅜–2¼	⅜–2¼	⅜–2¼	⅜–2¼
28	25	⅜–2¼	⅜–2¼	⅜–2¼	⅜–2¼
29	26	⅜–2¼	⅜–2¼	⅜–2¼	⅜–2¼
30	27	⅜–2	⅜–2	⅜–2	⅜–2

Values below dotted line are based on the minimum value for p' of 0.0075.

Above the dotted line the formula $p' = 0.45(R - 1)\frac{f'_c}{f'_s}$ governs.

DIAGRAM 1.—LOCATION OF POINTS WHERE BARS MAY BE BENT UP FOR UNIFORMLY LOADED BEAMS

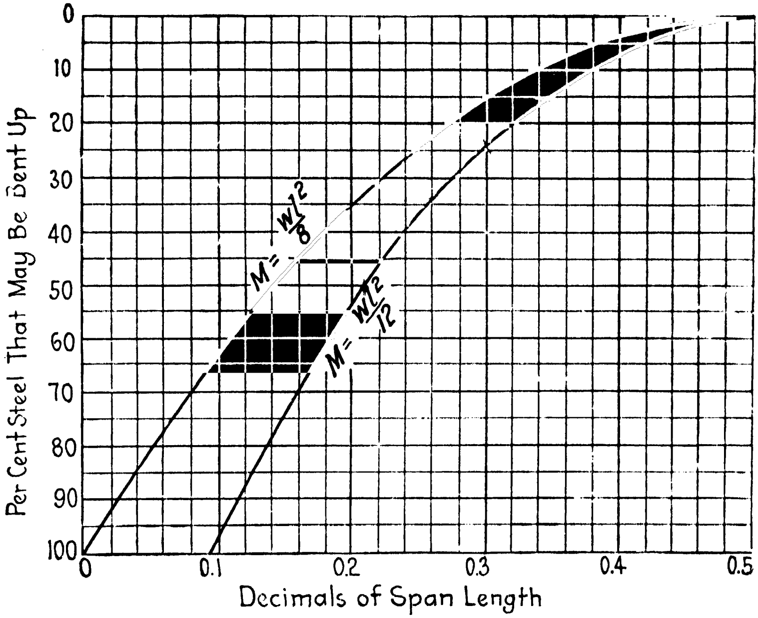


DIAGRAM 2.—T-BEAM REVIEW

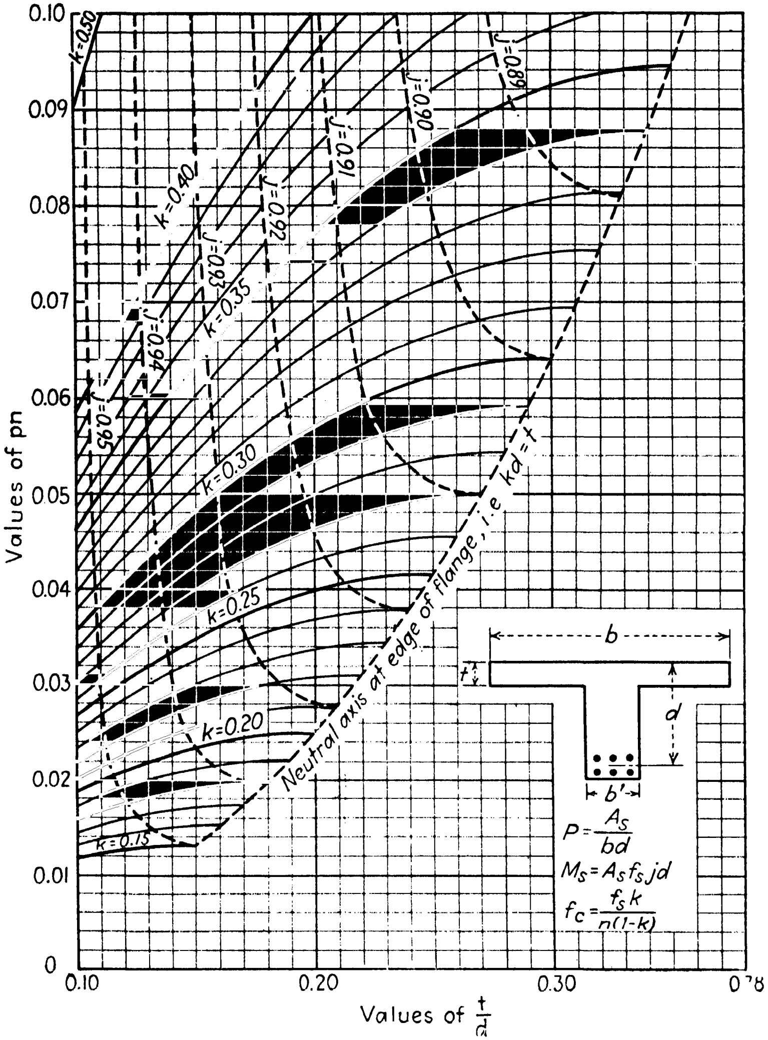


DIAGRAM 3.—RECTANGULAR BEAMS REINFORCED FOR COMPRESSION

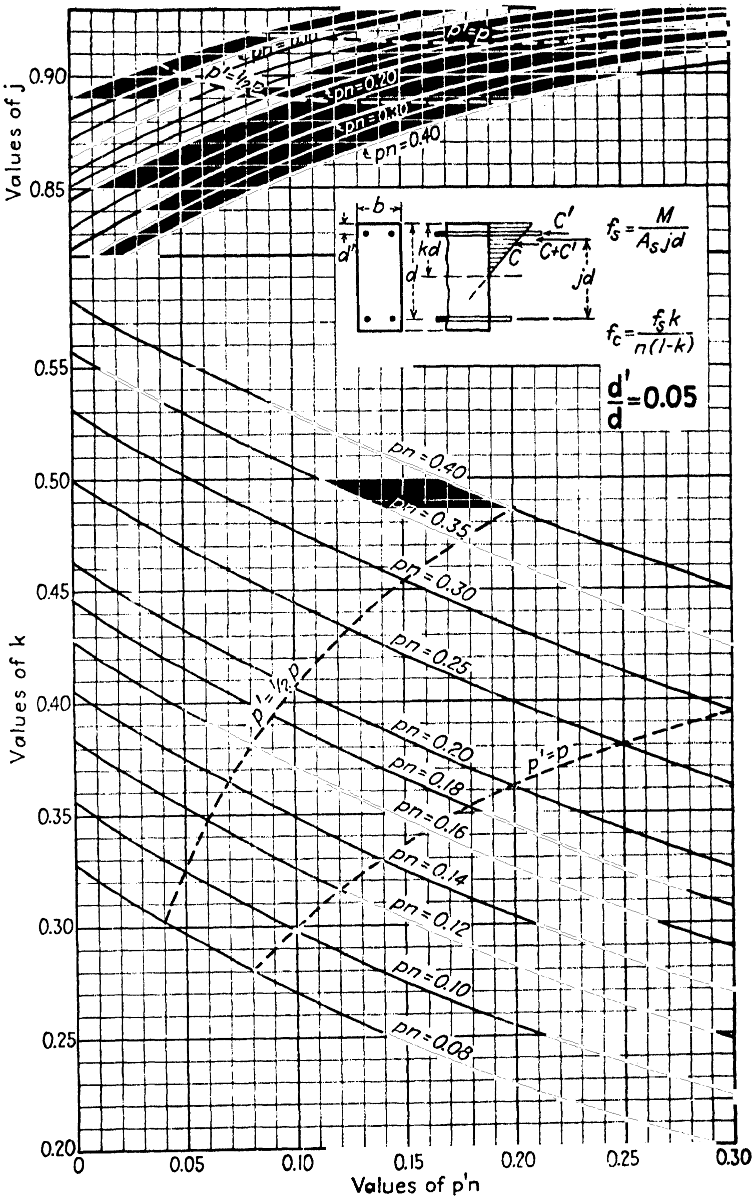


DIAGRAM 4.—RECTANGULAR BEAMS REINFORCED FOR COMPRESSION

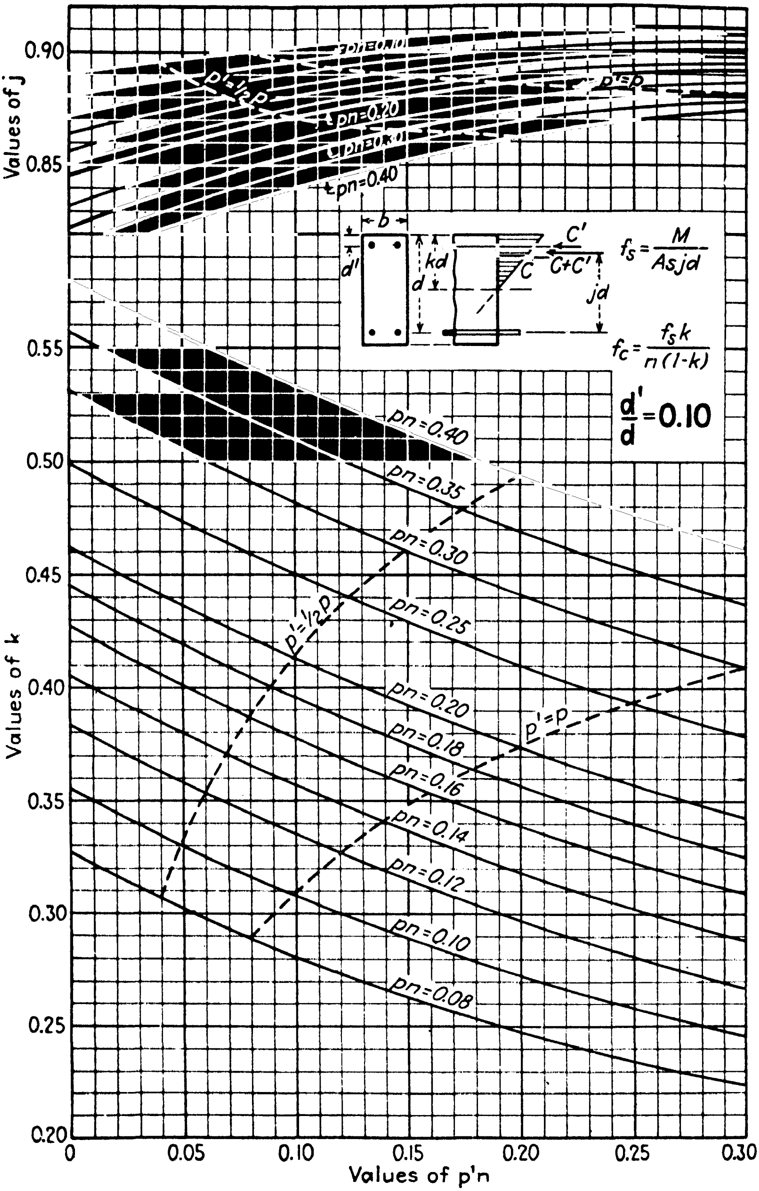


DIAGRAM 5.—RECTANGULAR BEAMS REINFORCED FOR COMPRESSION

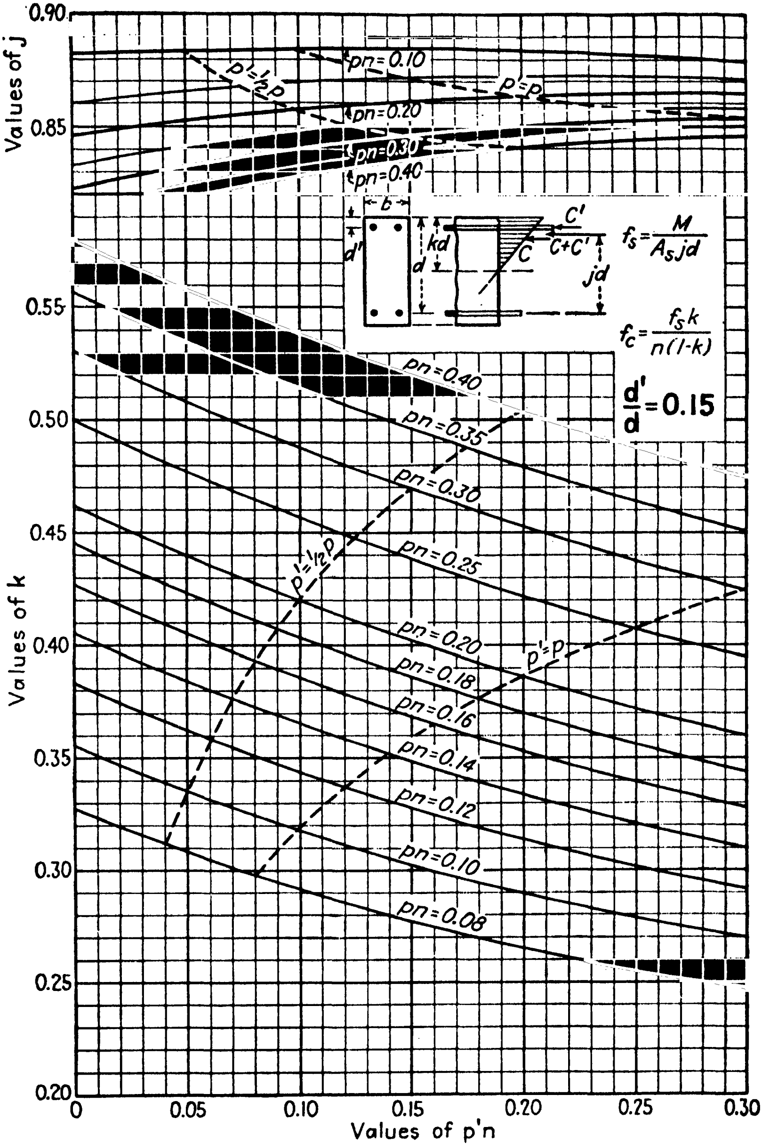


DIAGRAM 6.—RECTANGULAR BEAMS REINFORCED FOR COMPRESSION

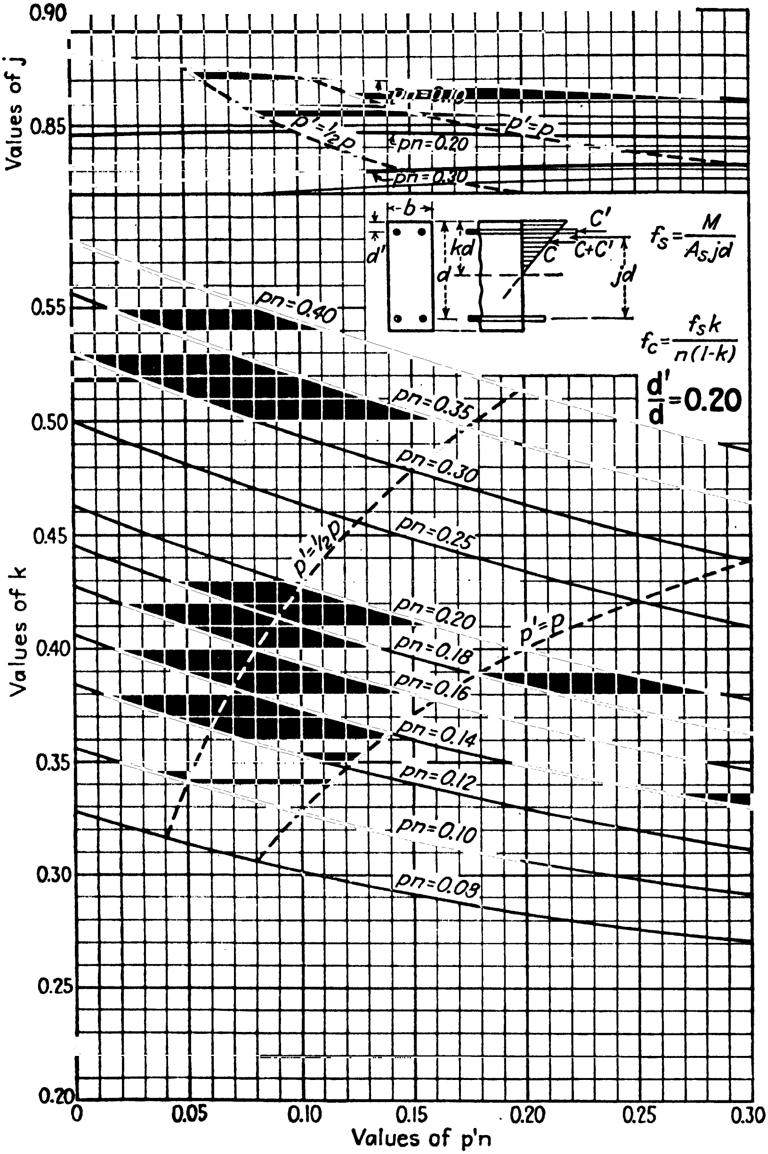


DIAGRAM 7.—ALLOWABLE UNIT STRESSES IN ECCENTRICALLY LOADED CIRCULAR SPIRAL COLUMNS

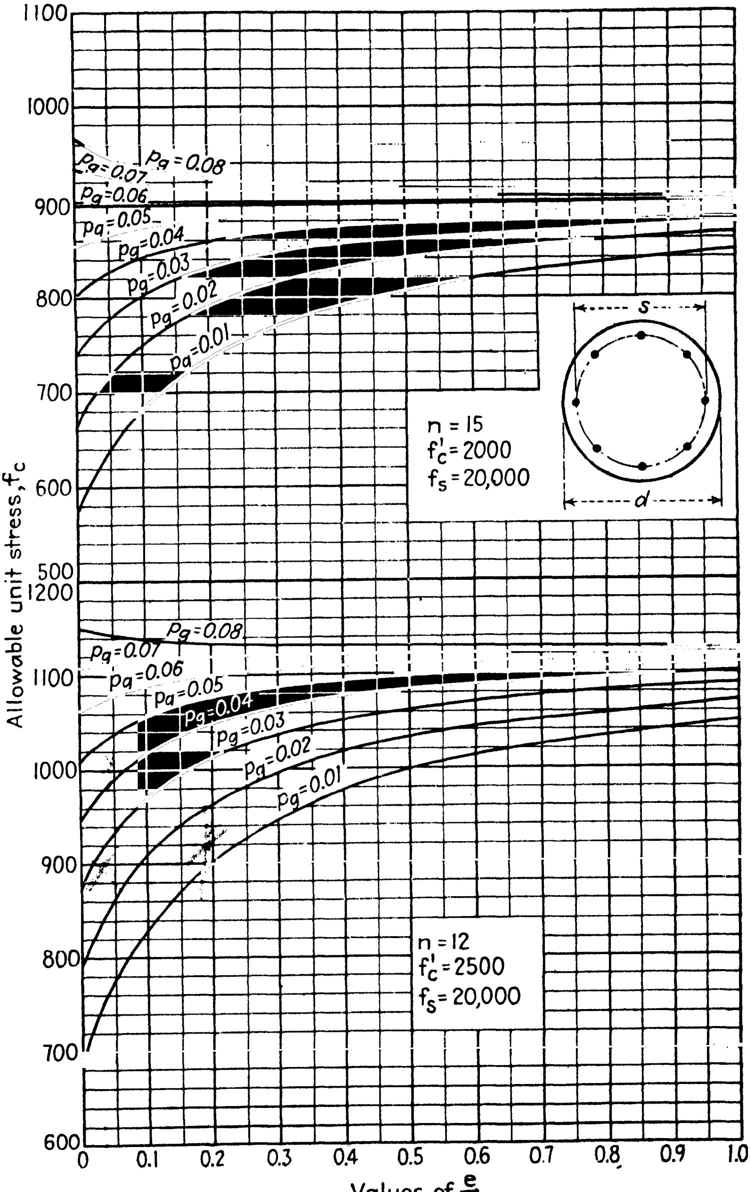


Diagram 8.—Allowable Unit Stresses in Eccentrically Loaded Circular Spiral Columns

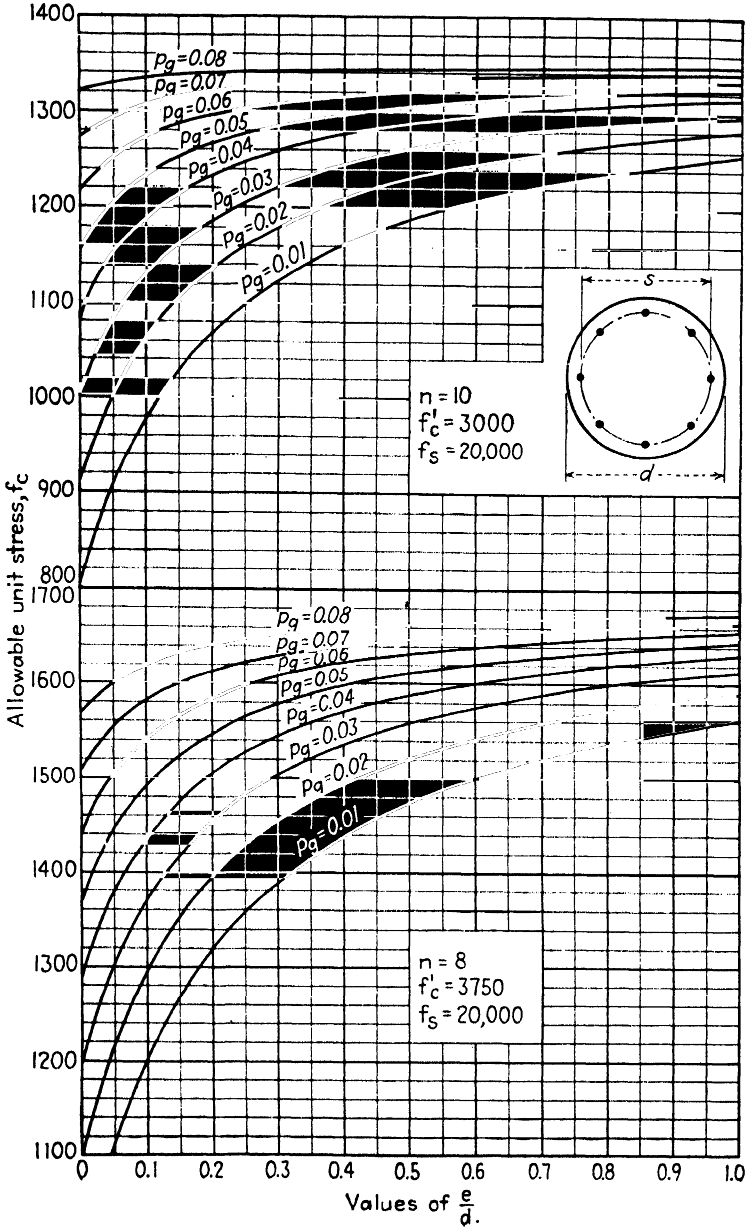


DIAGRAM 9.—ALLOWABLE UNIT STRESSES IN ECCENTRICALLY LOADED CIRCULAR SPIRAL COLUMNS

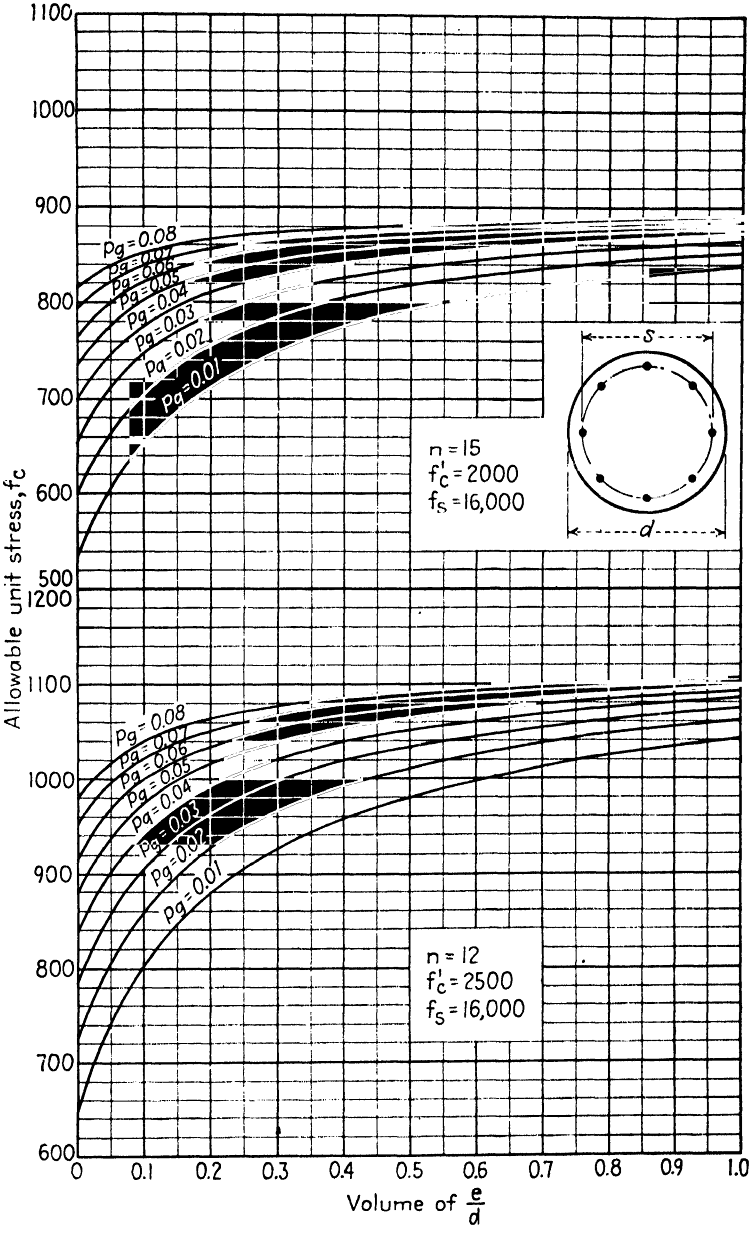


DIAGRAM 10.—ALLOWABLE UNIT STRESSES IN ECCENTRICALLY LOADED CIRCULAR SPIRAL COLUMNS

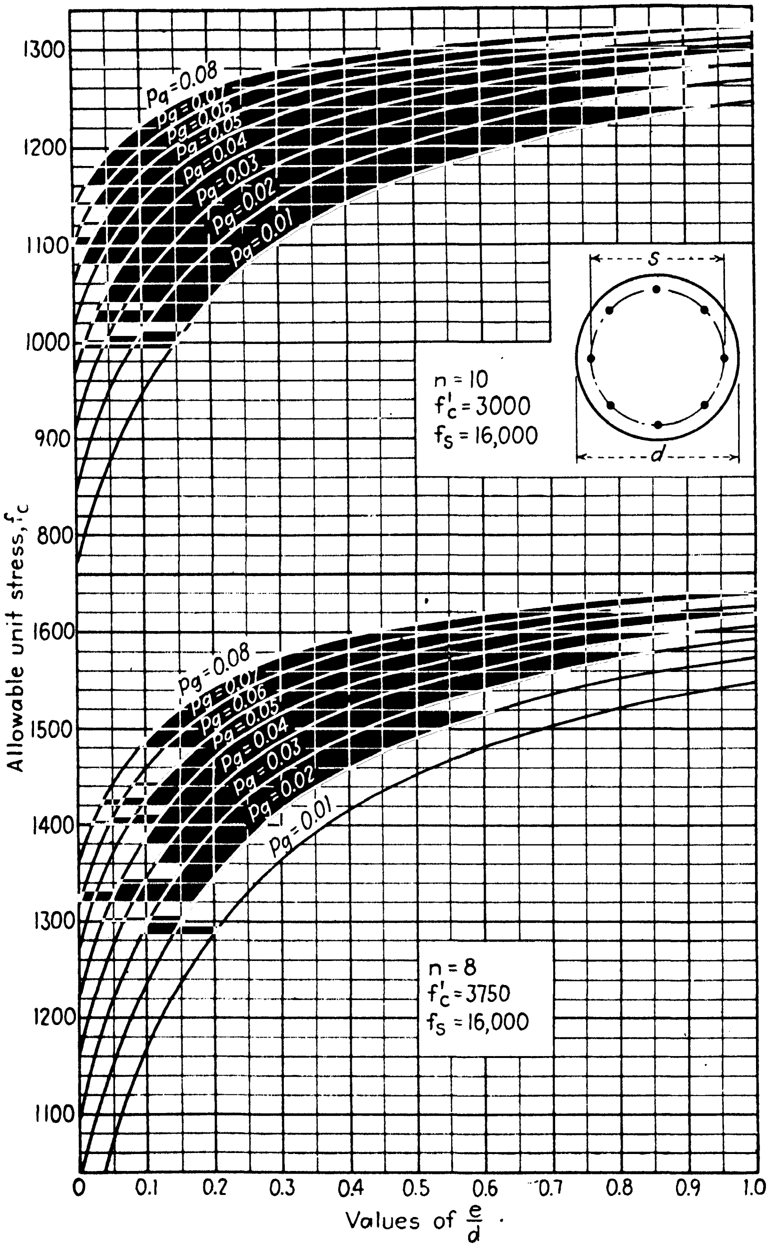


DIAGRAM 11.—ALLOWABLE UNIT STRESSES IN ECCENTRICALLY LOADED TIED COLUMNS

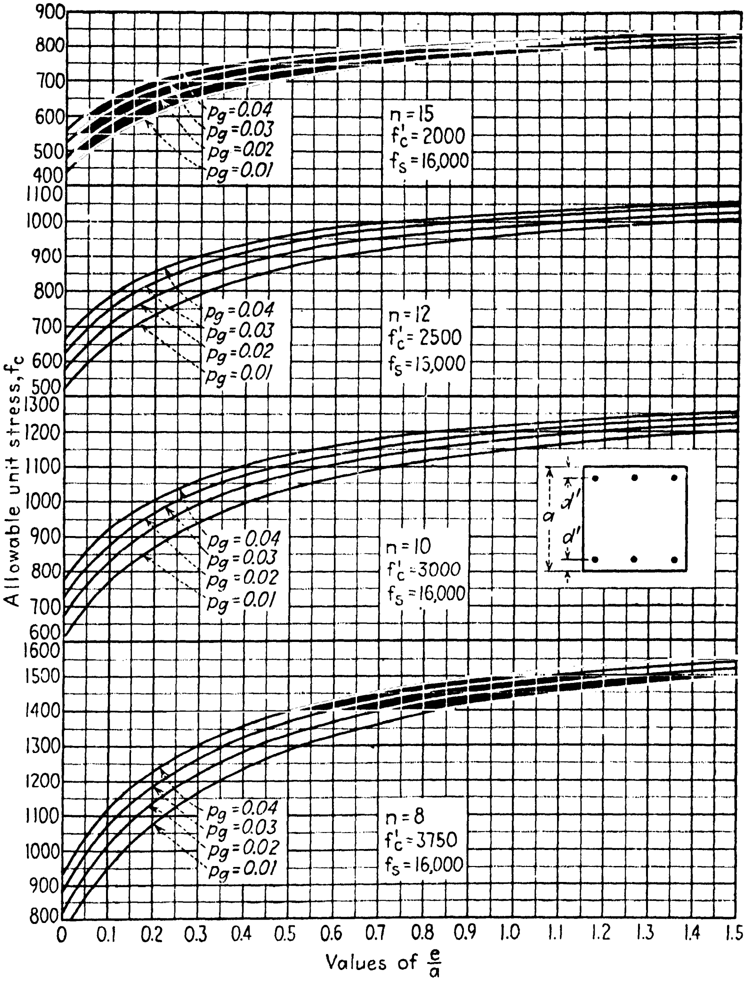


DIAGRAM 12.—ALLOWABLE UNIT STRESSES IN ECCENTRICALLY LOADED TIED COLUMNS

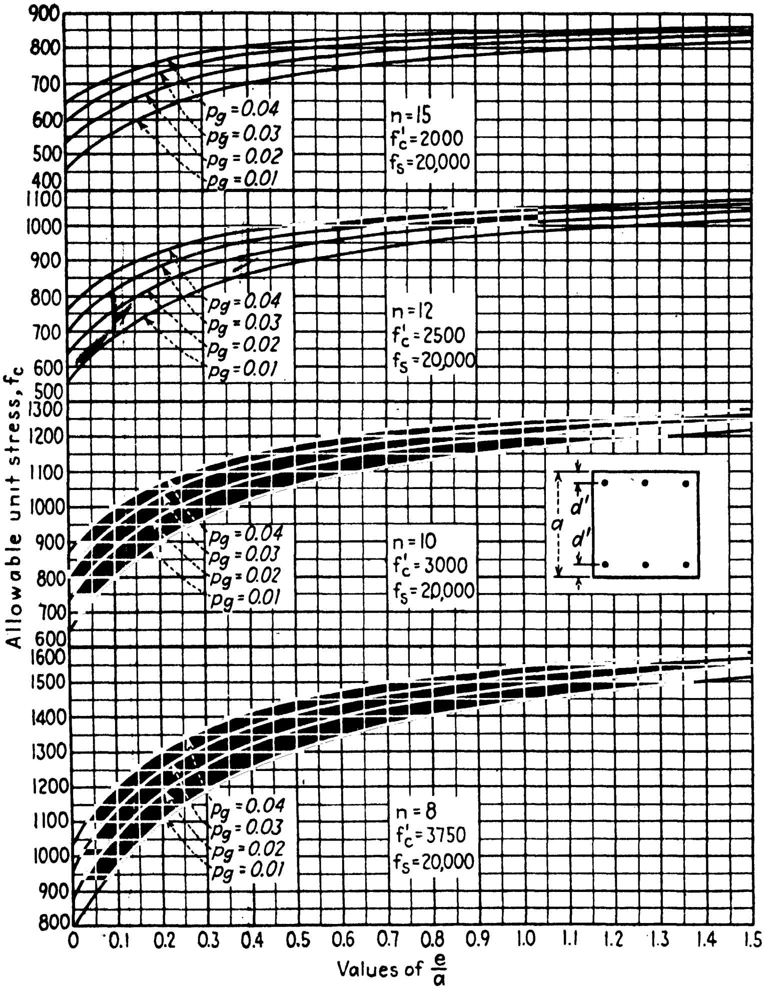


DIAGRAM 13.—BENDING AND AXIAL STRESS, CIRCULAR SECTIONS

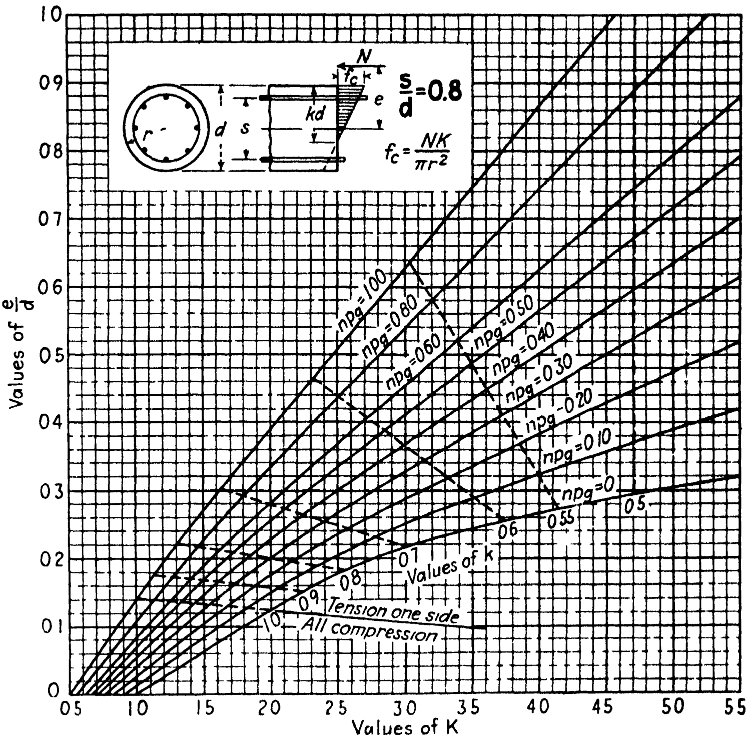


DIAGRAM 14.—BENDING AND AXIAL STRESS, CIRCULAR SECTIONS

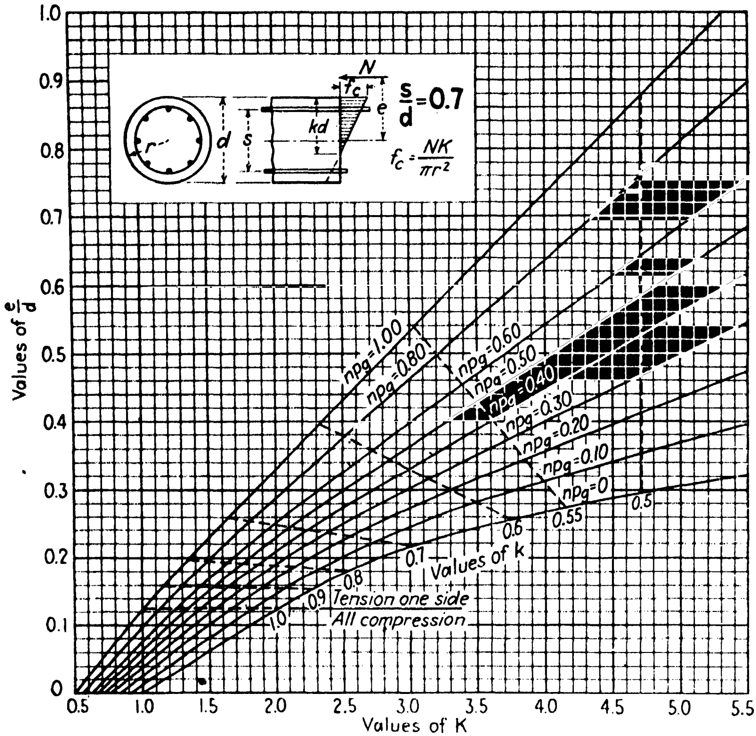


DIAGRAM 15.—BENDING AND AXIAL STRESS, CIRCULAR SECTIONS

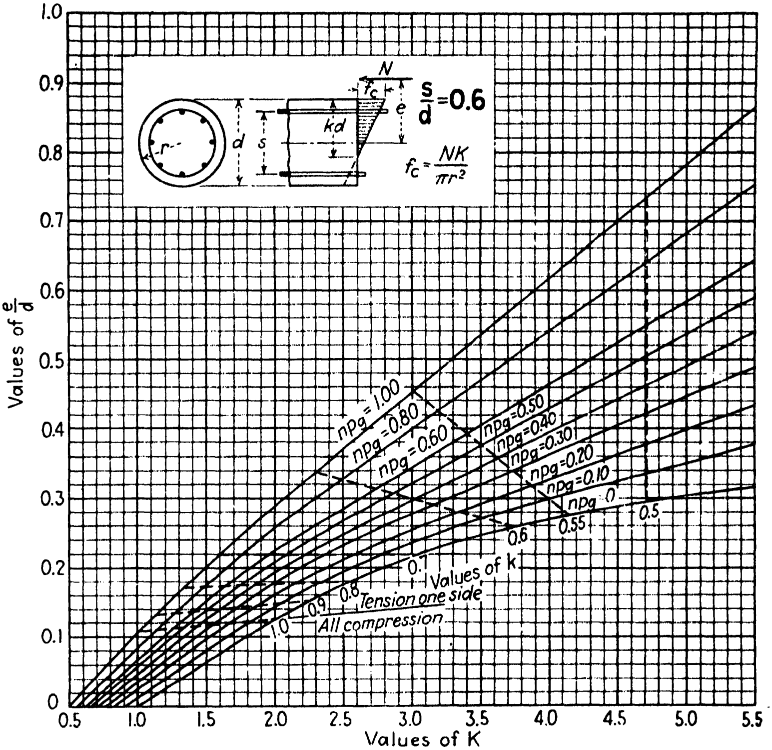


DIAGRAM 16.—BENDING AND AXIAL STRESS, RECTANGULAR SECTIONS

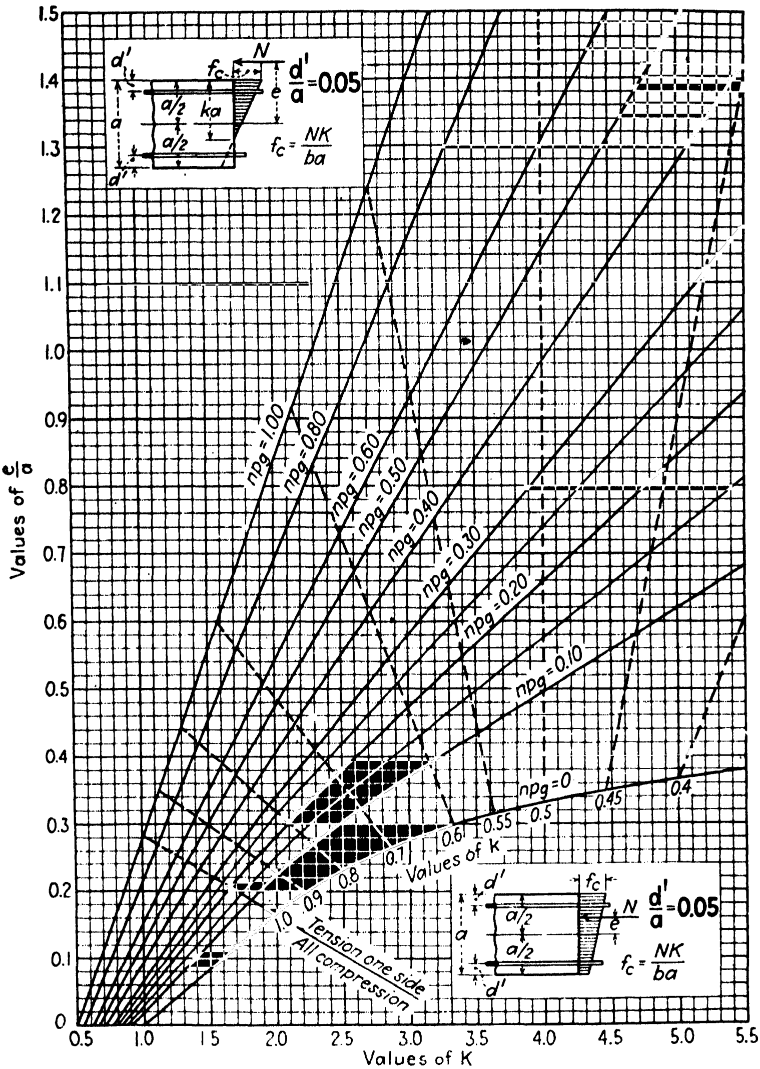


DIAGRAM 17.—BENDING AND AXIAL STRESS, RECTANGULAR SECTIONS

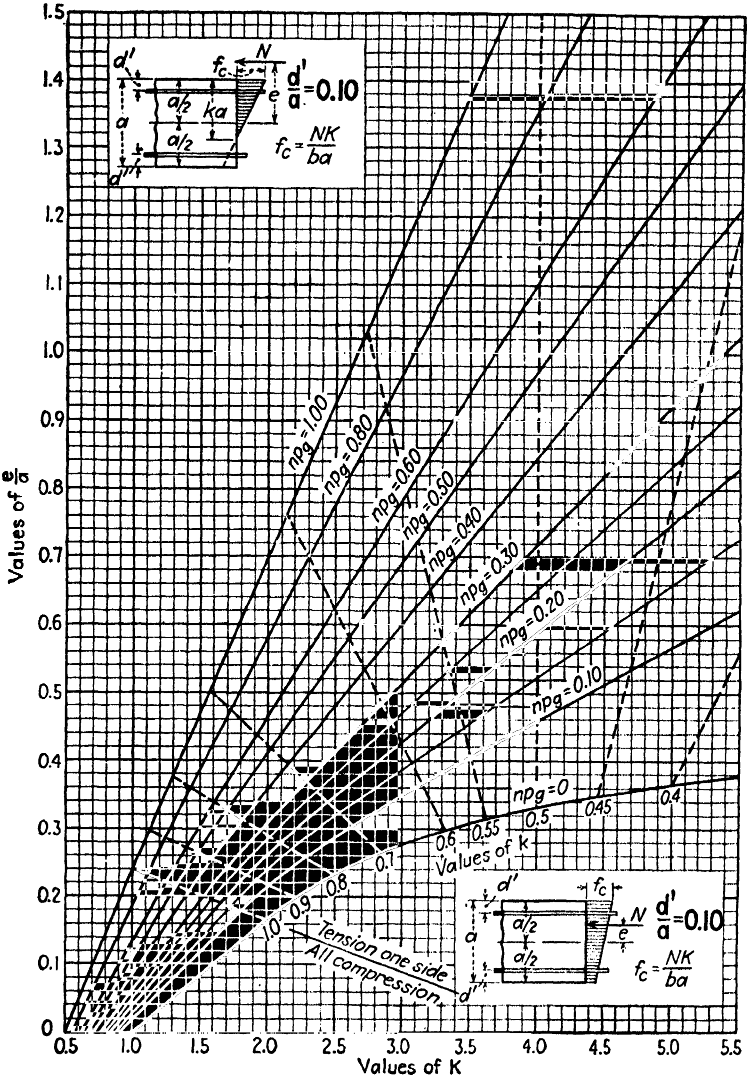


DIAGRAM 18.—BENDING AND AXIAL STRESS, RECTANGULAR SECTIONS

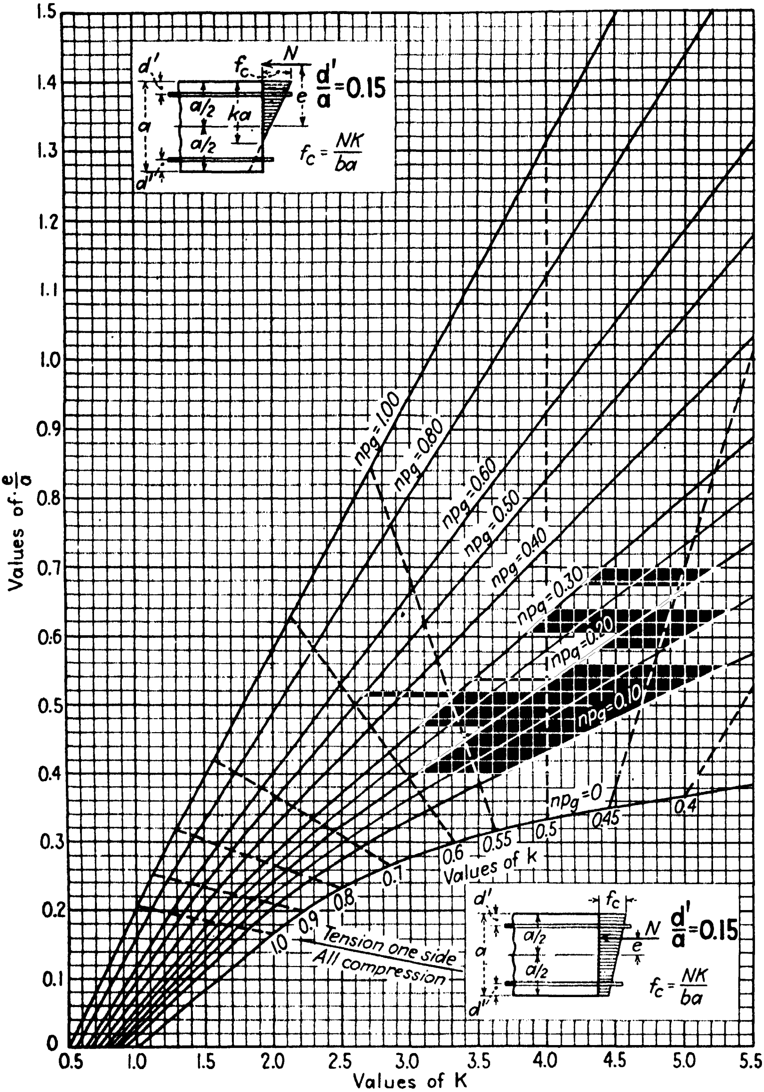


DIAGRAM 19.—BENDING AND AXIAL STRESS, RECTANGULAR SECTIONS

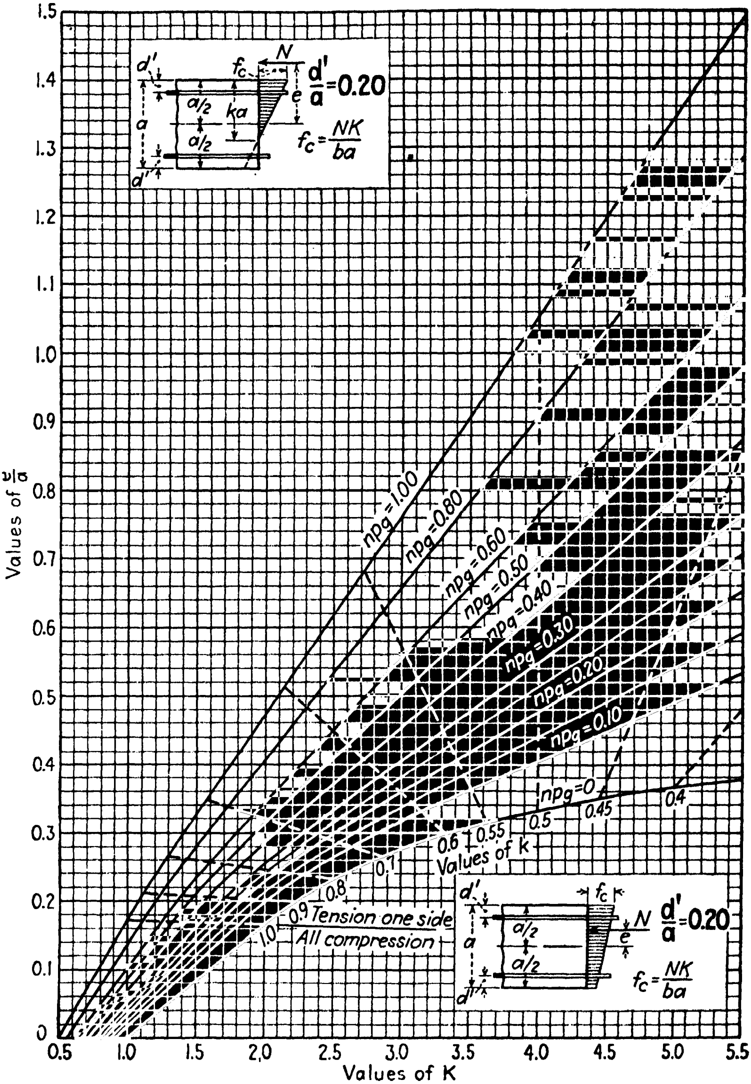


DIAGRAM 20.—BENDING AND AXIAL STRESS, RECTANGULAR SECTIONS

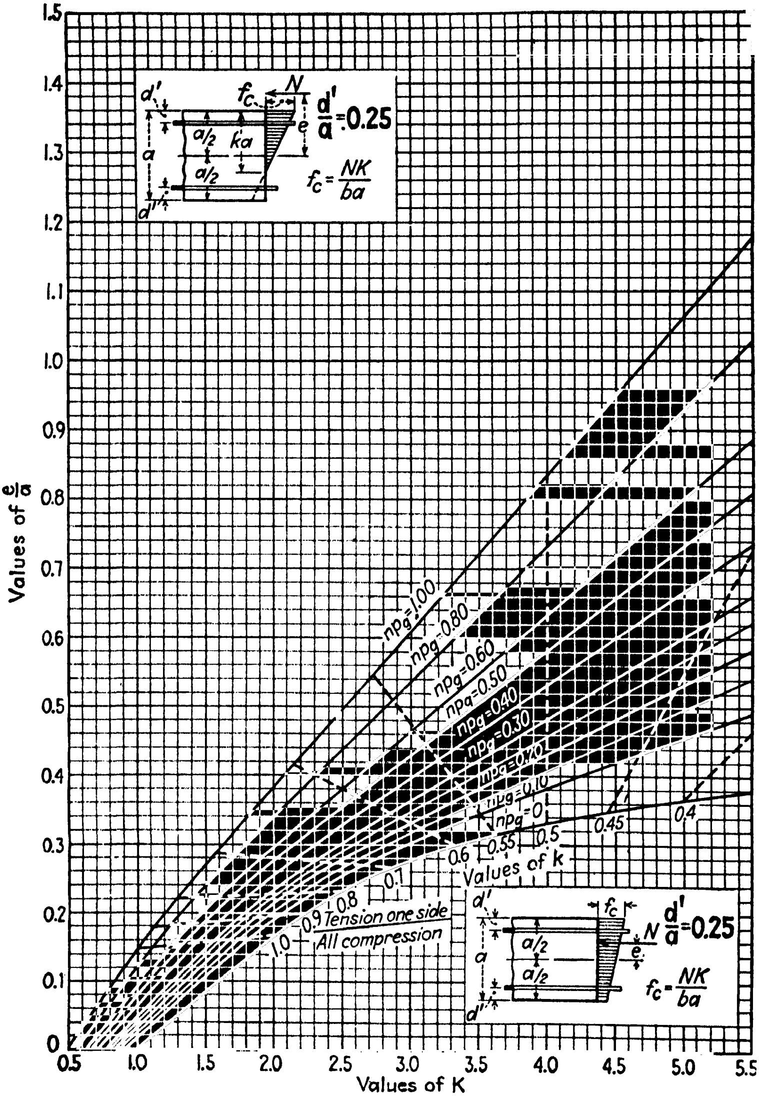
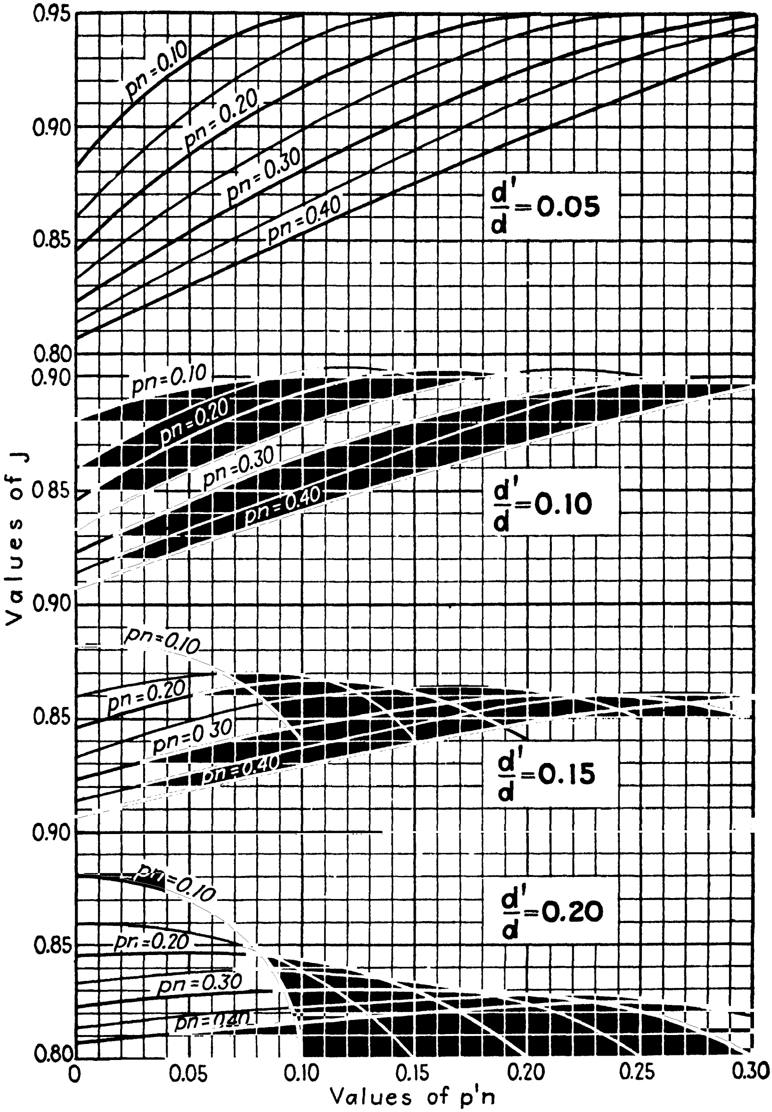


DIAGRAM 21.—RECTANGULAR BEAMS REINFORCED FOR COMPRESSION (BASED ON $f'_s = f_s$)



INDEX